Field Exam – Autonomy

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All students are expected to answer all parts of both questions during the exam. As a guide to using your time wisely, you should plan to spend approximately 2/3 of your preparation time on the first question and 1/3 on the second question. Similarly, plan to spend about 25 minutes of the exam time on the first question and 15 minutes on the second.

Question 1

In this question we ask you to design an algorithm to solve a class of constraint optimization problems, called Maximum Satisfiability Problems (MAX-SAT). MAX-SAT takes as input a propositional logic formula PHI in conjunctive normal form, containing propositions P. The MAX-SAT problem is to determine the maximum number of clauses in PHI that can be satisfied by an assignment of True or False to the propositions P in PHI. MAX-SAT is a generalization of the propositional satisfiability problem (SAT), which asks whether there exists a truth assignment that makes ALL clauses in PHI true.

As an example, consider the formula PHI-A, in conjunctive normal form:

$$(x_0 \vee x_1) \wedge (x_0 \vee \neg x_1) \wedge (\neg x_0 \vee x_1) \wedge (\neg x_0 \vee \neg x_1)$$

PHI-A contains two propositional variables, $\{x_0, x_1\}$, and four clauses. If PHI-A was given as a SAT instance, it would not be satisfiable: no matter which truth values were assigned to x_0 and x_1, at least one of PHI-A?s four clauses would be false. However, as a MAX-SAT problem, it is possible to assign truth values to x_0 and x_1 so that three out of four clauses are true; for example, the assignment $\{x_0 = \texttt{true}, x_1 = \texttt{true}\}$ makes the first three clauses true, and the fourth clause false. Hence the solution of MAX-SAT for PHI-A is three.

1. State MAX-SAT precisely as a constraint optimization problem.

2. Apply Max-Sat: Consider the following automobile configuration problem:

   A car manufacturer has released a new model. The model includes required and optional features, and a list of options for each feature. The customer must select one option for each required feature, while they may select either one or no option for each optional feature.

   The required features and their options are:

Engine:	E1, E2, or E3;
Gearbox:	G1, G2, or G3;
Control unit:	C1, C2, C3, C4, or C5;
Dashboard:	D1, D2, D3, or D4;

   The optional features and their options are:

Navigation system:	N1, N2, or N3;
Air conditioner:	AC1, AC2, or AC3;
Alarm system:	AS1, or AS2;
Radio:	R1, R2, R3, R4, or R5;

In addition, the following constraints restrict the ways in which options can be combined.

If N1 or N2 is selected, then D1 must be selected.
If N3 is selected then D2 or D3 must be selected.
If AC1 or AC3 is selected, then D1 or D2 must be selected.
If AC2 is selected, then C3 must be selected.
If AS1 is selected, then D2 or D3 must be selected.
If R1 or R2 or R5 is selected, then D1 or D4 must be selected.

A customer has selected E1, G2, C2 and D3 for the required features, and would like to know the maximum number of optional features she/he can select. Can you formulate this problem as a MAX-SAT problem? If yes, sketch the MAX-SAT problem. If no, explain why.

(Example taken from "Applications of MaxSAT in Automotive Configuration" by Rouven Walter and Christoph Zengler and Wolfgang Kuchlin).

3. Sketch an algorithm for finding the solution to MAX-SAT. Assume that memory is limited.
4. Sketch an anytime algorithm for solving MAX-SAT.

Question 2

Assume a 2D system $\mathbf{x} = (x, y)$. Imagine that you have a prior estimate for its state $\mathbf{x}_0 = [0\ 0]^T$ and covariance $Q_0 = \mathbf{I}_2$ and you receive a single measurement according to the measurement model

$$y = H\mathbf{x} + \mathbf{v}, \qquad \mathbf{v} \sim N(0, \sigma^2).$$

You want to compute a posterior estimate from this measurement.

1. Please give H and σ that would lead to a posterior covariance, Q_1, such that the variance in the x co-ordinate is decreased by half and the variance in the y co-ordinate is unchanged.
2. Consider the same H, but the limiting cases of $\sigma = 0$ and $\sigma = \infty$. How does the estimate behave?