Field Exam – Autonomy

All students are expected to answer all parts of both questions during the exam. As a guide to using your time wisely, you should plan to spend approximately 30 minutes of your preparation time on each of the two questions, and 20 minutes of the exam time on each of the two questions.

Question 1: Planning

Let us assume a vehicle is restricted to the 2D plane and knows its position (x, y). Its orientation is fixed at $\theta = 0$ and never changes. At each time t, the vehicle can take one of four control actions: `up`, `down` `right`, `left`. These actions do what you expect: for example, the `up` action moves the vehicle in the positive y direction by 1m. The dynamics of `down`, `left` and `right` are similarly defined.

Formally, if the vehicle chooses the `up` action, it has the following dynamics:

$$\begin{bmatrix} x \\ y \end{bmatrix}_t = \begin{bmatrix} x \\ y+1 \end{bmatrix}_{t-1} \tag{1}$$

$$\tag{2}$$

Assume the vehicle is attempting to generate a plan from a start position at $(0, 0)$ to a goal position $(10, 10)$. The vehicle considers itself to have arrived at the goal pose if it is closer than .5m to the goal position. There are two $2m \times 2m$ square obstacles centered at $(10, 5)$ and $(5, 10)$. If the vehicle collides with either of these obstacles, the vehicle breaks and it can no longer move (i.e., the mission is over).

Each action has cost -1. Your goal is to describe a planning approach that can generate a plan to reach the goal with minimum cost.

- Please identify what planning or optimization algorithm you will use, and justify your choice. Note that multiple algorithms are suitable.
- Please give a formal description of this planning problem that can be given to a planning algorithm.
- Please specify the objective function of the planning problem.
- Please describe your planning algorithm.
- Please identify whether or not your planning algorithm can guarantee that the vehicle will reach the goal with probability 1?
- Let us now add noise to the actions where the noise is distributed according to a Gaussian, so that if the vehicle chooses the `up` action, it has the following dynamics:

  $$\begin{bmatrix} x \\ y \end{bmatrix}_t = \begin{bmatrix} x \\ y+1 \end{bmatrix}_{t-1} + \begin{bmatrix} w_x \\ w_y \end{bmatrix} \tag{3}$$

  where

  $$w_x \sim \mathcal{N}(0, \sigma_x^2) \qquad \text{and} \qquad w_y \sim \mathcal{N}(0, \sigma_y^2) \tag{4}$$

  where σ_x^2 and σ_y^2 are known ahead of time (You may assume any values of these two variables you like. Plausible values are $\sigma_x^2 = \sigma_y^2 = 1$.)

  Please describe how your formal description of the problem, your objective and planning algorithm would change. Can your planning algorithm guarantee that the vehicle will reach the goal with probability 1?
- Please give an algorithm that can generate a plan that guarantees that the vehicle will reach the goal with some minimum probability.

Question 2: Inference

Let us assume a different vehicle is restricted to the 2D plane but does not know its position $\vec{x}$ in the plane or its orientation θ. The vehicle dynamics are not the same as the previous question, and are not known. The vehicle can estimate its position and orientation from the distances to a set of active beacons distributed in the plane, each with known position.

The vehicle has an initial position and orientation estimate given by a Gaussian with mean μ and covariance Σ. At each time step t, the vehicle receives some number n_t of range measurements, where the range r_i is to the i^th beacon. The range measurements are slightly noisy, with the noise of all measurements distributed according to a Gaussian with mean 0 and variance w. The vehicle knows which beacon generated each range, and the position of each beacon. You may assume that all beacons can be heard on every step, or you may assume that ranges are only received for beacons within some maximum distance r_{max}.

- Please draw the model that describes the probability distribution of $p(\vec{x}_t|r_{0:t})$, where $r_{0:t}$ is the history of received ranges from time 0 to time t.
- Please specify which variables are hidden or observed.
- Please specify each conditional probability or potential in the model, and how you derived the parameters from the problem statement. If the problem statement does not provide some necessary information to allow a fully-specified graphical model, please identify what is missing and provide an explanation for why you need it and how you might go about collecting that information.
- Please identify an algorithm for inferring the distribution $p(\vec{x}_t|r_{0:t})$.
- Please identify an algorithm for inferring the distribution $p(\vec{x}_t|r_{0:T}), t \in \{0, T\}$, that is, the position of the vehicle at any point in the past. Assume t is a discrete index, not a continuous value.
- Imagine that the range measurements no longer uniquely identify each beacon, that is, the vehicle cannot tell *which* beacon generated which range. How does your model change? Do your inference algorithms change?