Autonomy Field Exam

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1 Autonomy and Decision Making

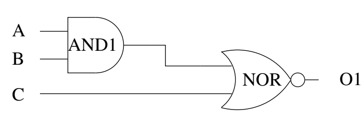
1. Consider a UAV flying in a desert environment. The problem with the desert is that it is hot — when the UAV flies fast, its speed controllers can overheat and crash the UAV. The UAV can fly with two different speeds: slow and fast. If it moves fast, it gets a reward of 10; if it moves slowly, it gets a reward of 4. We can model as an MDP with three states: cool, warm, and crashed. The transitions are shown in below. Assume that the discount factor is 0.9 and also assume that when we reach the state crashed, we remain there without getting any reward.

s	a	s'	$p(s' \mid s, a)$
cool	slow	cool	1
cool	fast	cool	.25
cool	fast	warm	.75
warm	slow	cool	.5
warm	slow	warm	.5
warm	fast	cool	.875
warm	fast	crashed	.125

   (a) Consider the fixed policy of always moving slowly. What is the value of $V(\text{cool}, \text{slow})$ under this policy?

   (b) What is the optimal policy for each state?

2. Consider the following circuit:



The inputs, A, B, C are all $True$, and the output $O1$ is also $True$. Let us determine if this is a fault condition, and which components may have failed.

   (a) Please provide a propositional logic theory of the circuit in terms of A, B, C and $O1$ in CNF form.

   (b) We know that the inputs A, B, C are all $True$, as is the output $O1$. Please use unit propagation to determine if this is a failure condition. Show the result of each step of unit propagation. If inconsistent, show the steps that lead to an inconsistency. If consistent, just show the first five steps of propagation. (You do not need to provide a diagnosis, only a statement of whether this is a failure.)

2 Estimation

3. Given the following discrete time system:

$$\begin{bmatrix} x_{1,k+1} \\ x_{2,k+1} \end{bmatrix} = \begin{bmatrix} 1 & a \\ a & 1 \end{bmatrix} \begin{bmatrix} x_{1,k} \\ x_{2,k} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_k$$

$$y_k = \begin{bmatrix} c_1 & c_2 \end{bmatrix} \begin{bmatrix} x_{1,k} \\ x_{2,k} \end{bmatrix} + [0] u_k$$

   (a) Please state the conditions under which the system is observable.

   (b) Please give specific values for a, c_1 and c_2 that make the system unobservable.

4. Consider a vehicle with a known, discrete time, linear dynamics model, such that

$$\mathbf{x}_{k+1} = A\mathbf{x}_k + B\mathbf{u}_k + \mathbf{w}_k$$

   where the state of the vehicle at time k is $\mathbf{x}_k$, the control is $\mathbf{u}_k$ and the state is perturbed by some random noise $\mathbf{w}_k \sim N(0, W)$. The vehicle receives measurements according to a known, discrete time, linear measurement model

$$\mathbf{z}_k = C\mathbf{x}_k + \mathbf{v}_k$$

   The vehicle follows some trajectory of length T, such that it has taken $vecu_{0:T}$ controls and received $\mathbf{z}_{0:T}$ measurements.

   The vehicle's camera took an image at some time t during that trajectory. Once the vehicle has completed its trajectory, the operator wishes to know the most likely position of the vehicle (and by extension its camera) at time t.

   (a) Please draw the graphical model for this problem.

   (b) Is $\max p(\mathbf{x}_t | \mathbf{u}_{0:T}, \mathbf{z}_{0:T})$ the same as $\max p(\mathbf{x}_t | \mathbf{u}_{0:t}, \mathbf{z}_{0:t})$? Why or why not?

   (c) Please sketch an algorithm for solving for $p(\mathbf{x}_t | \mathbf{u}_{0:T}, \mathbf{z}_{0:T})$. (You do not need to give the exact equations for solving this – the structure of the algorithm is sufficient.)