Field Exam – Controls

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All students are expected to answer all parts of both questions during the exam. As a guide to using your time wisely, you should plan to spend approximately 2/3 of your preparation time on the first question and 1/3 on the second question. Similarly, plan to spend about 25 minutes of the exam time on the first question and 15 minutes on the second.

Question 1

Consider a dynamical system of the following form:

$$\dot{x}(t) = a\,x(t) + u(t),$$

where $a \in \mathbb{R}$. We would like to design a feedback controller of the following form:

$$u(t) = kx(t),$$

where $k \in \mathbb{R}$, such that the following cost function is minimized:

$$J = \int_0^\infty e^{-\alpha t} \left(x(t)^2 + \rho u(t)^2 \right) dt$$

where α and ρ are positive scalars.

Please find optimal value of k in terms of a. *[Hint: First, find a solution to the differential equation and substitute your solution to the cost function.]*

Question 2

Assume a 2D system $\mathbf{x} = (x, y)$. Imagine that you have a prior estimate for its state $\mathbf{x}_0 = [0\ 0]^T$ and covariance $Q_0 = \mathbf{I}_2$ and you receive a single measurement according to the measurement model

$$y = H\mathbf{x} + \mathbf{v}, \qquad \mathbf{v} \sim N(0, \sigma^2).$$

You want to compute a posterior estimate from this measurement.

1. Please give H and σ that would lead to a posterior covariance, Q_1, such that the variance in the x co-ordinate is decreased by half and the variance in the y co-ordinate is unchanged.
2. Consider the same H, but the limiting cases of $\sigma = 0$ and $\sigma = \infty$. How does the estimate behave?