

**Communications and Networks Field Exam**  
**January 2011**

A) Markov Chains

The M/M/2 queueing system is a queueing system with two servers. Customers arrive to the system at a Poisson rate of  $\lambda$  per second and are served at a rate of  $\mu$  packets per second.

- i) Show the Markov chain for this system with the flow rates between the states
- ii) Write the local balance equations, and obtain an expression for the steady state probability of being in the different states
- iii) What is the probability that an arriving customer has to wait in line before being served?
- iv) Consider now a bank with multiple tellers. Discuss the tradeoff between having a single queue for all tellers and different queues one for each teller. This discussion can be qualitative. You don't need to obtain analytical results to justify your answers.

B) Maximum capacity path algorithms

Suppose that the weights on the links in a network represent the link's capacity. We would like to devise an algorithm that finds paths with greatest end-to-end capacity where the capacity of the path is limited to the minimum capacity link along the path.

How would you change the Bellman-Ford algorithm so that it can find the maximum capacity path from the source node to all other nodes? Clearly describe your new version of the algorithm including the initialization stage and the update equations.

Bellman-Ford shortest path algorithm:

$d_{ij} = \infty$  if  $(i,j)$  is not an arc;  $d_{ij} = 0$ .

Let  $D_i(h)$  be the shortest distance from 1 to  $i$  using at most  $h$  arcs.  $D_i(1) = d_{1i}$  ;  
 $i \neq 1$  ;  $D_1(1) = 0$

For  $h=1$  to  $n-1$ :  $D_i(h+1) = \min_j [D_j(h) + d_{ji}]$  ; for all  $i \neq 1$ .