Communications and Networks Qualifier Question

Please answer one of the following two questions. The first question is in the area of networks, and the second is at the intersection of communications and probability/statistics. You are free to select either question. Good luck!

Question 1: Networks

You are flying out of a small airport terminal with a single security checkpoint. While you're waiting in line at security, you decide to use your queuing knowledge to estimate how many other passengers you will see and how long you will have to wait.

Assume that on average, 330 passengers arrive to the checkpoint each hour and that these arrivals are Poisson distributed. Each passenger must pass through two (possibly three) stages:

Stage 1: Passengers first wait in a single line for ticket and ID check. The check time is exponentially distributed with mean 10 seconds, and everyone passes the check.

After the ticket and ID check, passengers are independently sent to one of the two X-ray machines, each with equal probability.

Stage 2: Passengers go through the X-ray machines, with time exponentially distributed with mean 12 seconds. After passing through the machine, exactly one of the following three things happens:

- With probability 0.1, the passenger fails the X-ray screen and is returned to the end of the same X-ray line to repeat the screen.
- With probability 0.1, the passenger is randomly selected for additional security screening and proceeds to stage 3 below.
- All other passengers leave the security checkpoint after a successful X-ray screen.

Stage 3: Passengers selected for additional security screening go through a single screening station that is shared by both X-ray machines. Screening time is exponentially distributed with mean 90 seconds. After this additional screen, all passengers are released from the security checkpoint.

Questions:

a) Draw the security checkpoint system as an open Jackson network, labeling external arrival rates and transition probabilities.

b) What is the average number in queue at each of the stations?

c) What is the average delay in the system?

d) What is the probability that a passenger arrives to an empty system?

Question 2: Communications, probability and statistics

Let $S \in \{-3, -1, +1, +3\}$ denote a transmitted symbol. Suppose that the signals are received through N independent channels. The received signals are denoted by R_1, R_2, $\cdots$, R_N with

$$R_k = S + N_k$$

where N_k's are independent and identically distributed Gaussian random variables, each with mean 0 and known variance σ^2. For part (a) and (b), assume that the symbol S has equal a priori probability.

(a) Design an optimum receiver that minimize the probability of error. [*Hint*: The probability of error is given by $\mathbb{P}\{\hat{S} \neq S\}$, where $\hat{S}$ is a symbol determined by the receiver based on received signals.]

(b) Find the probability of error for the receiver designed in part (a).

(c) Redo part (a) and (b) assuming that the symbol S has the following a prior probability:

$$\mathbb{P}\{S = -1\} = \mathbb{P}\{S = +1\} = 1/3$$
$$\mathbb{P}\{S = -3\} = \mathbb{P}\{S = +3\} = 1/6.$$