Communications and Networks

Part 1)

Let $X_1, X_2, ..., X_N$ be independent identically distributed (i.i.d.) random variables (r.v.'s) with common probability density function (p.d.f.) $f_X(x)$. Let $X_{[i]}$ be the ordered X_i , i.e., $X_{[1]} > X_{[2]} > ... > X_{[N]}$. In particular:

$$X_{[1]} = \max\{X_1, X_2, ..., X_N\}$$

$$X_{[N]} = \min\{X_1, X_2, ..., X_N\}$$

- Find the p.d.f. of $X_{[1]}$.
- Find the p.d.f. of $X_{[N]}$.
- Let $W = \sum_{i=1}^{L} X_{[i]}$ with $L \leq N$, where X_i are i.i.d. exponential r.v.'s with common mean Γ. Find the mean and the variance of W.

Part 2)

Consider a slotted aloha system with perfect capture. That is, if more than one packet is transmitted in a slot the receiver receives one of them correctly. The receiver provides immediate feedback to each of the transmitters about the success or failure of their transmission; and unsuccessful packets are transmitted later.

- Give a convincing argument why expected delay is minimized if all waiting packets attempt transmission in each slot.
- Find the expected delay assuming Poisson arrivals with overall rate λ.