

## Communications and Networks

Part 1)

Let  $X_1, X_2, \dots, X_N$  be independent identically distributed (i.i.d.) random variables (r.v.'s) with common probability density function (p.d.f.)  $f_X(x)$ . Let  $X_{[i]}$  be the ordered  $X_i$ , i.e.,  $X_{[1]} > X_{[2]} > \dots > X_{[N]}$ . In particular:

$$X_{[1]} = \max \{X_1, X_2, \dots, X_N\}$$

$$X_{[N]} = \min \{X_1, X_2, \dots, X_N\}$$

- Find the p.d.f. of  $X_{[1]}$ .
- Find the p.d.f. of  $X_{[N]}$ .
- Let  $W = \sum_{i=1}^L X_{[i]}$  with  $L \leq N$ , where  $X_i$  are i.i.d. exponential r.v.'s with common mean  $\Gamma$ . Find the mean and the variance of  $W$ .

Part 2)

Consider a slotted aloha system with perfect capture. That is, if more than one packet is transmitted in a slot the receiver receives one of them correctly. The receiver provides immediate feedback to each of the transmitters about the success or failure of their transmission; and unsuccessful packets are transmitted later.

- Give a convincing argument why expected delay is minimized if all waiting packets attempt transmission in each slot.
- Find the expected delay assuming Poisson arrivals with overall rate  $\lambda$ .