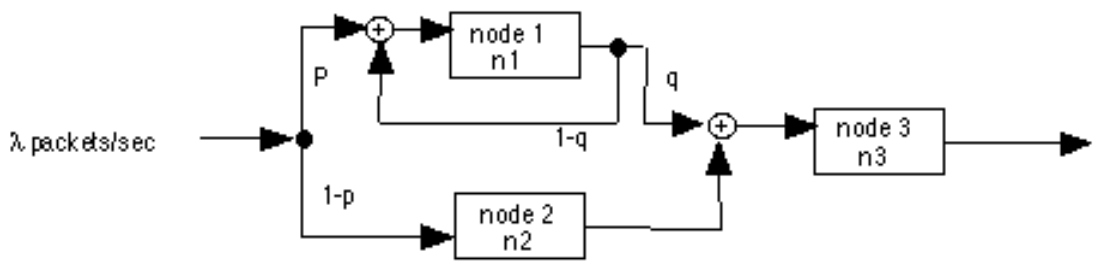
Communications and Networks Field Exam – January 2010

Part 1: Data Networks

A) Consider the network shown in the figure below. Packets enter the network at a rate of $\lambda$ packets per second. Each entering packet follows the upper route with probability p, and the lower route with probability 1-p. At the output of node 1, a packet proceeds to node 3 with probability q or with probability 1-q loops back to node 1. The system is in the steady state and the average number of packets at each node is n1, n2, n3, as shown. Assume zero propagation delays between nodes. Expressed in terms of n1, n2, and n3, what is the average amount of time that a packet spends at each node (t1, t2, t3) and in the entire system (ts)?




B) Now assume that external arrivals are Poisson and that the service times of a packet at each node are independent, even if the packet has previously visited the same node, and service times are exponentially distributed with rates ,$\mu_1$, $\mu_2$, and $\mu_3$, respectively.

i) Under what conditions are each of the queues stable?

ii) What is the steady-state, joint occupancy distribution (distribution of number of packets at each node) for nodes 1, 2 and 3?

P(n1,n2,n3) =

iii) Calculate the steady-state average occupancies (transmitted plus queue) n1, n2, n3 and the average amount of time that a packet spends in the system, ts.

## Part 2: Probability and statistics

A random variable $X$ is generated from one of the following two hypotheses:

$$\begin{aligned} H_0 &: X = \alpha\sqrt{E_{s0}} + N \\ H_1 &: X = -\alpha\sqrt{E_{s_1}} + N\,, \end{aligned}$$

where $\alpha > 0$ and $N$ is Normal $\mathcal{N}(0, N_0)$ random variable, and $E_{s_0}$, $E_{s_1}$, and $N_0$ are known constants. Let $P_i$ denote the probability that $X$ is generated from hypothesis $H_i$, for $i \in \{0, 1\}$.

1. Assume that $\alpha$ is a known constant. Obtain a likelihood ratio test, which decides whether $X$ is generated from hypothesis $H_0$ or $H_1$. Select the threshold that minimizes the probability of error.

2. Assume that $\alpha$ is a known constant. Obtain the probability of error for the decision rule in part 1.

3. Assume that $\alpha$ is a known constant and consider a special case when $P_0 = P_1 = 1/2$ and $E_{s0} = E_{s_1}$.

   (a) Repeat part 1.

   (b) Repeat part 2.