Communications and Networks: Data Networks Option

Probability: Sum of Independent Gaussian Variables

Let $Z = X + Y$ where X and Y are two independent Gaussian r.v.'s with means μ_X and μ_Y, and variances σ_X^2 and σ_Y^2, respectively, i.e., $X \sim \mathcal{N}(\mu_X, \sigma_X^2)$ and $Y \sim \mathcal{N}(\mu_Y, \sigma_Y^2)$.

1. Determine the distribution of Z.
2. Determine the mean μ_Z and the variance σ_Z^2 of Z.

If you want to use characteristic functions, then the characteristic function of a Gaussian r.v. θ with mean μ_θ and variance σ_θ^2 is $\phi_\theta(w) = \mathbb{E}\left\{e^{jw\theta}\right\} = \exp\left(+jw\mu_\theta - \frac{1}{2}w^2\sigma_\theta^2\right)$.

Data Networks Option

Consider a simple communication system with two links. Packets arrive for transmission through link i according to an arrival process $\{A_i(t)\}$, $i = 1, 2$. The arrival processes are assumed IID over time slots, mutually independent, with arrival rates $\mathbb{E}[A_i(0)] = \lambda_i$, and finite second moments. Packets are buffered in dedicated queues until they are transmitted; let $Q_i(t)$ be the number of packets that await to be transmitted through link i, at time slot t. At each time slot a scheduler takes one of the following four actions: (i) serves only the queue of link 1 at rate 3 packets per slot (schedule π_1); (ii) serves only the queue of link 2 at rate 3 packets per slot (schedule π_2); (iii) serves both queues at rate 2 packets per slot each (schedule π_3); (iv) does not serve any queue (schedule π_4). Thus, the service rate vector $R(t) = (R_1(t), R_2(t))$ belongs to the set $\mathcal{R} = \{(3,0), (0,3), (2,2), (0,0)\}$, at each time slot.

(a) Derive the stability region of the system, i.e., the necessary conditions that the arrival rates have to satisfy for the system to be stable;

(b) Write the difference equations that govern the evolution of the system, and provide an upper bound on the drift of the Lyapunov function $L(t) = \sum_{i=1,2} Q_i^2(t)$ under these dynamics. The drift of $L(\cdot)$ is defined as $\Delta(t) = \mathbb{E}[L(t+1) - L(t) \mid Q(t)]$;

(c) Propose a scheduling policy that stabilizes the system whenever the arrival rates are in the interior of the stability region.