Digital Communication

Let $X \in \{-1, +1\}$ denote the transmitted symbol with transmission probability

$$\mathbb{P}\{X = x\} = \begin{cases} p, & x = +1 \\ 1 - p, & x = -1. \end{cases}$$

Suppose that the signals are received through n independent channels. The received signals are denoted by $Y_1, Y_2, \ldots, Y_n$ with

$$Y_k = X + N_k$$

where N_k is Gaussian random variable with mean 0 and known variance σ_k^2.

(a) Design a hypothesis test to determine X based on the sample of the signals $Y_1, Y_2, \ldots, Y_n$, denoted by $y_1, y_2, \ldots, y_n$.

(b) Assume that the likelihood ratio test is used in part (a). Find the threshold that minimizes the probability of error.

(c) Find the probability of error for the threshold derived in part (b).

(d) Assume $p = 1/2$ and $\sigma_k = \sigma$ for $k = 1, 2, \ldots, n$. Repeat part (b) and (c).

Probability

Consider n independent Gamma random variables $X_1, X_2, \ldots, X_n$ with the probability density function of X_k given by

$$f_{X_k}(x) = \begin{cases} \frac{\beta^{\alpha_k}}{\Gamma(\alpha_k)} x^{\alpha_k - 1} e^{-\beta x}, & x > 0 \\ 0, & \text{otherwise.} \end{cases}$$

Show that $S = \sum_{i=k}^{n} X_k$ is a Gamma random variable, and find the probability density function of S.

[*Hint*: The characteristic function for X_k is

$$\phi_{X_k}(t) = \left(1 - \frac{it}{\beta}\right)^{-\alpha_k}.$$

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