Field Exam — Controls

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Problem 1

Consider a spacecraft composed of a rigid core and a flexible appendage (e.g., a solar panel), and its attitude dynamics about one of its principal axes of inertia. It is desired to design control laws to rapidly change the orientation of the spacecraft by a given angle Δ. Initial and final conditions are at rest.

- Describe briefly your selection of a control architecture, including options for actuators and sensors.
- Describe qualitatively the nature of the control law for a minimum-time maneuver, and explain how you would compute the necessary parameters.
- How would your solution change if it was desired to minimize a linear combination of the maneuver completion time, and of the total amount of propellant consumed?

Problem 2

Consider two single-input, single-output (SISO) linear time-invariant (LTI) systems described by the following state-space models:

$$G_1 : \frac{d}{dt}x_1(t) = A_1 x_1(t) + B_1 u_1(t), \qquad y_1(t) = C_1 x_1(t) + D_1 u_1(t),$$

$$G_2 : \frac{d}{dt}x_2(t) = A_2 x_1(t) + B_2 u_2(t), \qquad y_2(t) = C_2 x_2(t) + D_2 u_2(t),$$

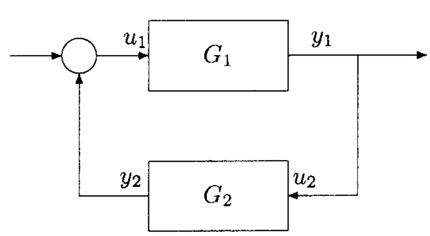
with

$$A_1 = \begin{bmatrix} -2 & 0 \\ 1 & 0 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad C_1 = \begin{bmatrix} -3 & -2 \end{bmatrix}, \quad D_1 = 1$$

and

$$A_2 = \begin{bmatrix} 0 & 4 \\ 1 & 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 0 & 2 \end{bmatrix}, \quad D_2 = 0.$$

Connect the two systems in feedback as shown in the figure below.



Discuss the stability of the feedback system. In particular, is there a bounded signal or initial condition that would cause the states of the system (as defined in the realizations above) to diverge? If not, prove so. If yes, find an example.