

**DEPARTMENT OF AERONAUTICS AND ASTRONAUTICS**  
**Controls Field Exam, January 25–26, 2010**

Consider the linear system with dynamics given by

$$\dot{x} = Ax + Bu$$

The linear quadratic minimum-time (LQMT) problem is to find the control and final time that minimizes the cost

$$J = \frac{1}{2}x^T(T)Sx(T) + \int_0^T (\rho + \frac{1}{2}x^T Qx + \frac{1}{2}ru^2(t)) dt$$

1. Find the necessary conditions for an optimal solution.
2. What are the conditions that determine the final time,  $T$ ?
3. Suggest an algorithm for efficiently determining the optimal solution.