Field Exam — Controls

Aeronautics and Astronautics Department,
Massachusetts Institute of Technology

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Part 1

An uncertain SISO plant P is known to be linear, time-invariant, and causal. A state-space model of the plant P can be written as

$$\begin{aligned} \dot{x} &= \begin{bmatrix} 0 & 1 \\ -100 & \alpha \end{bmatrix} x + \begin{bmatrix} 0 \\ \beta \end{bmatrix} u, \\ y &= [1 \quad 1]\, x, \end{aligned}$$

where the uncertain parameters α and β are known to satisfy

$$-10 \leq \alpha \leq 10, \qquad 1 \leq \beta \leq 2.$$

In order to determine more accurately α and β, it is desired to conduct a test campaign to collect experimental data. It is your job to plan and execute this test campaign. Discuss your plans: what experiments would you perform? What will drive your selection of hardware for supporting the experiments?

Note that the internal state of the system is not available, and you cannot modify the system (internally), e.g., by installing additional sensors, actuators, etc.

Part 2

Assume now that $\alpha > 0$ and β are known, e.g., as a result of successful data collection and processing from Part 1. It is desired to design a controller that would bring the system from initial conditions at rest to the state $x_f = 1/\sqrt{2}\,[1 \quad 1]$ within 1 second, minimizing the control effort

$$\int_0^1 u^2 \, dt.$$

What is the optimal control law? How would you solve this problem in the case in which the system is unstable?

Among all final states such that $\|x_f\|_2^2 = 1$, which ones are the most expensive to reach in 1 second? Which ones are the least expensive?

Part 3

Now consider the bounded-input case, i.e., $|u| < 1$. How would your answers in Part 1 and 2 change?

If it is desired to reach the state $x_f = 1/\sqrt{2}\,[1 \quad 1]$ in minimum time, what is the structure of the optimal control law?