Field Exam – Controls

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Question 1

We would like to design a cruise control system for a car. The cruise control system will be designed to regulate the speed of the car around a speed, which we denote by $\bar{v}$. We are able measure the velocity of the car, which we denote by $v(t)$. The acceleration of the car, denoted by $a(t)$, evolves as follows:

$$\dot{a}(t) = -(v(t) - \bar{v})^2 - a(t)^2 + u(t),$$

where $u(t)$ is the fuel input to the engine.

Please answer the following questions:

1. Develop a state space model for the car. Linearize the model (around an equilibrium point you find reasonable), and describe the stability characteristics of the linearized model. Is the system stable? What can you say about its dynamic behavior?

2. Design a control system using eigenvalue placement techniques, so that the settling time of the closed-loop system is no more than a few seconds. Note that you can measure the speed of the car, but not the acceleration.

3. How would you design a control system using optimal control techniques such that: you can balance between state error and control effort; you can make sure that the resulting controller response time is not faster than the controller you designed in the previous step?

   (In this step, please assume you have access to full state information, i.e., you can measure speed and acceleration both. Please do not attempt to solve Riccati equations numerically; you only need to describe your approach.)

Question 2

1. Given the following discrete time system:

$$\begin{bmatrix} x_{1,k+1} \\ x_{2,k+1} \end{bmatrix} = \begin{bmatrix} 1 & a \\ a & 1 \end{bmatrix} \begin{bmatrix} x_{1,k} \\ x_{2,k} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_k$$

$$y_k = \begin{bmatrix} c_1 & c_2 \end{bmatrix} \begin{bmatrix} x_{1,k} \\ x_{2,k} \end{bmatrix} + [0] u_k$$

   (a) Please state the conditions under which the system is observable.

   (b) Please give specific values for a, c_1 and c_2 that make the system unobservable.

2. Consider a vehicle with a known, discrete time, linear dynamics model, such that

$$\vec{x}_{k+1} = A\vec{x}_k + B\vec{u}_k + \vec{w}_k$$

   where the state of the vehicle at time k is $\vec{x}_k$, the control is $\vec{u}_k$ and the state is perturbed by some random noise $\vec{w}_k \sim N(0, W)$. The vehicle receives measurements according to a known, discrete time, linear measurement model

$$\vec{z}_k = C\vec{x}_k + \vec{v}_k$$

   The vehicle follows some trajectory of length T, such that it has taken $vecu_{0:T}$ controls and received $\vec{z}_{0:T}$ measurements.

   The vehicle's camera took an image at some time t during that trajectory. Once the vehicle has completed its trajectory, the operator wishes to know the most likely position of the vehicle (and by extension its camera) at time t.

   (a) Please draw the graphical model for this problem.

   (b) Is $\max p(\vec{x}_t | \vec{u}_{0:T}, \vec{z}_{0:T})$ the same as $\max p(\vec{x}_t | \vec{u}_{0:t}, \vec{z}_{0:t})$? Why or why not?

   (c) Please sketch an algorithm for solving for $p(\vec{x}_t | \vec{u}_{0:T}, \vec{z}_{0:T})$. (You do not need to give the exact equations for solving this – the structure of the algorithm is sufficient.)