Field Exam – Controls

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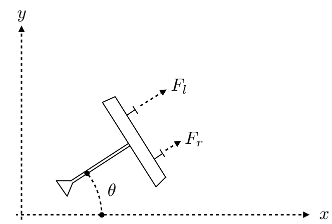
Question 1

We are given a seaplane that can land on water. The plane can navigate while on water using its two engines. See the figure on the right. We are interested in designing trajectory tracking controllers for when the seaplane is on the water. In the figure, x and y denote the Cartesian coordinates of the center of mass of the seaplane, θ denotes the angle that the plane makes with the x-axis, and F_l and F_r denote the forces generated by the left and right propellors, respectively. Please answer the following questions, devoting most of your time to Parts 2 and 3.



1. First, derive a state space model for this system. To simplify this modeling exercise, we assume that: 1. the rotation rate of the seaplane is proportional to the difference between the two propellor forces, *i.e.*, $F_r - F_l$; 2. the longitudinal speed of the aircraft, denoted by v, is proportional to the sum of the two forces, *i.e.*, $F_r + F_l$. You can assume that the propellors can turn in either direction to generate positive and negative thrust. The model you derive will determine the evolution of the state variables x, y, θ, v, assuming the inputs are F_r and F_l. *[Hint: Notice that this aircraft behaves much like a simple car model without skidding, for which* $\dot{x} = v\,\cos(\theta)$, $\dot{y} = v\,\sin(\theta)$, $\dot{\theta} = \delta$, *where* v *and* δ *denote speed and steering angle.]*

2. For all of the following questions you can assume that the vehicle starts relatively close to the x-axis, *i.e.*, $y \approx 0$, while relatively well aligned with the same axis, *i.e.*, $\theta \approx 0$. *[Note: If you are not sure whether the model you derived for Part 1 is correct, then please use the simple car model described in the hintfor Part 1.]* In all parts below, please describe the control design process only, please do *not* attempt numerical calculations.

   (a) We would like to design a linear controller that allows the vehicle to track the x axis at a constant longitudinal speed, say at speed $\bar{v}$. First, assume that you have full state feedback on x, y, θ, and v. How would you design this control system using the eigenvalue placement method? Please consider linearizing the system model, assess controllability, and describe the eigenvalue placement control design process including how you would choose the eigenvalues and how you would go about computing the controller.

   (b) Suppose we can observe the speed of the aircraft (*i.e.*, the variable v) and its distance to the x axis (*i.e.*, variable y), but we can not observe the variables x and θ. Can you design a compensator with output feedback using the eigenvalue placement technique?

   (c) Now suppose we can only observe the variables v, θ? Can you design a compensator with output feedback using the eigenvalue placement technique in this case?

   (d) Suppose we would like to use the LQR design methodology to design a controller that can track the x-axis at constant speed, assuming full state feedback. How can you formulate an LQR design problem that can balance control effort and settling time?

3. What are some of the limitations for the linear control systems that you designed in Part 2?

Question 2

Let us assume a simplified Dubins car in 2D that does not know its pose $\vec{x} = (x, y, \theta)$. The vehicle dynamics model is given according to

$$\begin{aligned} x_{k+1} &= x_k + \cos(\theta_k) + w_{x,k+1} \\ y_{k+1} &= y_k + \sin(\theta_k) + w_{y,k+1} \\ \theta_{k+1} &= \theta_k + \tan(\phi_k) + w_{\theta,k+1} \end{aligned}$$

where ϕ_k is the steering angle. Notice that this is a simplified Dubins car that is travelling at a constant velocity of 1 m/s. The noise terms $w_{\cdot,k}$ are distributed according to a zero-mean Gaussian with known covariance W,

$$\begin{bmatrix} w_x \\ w_y \\ w_\theta \end{bmatrix}_k \sim \mathcal{N}(0, W).$$

The vehicle can estimate its position and orientation from the bearing to a set of active beacons distributed in the plane, each with known position. The vehicle has an initial position and orientation estimate given by a Gaussian with mean $\hat{\vec{x}}_0$ and covariance Q_0. At each time step k, the vehicle receives some number n_k of bearing measurements, where the bearing $b_{i,k}$ is to the i^{th} beacon at position $(l_{x,i}, l_{y,i})$ with Gaussian noise such that

$$\begin{aligned} b_{i,k} &= atan2(l_{y,i} - y_k, l_{x,i} - x_k) - \theta_k + v_k \\ v_k &\sim \mathcal{N}(0, r^2), \end{aligned}$$

where r^2 is known and the same for every beacon.

The vehicle knows which beacon generated each bearing, and the position of each beacon. You may assume that a signal is received from every beacon on every step.

1. Please identify an algorithm for inferring the distribution $p(\vec{x}_t | b_{0,t})$. (Any derivatives you may need, you can leave unsolved.)

2. Does your answer change if the vehicle trajectory keeps the vehicle very far from the beacons? Does your answer change if the vehicle trajectory causes the vehicle to travel very close to the beacons?

3. Please identify an algorithm for inferring the distribution $p(\vec{x}_t | b_{0:T}), t \in \{0, T\}$, that is, the position of the vehicle at any point in the past.

4. Imagine that you no longer know the beacon positions. How does your model change? Do your inference algorithms change?