Controls Field Exam Question

January 2008

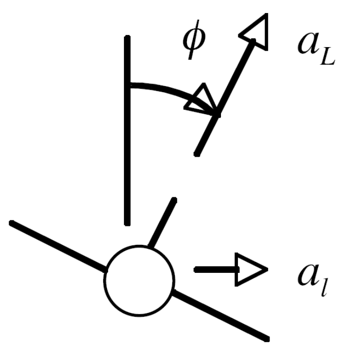
The roll rate $\dot{\phi}$ of an aircraft satisfies the following differential equation

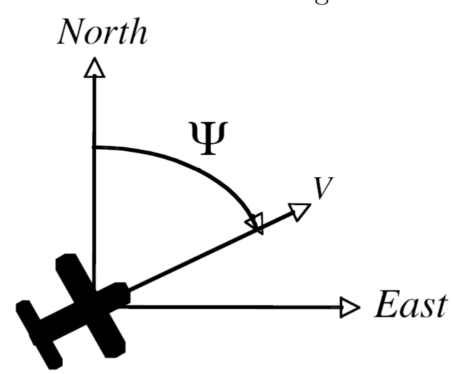
$$\dot{\phi} = u$$

where u is the input command to the aircraft flight control system. The roll angle produces a lateral acceleration in the horizontal plane, given by:

$$a_l = a_L \sin \phi$$

where a_l is the lateral acceleration and a_L is the vertical force due to the wing lift.





The heading angle of the aircraft velocity vector Ψ satisfies the differential equation

$$\dot{\Psi} = \frac{a_l}{V}$$

where V is the aircraft speed, which is assumed to be constant. Assume for now that $a_L = 10\text{m/s}^2$, $V = 20\text{m/s}$, and that only the heading information can be measured.

1. Develop a nonlinear model of the system, and linearize it assuming that the roll angle is small. Create a state space model of the resulting linear system.

2. Use LQG techniques to design a dynamic output feedback (DOFB) controller for this system. Focus the regulator design on controlling the heading angle. Assume that the dominant process noise enters the system through the roll rate dynamics. For simplicity assume that the intensity of the process noise noise w and sensor noise v are the same, i.e., $w \sim N(0, q)$ and $v \sim N(0, q)$ and they are independent.

   - What weighting matrices did you use in the design, and why.
   - Are all observability and controllability assumptions satisfied for this optimal compensator problem?

3. Give a classical interpretation of your resulting DOFB controller.

4. What are your thoughts on the likelihood that this design would work on the real system? In particular consider:

   - What margins (guaranteed and actual) you might expect for this linearized system and your controller.
   - The potential impact of the nonlinearity in the original system dynamics on the stability and performance of the closed-loop system.