# Controls Field Exam

## Question 1

Consider a satellite orbiting the earth. The satellite's distance from earth is denoted by $r(t)$. The satellite is put on orbit such that its nominal distance to earth is $R$, and we would like to design a controller to regulate $r(t)$ around $R$ using the thrusters on the satellite. The dynamics of the satellite can be described the following differential equation:

$$\ddot{r}(t) = \omega^2 \, r(t) - \frac{\beta}{r(t)^2} + u(t),$$

where $\omega$ is the angular velocity of the satellite and $\beta$ is a constant that depends on the mass of the satellite. We chose the orbit radius $R$ and the angular velocity $\omega$ such that

$$\omega^2 \, R^3 = \beta.$$

(Please note that this is a grossly simplified model of a satellite orbiting earth. We arrive at this model after making various assumptions and simplifications. The behavior of this simple model may not closely match more complex models of satellite dynamics. This question is intended only to test your control design skills. It can be answered without any knowledge of satellite dynamics.)

Please answer the following questions:

1. Design a regulator that regulates the altitude of the satellite. Use the infinite-horizon LQR control design methodology. Equally weigh the deviation of $r(t)$ and $u(t)$ from their nominal values, and do *not* (directly) include the deviation of $\dot{r}(t)$ from its nominal value in the cost function. Please compute an LQR controller that fulfills this specification. For simplicity, suppose $\omega = 1$.

2. Design a regulator (for the **nonlinear** system) such that the closed-loop dynamics behaves exactly like a **linear time-invariant** system with two poles at $-2$. For simplicity, suppose $\omega = \beta = R = 1$.

## Question 2

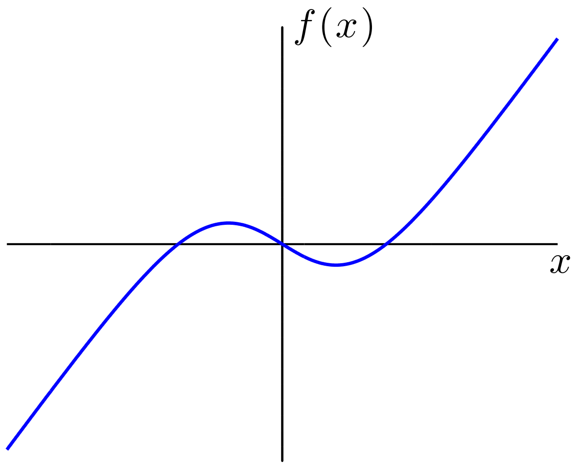
Consider a stochastic first-order system with linear dynamics and nonlinear measurement given by

$$\dot{x} = -x + w$$
$$y = f(x) + v$$

where

$$f(x) = \frac{12}{7}x - 5\,\frac{e^x - 1}{e^x + 1}$$

The process noise $w$ and measurement noise $v$ are independent, zero mean, Gaussian white noise processes with intensities $W$ and $R$, respectively. The initial state of the system is known to be $x(0) = 0$. A plot of $f(x)$ is below. For reference, the slope of $f$ at $x = 0$ is approximately $-0.252$, and $f(x)$ crosses the $x$ axis at approximately $x = \pm 2.48$.




Describe how you would develop a state estimator for this problem, and the problems you might encounter. You might consider some limiting cases:

1. $W \ll 1$, $R \ll W$
2. $W \ll 1$, $R \gg W$
3. $W \gg 1$, $R \ll W$
4. $W \gg 1$, $R \gg W$