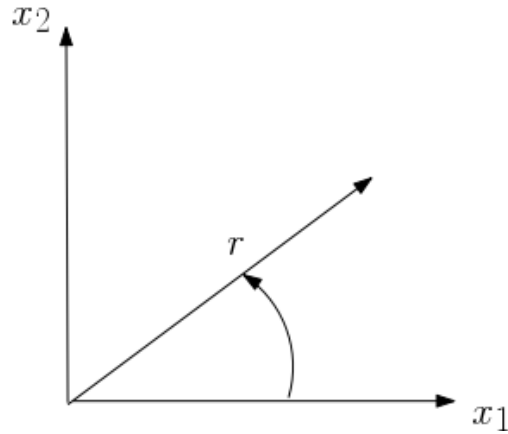


Field Exam
2014
Materials and Structures Question

January 22, 2014

Question	Points	Score
1	10	
2	20	
3	25	
4	45	
Total:	100	



Consider the stress field given in polar coordinates (see reference frames considered in figure) by the expressions:

$$\sigma_{rr} = \frac{\sigma^\infty}{2} \left[\left(1 - \frac{a^2}{r^2}\right) + \left(1 + 3\frac{a^4}{r^4} - 4\frac{a^2}{r^2}\right) \cos 2\theta \right] \quad (1)$$

$$\sigma_{\theta\theta} = \frac{\sigma^\infty}{2} \left[\left(1 + \frac{a^2}{r^2}\right) - \left(1 + 3\frac{a^4}{r^4}\right) \cos 2\theta \right] \quad (2)$$

$$\sigma_{r\theta} = -\frac{\sigma^\infty}{2} \left(1 - 3\frac{a^4}{r^4} + 2\frac{a^2}{r^2}\right) \sin 2\theta \quad (3)$$

Question 1 [10 points]

This stress field was obtained from a stress function which **satisfies the biharmonic equation** $\nabla^4\phi = 0$. Does this field:

- (a) (5 points) satisfy the differential equations of local equilibrium of stress? why/why not?
- (b) (5 points) relate to a compatible strain field for a linear isotropic material? why/why not? what about for an orthotropic material? why/why not?

Question 2 [20 points]

Evaluate the stress field at:

(a) (5 points) $r = a$, what does this represent as a traction boundary condition?,

(b) (5 points) $r \gg a$, what does this represent as a traction boundary condition?.

(c) (5 points) Based on these two boundary conditions, describe in words and in a sketch what specific linear elasticity problem (physical situation: geometry, loads) this solution pertains to.

(d) (5 points) It is observed that the stresses do not depend on the material properties, what aspect(s) of the problem do?

Question 3 [25 points]

Evaluate the distribution of the stress components $\sigma_{11}, \sigma_{22}, \sigma_{12}$ in the cartesian reference frame x_1, x_2 on the axis $x_1 = 0$ as a function of x_2 . According to the expression obtained, what is the maximum amplification of the remote stress σ^∞ (stress concentration factor) and where and for which stress component is it realized?

Question 4 [45 points]

Use the solution obtained in question 1 to obtain the stress distribution as a function of θ for $r = a$ for the following two different loading cases. (Hint: $\cos(a + \pi) = -\cos a$) :

- (a) (15 points) a remote hydrostatic stress field $\sigma_{11}^{\infty} = \sigma_{22}^{\infty} = p$. Sketch the remote loading on the plate with the hole for this case.

- (b) (15 points) a remote pure-shear stress field $\sigma_{12}^{\infty} = \sigma_{21}^{\infty} = \tau$. What is the stress concentration factor in this case? Sketch the remote loading on the plate with the hole and the location of the maximum hoop stresses adjacent to the hole surface.

- (c) (15 points) What are the stress concentration factors for the hoop stresses $\sigma_{\theta\theta}$ in these last two cases and where are they realized?