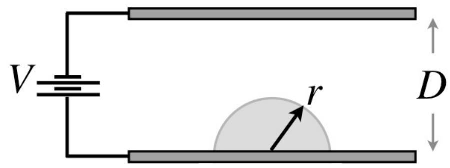
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Aeronautics and Astronautics

Field Exam in Space Propulsion
January - 2011

- You need to solve **TWO** of the three problems in this exam
- Read carefully the problem before writing your solution
- Make sure to state your assumptions
- Identify clearly the process you will use to solve each problem
- Manage your time with care

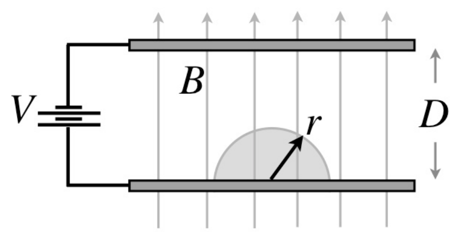
Question #1

Suppose you have a parallel metallic plate setup with a small liquid droplet of radius r deposited on the bottom plate. The liquid has conductivity *K*, surface tension γ and density ρ. The experiment is carried out in air.



This setup is typical for generating the electrostatic liquid instability that forms Taylor cones when the electric pressure balances the liquid surface tension at some critical voltage V_0.

It is postulated that a new variety of conductive liquids with non-negligible magnetic susceptibility χ is available.* We hypothesize that a magnetic field *B* would reduce the critical voltage to a value $V < V_0$.



Given the (normal) electric and magnetic components of the Maxwell stress tensor,

$$T_E = \frac{1}{2}\varepsilon E^2 \quad \text{and} \quad T_B = \frac{1}{2}\mu H^2$$

a) Derive an expression for the voltage reduction in the form $V/V_0 = f$, where $f < 1$ is a function of *B* and other model parameters. Assume the *electromagnetic* Taylor instability will occur after forces at <u>any</u> point on the interface balance out. (Hint: where will this first happen on the meniscus?)

b) For a magnetic susceptibility of $\chi = 5\times10^{-3}$, what magnetic field *B* would be required to reduce the required critical voltage by 50%? Assume a liquid meniscus with r = 0.5 mm, γ = 0.05 N/m, K = 1 Si/m and ρ = 1 g/cm^3. Comment on the practicality of applying such a magnetic field in this configuration.

c) Comment on the potential impact (positive or negative) of this configuration on electrospray propulsion in the droplet and pure ionic regimes.

d) Calculate the resulting thruster voltage efficiency as a function of the original voltage efficiency η_0 and *f*. What is the new voltage efficiency for the case in b) if η_0=0.95 and 0.5? Assume the charged particle emission cost (V_{loss}) is independent of *B*.

* Recall that $\mu = \mu_0(1+\chi)$.

Question #2

In a Hall thruster experiment, one measures the flow rate $\dot{m}$, the anode voltage and current (V_a, I_a), and the thrust F. In addition, from spectroscopic measurements, one can get reasonably good estimates of the density fractions of single and double ionized ions (r_1, r_2, with $r_1 + r_2 = 1$) at some station in the plume. Finally, from stopping potential data, one can get the potentials $V_1 = V_a\varphi_1$ and $V_2 = V_a\varphi_2$ at which single and double ionized ions originate.

Propose a data reduction scheme to estimate:

(a) The beam current I_B, the current efficiency $\eta_a = I_B/I_a$, the utilization efficiency $\eta_u = \dot{m}_{ion}/\dot{m}$ and the voltage efficiency $\eta_E = \eta/(\eta_a\eta_u)$.

(b) The fractions of the mass flow, ion current and thrust that are carried by single and double ionized ions.

(c) In order to isolate effects, assume $\eta_a \cong 1$, $\eta_u \cong 1$ and $\varphi_1 \cong \varphi_2 \cong 1$. Derive an expression for the efficiency η under these conditions, and explain why η decreases with r_2, the double ion fraction. Similarly, derive an expression for the specific impulse and explain the variation with r_2.

State any assumptions or approximations you make in your analysis, and comment on how they might be avoided.

Question #3

In a Hybrid Rocket, a gaseous oxidizer stream flows parallel to a solid fuel surface. Heat from a "flame" surface located near the surface pyrolizes and evaporates the fuel, and the fuel gas flows towards the flame. Oxidizer gas flows towards the flame from the outside, and both streams react at the flame surface, the products then diffusing away from it in both directions. The solid surface blocks the product gases, which are therefore stagnant between the solid and the flame. Above the flame, the product gases diffuse and convect against the incoming oxidizer gas, up to the point where the local composition corresponds to pure oxidizer. Over time, this boundary moves, as more fuel evaporates and more products are generated.

The object of this problem is to formulate a diffusion-limited model for prediction of the flux γ of fuel (kg/m^2/s), the flame stand-off distance x_F, the flame temperature T_F and the product layer thickness δ.

Assume the effect of the flow parallel to the surface can be ignored for this analysis, and superimposed later. Assume also that the time evolution of the layer thickness δ, and anything that depends on it, is slow compared to the gas transit time across the layer, so that the transverse analysis can be treated as if it were in steady state, with the instantaneous $\delta(t)$ corresponding to the current time t.

Ignore density variations, and assume the mass diffusivity D, thermal conductivity k, and specific heat c_p are the same for all species. The pyrolization/evaporation process at the solid surface consumes an amount L of energy per kg of gas generated, and the combustion at the flame generates an amount Q of heat per kg of fuel consumed. Fuel and oxidizer are consumed in their stoichiometric proportion $i = (O/F)_{stoich}$. The surface temperature T_s and that of the outside oxidizer stream, T_{ox}, are assumed known.

(a) Outline the model formulation. State the balance equations that need to be written for the regions $0 < x < x_F$ (between surface and flame) and $x_F < x < \delta$ (outside the flame), the variables involved, and the boundary conditions to be imposed. For this part, assume δ is a known distance.

(b) Write down and integrate as many of these equations as you have time to.

(c) Formulate a physical condition to determine $\delta(t)$. Here, t is the "slow" time scale, as noted above.