MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Aeronautics and Astronautics

Field Exam in Space Propulsion
January – 2013

- There are three problems in this exam. Solve only **TWO**.
- Read carefully each problem before writing your solution.
- Make sure to state and be consistent with the problem assumptions.
- Identify clearly your line of thought in your solutions.
- Manage your time with care.

Problem #1

Consider the design of an Expander Cycle pressurization system for a Hydrogen-Oxygen rocket. For simplicity, assume the two liquids start from negligible pressure and temperature levels. All of the fuel (Hydrogen) flows through the nozzle cooling passages, where it picks up an amount of heat per kg that is a fraction f of the fuel heat value $h_F = 1.2\text{x}10^8$ J/kg. At the same time, the fuel undergoes a pressure loss by a fraction α_N of the chamber pressure, P_c. It then drives a turbine of efficiency η_T, the discharge of which is injected in the main combustion chamber with an injector pressure loss $\alpha_{inj}P_c$. Assume the gas driving the turbine has a ratio of specific heats γ. The Oxygen is pumped to the same pressure as the turbine exhaust, and is injected into the main chamber with the same pressure drop as the fuel. Both pumps are on a common shaft, and have the same efficiency, η_P. The rocket operates with an Oxidizer/Fuel mass ratio OF.

(a) Formulate an equation or a system of equations that would relate the chamber pressure P_c to the pressure rise ΔP_{FP} in the fuel pump. Solve for the chamber pressure as a function of the ratio $\delta = (\Delta P_{FP})/P_c$.

(b) Discuss the behavior of the solution in Part (a), in particular whether a value of δ' exists that will maximize P_c. If so, calculate $P_c(\delta')$ and $\Delta P_{FP}(\delta')$ for the following numbers:

$f = 0.01$ $\quad$ $\eta_T = \eta_P = 0.8$ $\quad$ $\alpha_{inj} = \alpha_N = 0.15$ $\quad$ $\rho_F = 69$ kg/m^3

$\rho_{OX} = 1100$ kg/m^3 $\quad$ OF = 5 $\quad$ $\gamma = 1.4$

Problem #2

The BGK model in kinetic theory replaces the full Boltzmann collisional operator by a relaxation term:

$$\left(\frac{df}{dt}\right)_{Coll.} = \nu(f_M - f)$$

where

$$f_M = n\left(\frac{m}{2\pi kT}\right)^{3/2} e^{-\frac{m(\vec{w}-\vec{v})^2}{2kT}}$$

is the Maxwellian distribution that has the same total density n, mean velocity $\vec{v}$ and temperature T as the actual distribution f, and where ν is a constant collision frequency. For this approximation to behave at least qualitatively correctly, the Boltzmann H theorem should be satisfied. Consider a uniform gas at rest and with a constant density n, but initially out of equilibrium, and show that, using the BGK model,

$$\frac{dH}{dt} \leq 0; \quad H \equiv \int f(\ln f) d^3w$$

where the equality holds only when $f = f_M$ for all velocities.

Problem #3

Different strategies could be utilized to circularize elliptical trajectories using low thrust maneuvers compatible with electric propulsion. In one such strategy, continuous thrust *F* is applied perpendicular to the apsidal line (line along the eccentricity vector) and co-planar with the orbit, with a direction opposite to the velocity vector at perigee.

a) For this to be a useful approach, the line of apsides should not rotate on average and the orbital period should remain constant during the maneuver. Argue from first principles that these conditions are indeed satisfied. Assume the thrust acceleration *F/m* remains constant throughout, and that the orbital elements change only slightly over one orbit.

b) From the angular momentum equation, averaged over each orbit, derive an equation that could be used to compute the long-term rate of change of eccentricity of the orbit.

c) Indicate how this can be used to calculate the propulsive *Delta-V* for the circularization maneuver.