From the outside, an Ion Engine and a Hall thruster are very similar devices: in both of them a Noble Gas is partially ionized, the ions are made to fall through a potential difference, and the electrons are captured by an anode and joined to the ions to form a plasma beam. The overall performance of the two is also similar, although not identical.

You are asked to comment on a few of the differences. Specifically,

1) For a given thrust and specific impulse, an ion engine is physically larger than a Hall thruster. Explain why.

2) Both devices use magnetic fields, even though they both are electrostatic ion accelerators. Explain the role of the magnetic field in each of them, and how this guides the layout of these $\vec{B}$ fields.

3) One advantage of ion engines over Hall thrusters is a better collimated beam (smaller plume divergence). Explain why this is so and what could be done to reduce the beam divergence in a Hall thruster to something approaching ion engine levels.

4) The efficiency has been historically higher for ion engines than for Hall thrusters (at a fixed specific impulse). Comment on possible reasons and remedies (in the Hall thruster case).
A satellite is initially in an elliptic Geosynchronous Transfer Orbit about Earth, and it is desired to use a low-thrust engine to raise continuously its energy to escape conditions, without rotating its orbit's line of apses or its plane. The thrusting acceleration $f = F/m$ ($F =$ thrust, $m =$ mass) can be modulated as a function of the true anomaly $\theta$, and the thrust vector $\vec{F}$ is to be aligned with the velocity vector $\vec{v}$.

Formulate a set of first-order time differential equations for $\theta$, $a$ (the instantaneous semi-major axis) and $e$ (the instantaneous eccentricity). These equations should be numerically (or perhaps analytically) solvable once $f(\theta)$ is specified. What should be the end condition indicating when escape is reached?

**Hint:** What is the rate of change of orbital energy?
Space Propulsion Field Exam: Rocket Propulsion elective

The schematic shows the pressurization system of a LOX-RP1 rocket, with some estimated pressures. The chamber pressure is 80 atm. The Oxidizer/Fuel ratio (OF) in the main combustor is 2.3, but a much higher value is expected in the Gas Generator (GG), in order to limit the turbine inlet temperature to 1000K. The thrust is to be $2 \times 10^6$ N, and the specific impulse is estimated at 350 s.

Elaborate on the set of procedures and equations you would need to answer the following questions:

(a) Calculate the OF value (or the fuel fraction) for the Gas Generator.
(b) Calculate the fraction $x$ of the overall mass flow rate that needs to be diverted to the Gas Generator (including LOX and RP1).
(c) For a turbine design with impulse stages and axial stage exit flow, with the mean linear blade speed or limited to less than 250 m/s, calculate the number of turbine stages needed.

Use the following data if you are providing numerical answers to the questions above:

Estimated liquid enthalpies at the GG inlet:
- LOX (100K): $-3.76 \times 10^5$ J/kg
- RP1 (350K): $-1.55 \times 10^6$ J/kg

Enthalpies of formation of gaseous species (at 298K):
- CO$_2$: $-393,500$ J/mol
- H$_2$O: $-240,800$ J/mol
- O$_2$: 0 J/mol

Average specific heats of gaseous species (up to 1000K):
- CO$_2$: $45.71$ J/mol/K
- H$_2$O: $37.35$ J/mol/K
- O$_2$: $28.47$ J/mol/K

Ignore other possible species.

Isentropic Efficiency of both pumps and of the turbine: 0.7

NOTE: For a quick estimate in part (a) you can use the “fuel heat value” approach rather than the more complete thermal balance. If so, use a heat value of 43 MJ/kg for the RP1.