- There are two problems in this exam.
- Read carefully each problem before writing your solution.
- Make sure to be consistent with the problem assumptions.
- Numerical answers are of course preferable, but you could leave your answers as equations: It is more important to show your line of thought and describe the steps required to arrive at your solution.
- **Manage your time with care.**
Problem #1

Almost \( \frac{1}{2} \) of input power to a Hall thruster does not show up as jet kinetic energy. Where does it go?

A typical Xenon Hall thruster with \( P = 1500 \) W, \( \eta = 0.5, \eta_u = 0.85, I_{sp} = 1600 \) s, and dimensions \( D_{out} = 0.1 \) m, \( D_{in} = 0.07 \) m, \( D_{ext} = 0.14 \) m, \( L = 0.03 \) m, is observed to operate at a wall temperature \( T = 600 \) K. Estimate:

a. Power lost to ionization and excitation (followed by prompt radiation).

b. Power lost to the walls by electrons and ions. Estimate \( T_e = 4 \) eV over the internal portion of the channel (\( T_e \) is larger just outside). Assume ion-sonic exit. Assume the average density is \( \frac{1}{2} \) of that at the exit.

c. Power radiated by the outer walls (estimate an emissivity \( \epsilon = 0.8 \) at the external diameter of 0.14 m).

d. By an overall power balance, thermal power conducted to the engine mount.

For extra credit: Is there a possibility for using the Xenon propellant as a regenerative coolant, as in chemical rockets?

Some useful data:

\[
\begin{align*}
m_i &= 2.17 \times 10^{-25} \text{ kg} \\
m_e &= 9.11 \times 10^{-31} \text{ kg} \\
k_B &= 1.38 \times 10^{-23} \text{ J/kg} \\
\sigma &= 5.67 \times 10^{-8} \text{ Wm}^{-2} \text{K}^{-4} \\
e &= 1.6 \times 10^{-19} \text{ C} \\
V_i &= 12.1 \text{ eV/ion}
\end{align*}
\]
Problem #2

Starting from Low Earth Orbit and using Electric Propulsion all the way (or coasting), how far from Earth can one send a payload in 20 years? Assume a nuclear power source with a specific power plant mass $\alpha = 5 \text{ kg/kW}$ and a thruster with an efficiency $\eta = 0.75$. The specific impulse should be no less than 600 s, but could be more.

To gain time, here are a few assumptions and approximations you may want to use:

- Since the mission is so long in time and, presumably in distance, you can ignore the initial stages involving Earth escape and Sun escape, and assume the spacecraft starts from “not far from” the Sun, having escaped its attraction, and with very small speed w.r.t. the Sun.

- Power should be constant, equal to the maximum provided by the reactor. It seems reasonable to expect that the best throttling law will be $a = F/m = \text{constant}$. This has consequences as to the variation of the jet speed $c$ with time.

- When fuel is exhausted, the whole mass can be regarded as payload, and since the power plant is likely to dominate, the specific mass $\alpha$ refers to this final mass.

- It is likely that there will be a powered phase, followed by a coasting phase once the propellant is exhausted at some cut-off time $t_{co}$. This time should be selected so as to maximize the final distance $r_f = r(t_f)$.

- This is optional, but your algebra will be cleaner, and you will more easily be able to generalize the results if you use non-dimensional variables from the start.

Expected results are:

- Propulsive acceleration, $a$
- Power per unit initial mass, $P/m_0$
- Mass fraction of the power plant, $m_{pp}/m_0$
- Cutoff time, $t_{co}$
- Specific impulse, $c(t)$
- Distance achieved, $r_f$