2017 Space Systems Qualifying Examination Question and Solutions

Useful constants:
\[ G = 6.67408 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2} \]
\[ M_{\text{Sun}} = 1.989 \times 10^{30} \text{ kg} \]
\[ M_{\text{Jupiter}} = 1.898 \times 10^{27} \text{ kg} \]
\[ M_{\text{Earth}} = 5.972 \times 10^{24} \text{ kg} \]
\[ \text{AU} = 1.496 \times 10^{11} \text{ m} \]
\[ g = 9.80665 \text{ m/s}^2 \]

NASA recently selected two Discovery missions to explore asteroids, teaching us about the early solar system. The mission names are Lucy and Psyche. The Lucy mission will fly by six Jupiter Trojan asteroids (visiting both "Trojans" and "Greeks"). The Psyche mission will fly to and orbit 16 Psyche, a giant metal asteroid in the main belt.

1) Draw a top-down perspective diagram of the solar system that includes the Sun, the Earth, and Jupiter; label it with distances in AU. If you do not know the distances exactly, make a reasonable guess. [Time estimate: 2 min]

Answer: See diagram in Figure 1. Jupiter is approximately 5 AU from the Sun; its semi major axis is ~5.2 AU. Mars is ~1.5 AU, and Earth at 1 AU.

![Figure 1: Diagram for question 1](http://lasp.colorado.edu/~bagenal/1010/)

*The Cosmic Perspective* by Bennett, Donahue, Schneider and Voit

http://lasp.colorado.edu/~bagenal/1010/
2) Update your diagram to include the approximate locations and distances to the Trojan asteroids. Your diagram should at minimum include the angle ahead/behind Jupiter of the Trojan asteroids. Qualitatively explain this geometry. [Time estimate: 3 min]

Answer: See diagram in Figure 2. The diagram at minimum should show the L4 and L5 Sun-Jupiter Lagrange points. If not already included, when asked, the student should be able to also draw L1, L2, L3. Student should know that the Trojan asteroids are in relatively stable orbits at L4 and L5. Students should know that the geometry is approximately equilateral triangles and thus 60 degrees ahead of and behind Jupiter. “Greeks” are at L4 and “Trojans” are at L5. The advanced student could qualitatively talk about how to derive the locations of the Lagrange points using a rotating frame of reference, and that this is a restricted three-body problem.

![Figure 2: Diagram for question 2](http://www.space.com/images/i/000/049/587/original/lagrange-points1.jpg?interpolation=lanczos-none&downsize=*.1000)

3) What is the angular velocity at which both the Sun and Jupiter orbit their mutual center of mass? [Time estimate: 5 min]

Answer: The student could give the “easy” answer and state that since they already know/remember that the barycenter is very close to the Sun (just outside the surface) the period should still be about the same as Jupiter’s orbital period, and just take $2\pi$ by divided by the period to get angular velocity in rad/s.
\[ T_{\text{Easy}} = 2\pi \sqrt{\frac{a^3}{\mu}} = 2\pi \sqrt{\frac{(5.2 \text{ AU converted to m})^3}{G M_{\text{Sun}}}} = 3.7417 \times 10^8 \text{s} = 11.8648 \text{ years} \]

\[ \omega_{\text{Easy}} = \frac{2\pi}{3.7417 \times 10^8 \text{s}} = 1.6792 \times 10^{-8} \text{rad/s} = 9.6212 \times 10^{-7} \text{deg/s} = 3.464 \text{ mas/s} \]

However, it would be desirable to have the student be able to set up the calculation for the barycenter (simple center of mass), and then to show the small difference between the “easy” answer and a more detailed answer. Can assume circular orbits. Find the barycenter, and then update the gravitation expression to include a second body, and solve for period, then angular velocity.

\[ T_{\text{TwoMasses}} = 2\pi \sqrt{\frac{a^3}{\mu}} = 2\pi \sqrt{\frac{(5.2 \text{ AU converted to m})^3}{G (M_{\text{Sun}} + M_{\text{Jupiter}})}} \]

\[ = 3.7399 \times 10^8 \text{s} = 11.8592 \text{ years, ~2 days shorter than the "easy" approach} \]

\[ \omega_{\text{TwoMasses}} = \frac{2\pi}{3.7399 \times 10^8 \text{s}} 1.6800 \times 10^{-8} \text{rad/s} = 9.6258 \times 10^{-7} \text{deg/s} = 3.476 \text{ mas/s} \]

The "a" is the sum of the semi-major axes of the ellipses (we assume circular), or equivalently, the semimajor axis of the ellipse in which one body moves in a frame of reference with the other body at the origin.

\[ F = \frac{m_1 v_1^2}{r_1} = \frac{m_2 v_2^2}{r_2} \]

\[ P = \frac{2\pi r_1}{v_1} = \frac{2\pi r_2}{v_2} \]

\[ \frac{r_1}{r_2} = \frac{v_1}{v_2} \]

\[ r_1 = \frac{m_2}{m_1} r_2 \]

Semimajor axis \( a = r_1 + r_2 \)

\[ F = \frac{G m_1 m_2}{a^2} = \frac{m_1 v_1^2}{r_1} = m_1 r_1 \left( \frac{v_1}{r_1} \right)^2 = m_1 r_1 \left( \frac{2\pi}{P} \right)^2 \]

use

\[ r_1 = \frac{m_2 a}{(m_1 + m_2)} \]

to get

\[ \frac{G (m_1 + m_2)}{a^3} = \left( \frac{2\pi}{P} \right)^2 = \omega^2 \]
Using the mass of the sun and Jupiter, with $a = 5.2\ AU = 7.7792e+11\ m$, the barycenter $r_1 = 7.42e+08\ m$ (~742,000 km, just outside surface of Sun which has radius ~695,700 km).

4) a) In the Sun-Earth system, where do we currently have spacecraft stationed to warn us in advance of impeding space weather events due to solar activity? [Time estimate: 3 min]

b) The SOHO spacecraft gives us about an hour of early warning. Using the definition of “Hill sphere” or “Roche sphere” as the location of the spacecraft, what is the approximate speed at which the energetic particles are traveling from the solar flare to Earth? [Time estimate: 5 min]

Answer: a) We currently have space weather warning spacecraft stationed at Sun-Earth L1, which is between the Earth and the Sun, such as SOHO, ACE, DISCOVR, and WIND. It makes the most sense to put a spacecraft at L1. Putting a spacecraft at L2 is behind Earth, so that doesn’t help us with space weather (good place to put space telescopes, though). L3 is on the far side of the Sun, which could monitor sunspots in advance, but because it would take ~2 weeks for the sunspots to rotate into view of the Earth and the uncertainty in evolution toward a coronal mass ejection (CME) limits its utility for space weather warnings. L4 would look at sunspots and potential flare locations even further in advance than L3 (or you could say, after they’ve rotated past the Earth). L5 could be another option (see diagram in Figure 4), because it could look at sunspots only a few days before they rotate to face the Earth, and ESA is considering sending a monitoring mission there. One could argue that the L5 and L4 perspectives could contribute a new angle to improve velocity estimates, but that still wouldn’t justify a mission to L4. Note the STEREO mission has two spacecraft with heliocentric orbits that are just a bit faster/slower than Earth’s orbit; they periodically pass near L4 and L5 but do not stay there.

Answer b) There may be some confusion for students here between Hill Sphere and Sphere of Influence. We mean Hill Sphere. They are different definitions:

Hill Sphere: $r = a \left(\frac{m}{3M}\right)^{1/3}$ where $a$ is the semi-major axis of the smaller mass $m$ about the larger mass $M$. Here, we use $a = 1\ AU$ (in $m$), $m = M_{Earth}$ and $M = M_{Sun}$ to get $r = 1.4964 \times 10^9\ m$ or ~0.01 AU. This is an approximation to the location of L1. The Hill sphere considers $m$ that orbits about $M$ – it includes consideration of centripetal force $mv^2/r = m r \omega^2$ as well as gravitational force $GMm/r^2$. Equating these, student can at least infer third order in $r$. Hill sphere addresses “what is the region where $m$ dominates the attraction of satellites.” The outer region of this shell is a zero-velocity surface. There is a more detailed derivation of the expression below, to aid if students do not have it committed to memory. With L1 ~0.01 AU or some students recalled as ~1.5 x 10^6 km, and using time of 1 hour, the particle speed is ~415 km/s.
Derivation of Hill Sphere (from http://www.jgiesen.de/astro/stars/roche.htm with less confusing variable names):

![Diagram](http://www.jgiesen.de/astro/stars/roche.htm)

**Figure 3: Diagram for Answer 4b.**

The equilibrium condition for the planet \( m \) is:

\[
m \omega^2 R = G \frac{m M}{R^2}
\]

\[
\omega^2 = \frac{GM}{R^3}
\]

The satellite feels the combined gravitational forces exerted by the star and the planet:

\[
m_{\text{sat}} \omega^2 (R+r) = G \frac{m_{\text{sat}} M}{(R+r)^2} + G \frac{m_{\text{sat}}}{m} m^2
\]

Inserting \( \omega^2 \):

\[
G \frac{m_{\text{sat}}}{m} M \frac{(R+r)}{R^3} \approx G \frac{m_{\text{sat}}}{m} M \frac{(R+r)^2}{R^3} + G \frac{m_{\text{sat}}}{m} \frac{m}{r}^2
\]

\[
M \frac{(R+r)}{R^3} \approx \frac{M}{(R+r)^2} + \frac{G m}{r}^2
\]

\[
M \frac{(R+r)^3}{R^3} \approx \frac{M}{R} \frac{r^2}{R} + m \frac{R^3}{r^2} (R+r)^2
\]

\[
m \frac{R^3}{r^2} (R+r)^2 \approx M \frac{r^2}{R^3} \left( R^2 + 3R^2 r + 3R^2 r^2 + r^3 \right) - M \frac{R^3}{r^2}
\]

\[
m \frac{R^3}{r^2} (R+r)^2 = M \frac{r^3}{R^3} (3R^2 + 3R^2 r + r^2)
\]

For \( r << R \): \( (R+r)^2 \approx R^2 \), and \( 3R^2 + 3R^2 r + r^2 \approx 0 \). The equation simplifies:

\[
m \frac{R^3}{r^3} = 3 \frac{M}{R^3} \frac{R^3}{r^3} R^2
\]

\[
m R^3 = 3 M r^3
\]

\[
r = R \left( \frac{m}{3M} \right)^{1/3}
\]

On the other hand, the Sphere of Influence: \( r = a \left( \frac{m}{M} \right)^{2/5} \) doesn’t take the orbital motion (centripetal force) into account. So here, for a test satellite, the force of the
larger body on the smaller body is balanced by the force of the smaller body on the larger body. This isn’t quite the same as the Hill sphere. The gravitational sphere of influence asks which of two gravitating bodies should be used as the origin for purposes of modeling the behavior of some third body such as a spacecraft. This is useful for answering questions such as, “In a patched conic approximation, what’s the right place to switch from one conic to another?” and “When a spacecraft is moving away from a large body and toward a smaller body, when should the spacecraft navigation switch from a large body centric to a small body centric point of view?”

Reference for a nice derivation of Sphere of Influence:

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Figure 4: Diagram for Answer 4a.

5) Define what an orbital resonance is. What kind of orbital resonance are the Trojans in with Jupiter? [Time estimate: 3 minutes]
Answer: The Trojan asteroids are in a 1:1 resonance with Jupiter. Diagram in Figure 5 can be used to support definition of orbital resonance, which is when orbiting bodies exert a regular, periodic gravitational influence on each other, because their periods are related by a ratio of two small integers.

http://www.mhhe.com/physsci/astronomy/fix/student/images/12f33.jpg
Figure 5: Diagram for Answer 5

6) a) Update your diagram from question (2) to include the approximate location of the main asteroid belt, where 16 Psyche resides. [Time estimate: 2 min]

b) Why didn’t planets accrete in the main asteroid belt; where did most of the protoplanets in this region go? [Time estimate: 2 min]

c) There are gaps in the main asteroid belt called Kirkwood gaps. Predict and calculate the semi-major axis of three Kirkwood gaps. [Time estimate: 8 min]

Answer:
(a) See diagram in Figure 1. The main asteroid belt is between Mars and Jupiter, from about 1.8 AU to 4.3 AU. 16 Psyche has an aphelion of 3.3 AU and perihelion of 2.5 AU, with \( a = 2.9 \) AU and \( e = 0.140 \). The period of 16 Psyche is about 5 years.

(b) Planetesimals were affected by gravitational perturbations from Jupiter, with violent collisions leading to shattering and ejections instead of accretion.

(c) Kirkwood gaps are due to orbital resonances, when the asteroid period around the Sun is in resonance with Jupiter; this causes perturbations and instabilities.
2:1 resonance results in high eccentricity Griqua asteroids, eccentricity continues to increase until the asteroid encounters a major planet.

3:1 resonance results in high eccentricity Alinda asteroids.

3:2 resonance results in the Hildas asteroids, between the main belt and Jupiter. Hildas move in their elliptical orbits so that their aphelia put them opposite Jupiter, or 60 degrees ahead of or behind Jupiter at the L4 and L5 Lagrangian points (but they are not in a 1:1 resonance like the L4 and L5 Trojans, they are ~4.0 AU).

Strong Kirkwood gaps
2.06 AU (4:1 resonance)
2.5 AU (3:1 resonance), Alinda asteroids
2.82 AU (5:2 resonance)
2.95 AU (7:3 resonance)
3.27 AU (2:1 resonance), Griqua asteroids

Weaker and/or narrower gaps
1.9 AU (9:2 resonance)
2.25 AU (7:2 resonance)
2.33 AU (10:3 resonance)
2.71 AU (8:3 resonance)
3.03 AU (9:4 resonance)
3.075 AU (11:5 resonance)
3.47 AU (11:6 resonance)
3.7 AU (5:3 resonance)

\[ \text{Gap (m)} = \sqrt[3]{\frac{GM_{\odot}}{\frac{T}{2\pi}}} \]

\[ T_{\text{Jupiter}} = 3.741701848574116 \times 10^8 \text{ seconds or } \sim 11.8 \text{ Earth years} \]

In the equation use the appropriate ratio with \( T_{\text{Jupiter}} \). For example, a 5:2 resonance would use \( T = (2/5) T_{\text{Jupiter}} \).

7) a) The Discovery mission call limited the cost cap (not including launch vehicle) to $450M per mission, and requires only the use of solar power. The missions therefore will rely on Solar Electric Propulsion. Explain what Solar Electric Propulsion means with a block diagram. [Time estimate: 2 min]

b) Qualitatively explain what the difference between an electrostatic gridded ion thruster and an electrostatic Hall Effect thruster are. [Time estimate: 5 min]

c) The Dawn spacecraft, a previous Discovery mission, went to main belt asteroid protoplanets Vesta and Ceres, and had an ion engine. Dawn had a dry mass of 741 kg, carried 425 kg of xenon onboard, with an Isp of 3100 s. What delta-V could Dawn achieve? Is that delta-V sufficient to orbit a main belt
asteroid? If not, where might the additional delta-V come from? [Time estimate: 2 min]

**d)** The Dawn spacecraft had a wet (launch) mass that was actually 1218 kg. Explain what the other mass was needed for. [Time estimate: 2 min]

**Answers:**

a) See SEP block diagram in Figure 6, basically solar panels and regulator (or, it could be unregulated) to thrusters, then a regulator down to PPU and s/c bus voltage. At minimum the diagram should have solar panels, energy storage (batteries) and power regulation, and a thruster. Also nice to include source of fuel for thruster. Can ask students about solar panel operating voltages (mid sized spacecraft at ~100V).

![SEP block diagram](image)

*From Oh et al., “Solar Electric Propulsion for Discovery-Class Missions” 2014 Figure 6: Diagram for Answer 7a*

b) Gridded electrostatic thrusters ionize xenon gas, and then extract the ions using >2 multi-aperture grids with a large potential between them (charged by the thruster’s power supply). Because they emit only positive ions, a cathode needs to be placed external to the thruster to emit the same charge in electrons to keep the spacecraft from charging, otherwise the ions would return to the spacecraft.

The Hall effect involves the fact that the presence of a magnetic field can affect (“curve”) the path of electrons (or holes, depending on what you are calling current). In a conductor the electrons pile up on one side resulting in a potential difference whose electric field magnitude will cancel out the effect of the magnetic field so that with this potential bias in place, current would again move straight. A Hall Effect sensor measures the potential of that electric field.

In a Hall Effect thruster, electrons are generated by a hollow cathode at the downstream end of the cylindrical thruster. The anode (positive electrode, or channel) is charged to a high potential by the thruster’s power supply. The electrons are attracted to the channel walls and accelerate in the upstream direction. As the electrons move toward the channel, they encounter a magnetic field produced by the thruster’s powerful electromagnets (an inner and outer magnetic coil). This high-
strength radial magnetic field traps the electrons, causing them to form into a circling ring at the downstream end of the thruster channel. The Hall thruster gets its name from this flow of electrons, called the Hall current. Xenon gas is in the channel, and collisions of xenon with the trapped electrons generate ions, and the ions accelerate outward due to the electric field between the channel and ring of electrons. A second hollow cathode at the exit of the thruster emits electrons that are attracted to the exiting ions and exit the thruster as well, leaving things charge neutral.

Figure 7: Diagram for Answer 7b: gridded ion thruster

Figure 8: Diagram for Answer 7b: Hall effect thruster
c) Rocket equation,

\[ \frac{m_f}{m_i} = e^{\frac{\Delta v}{g_{ecl}}} - \ln \left( \frac{741 \text{ kg}}{(741 + 425) \text{ kg}} \right) \cdot \left( 9.8 \frac{\text{m}}{\text{s}^2} \right) \cdot 3100 \text{ s} = 13.78 \text{ km/s} \]

Assume main belt is ~2 AU (4 Vesta is actually a bit further out with v = 19.34 km/s) and use

\[ \sqrt{\frac{(GM_{\text{Sun}})}{2 \text{ AU}}} = 21 \text{ km/s} \]

The heliocentric velocity of Earth is ~29 km/s. The answers here depend on what assumptions are made about the capability of the launch vehicle and the concept of operations for getting from Earth to the asteroid. If the launch vehicle achieves Earth escape velocity (~11 km/s) then the spacecraft can start in a heliocentric orbit at 1 AU. If not, then the spacecraft has to use some delta-V to escape Earth into heliocentric orbit as well. From there, the student could initially consider as a lower bound a Hohmann transfer to the asteroid but this is not practical because an ion engine is not capable of the idealized (or even close to it) thrust at the needed points. So the trajectory would be more like a spiral. Students should offer gravity assists using Earth and Mars as a way to increase the delta-V capability of the mission in order to reach orbit around a main belt asteroid.

d) The remaining ~50 kg was for hydrazine for attitude control thrusters.

8) The Psyche mission instruments include a (a) multispectral imager, (b) gamma ray and neutron spectrometer, and (c) a magnetometer. Explain what each would measure, how (a) might be implemented, and why both (b) and (c) need to be mounted at the end of a 2 m boom. [Time estimate: 6 min, 2 each]

Answers:

The Multispectral Imager provides high-resolution images using filters to discriminate between 16 Psyche’s metallic and silicate constituents. Key concepts here are making photometric measurements using filters.

The Gamma Ray and Neutron Spectrometer will detect, measure, and map 16 Psyche’s elemental composition. It works because cosmic rays bombard the surface of the
asteroid and dislodge nuclei; in addition to the neutrons, the energy state transitions generate gammas. For this measurement, we know the neutrons have to be generated by an energetic particle impact because their half-life is ~15 minutes otherwise. Gammas can also be formed if there are decaying radioactive materials on the surface. Semiconductor crystals are used to detect gammas (pulses are generated when gammas hit) and scintillators, which generate light (glow) pulses when hit by neutrons, are used to detect neutrons. The instrument is mounted on a 2-m boom to distance the sensors from background radiation created by energetic particles interacting with the spacecraft and to provide an unobstructed field of view.

The Psyche Magnetometer is designed to detect and measure the remnant magnetic field of the asteroid. It is composed of two identical high-sensitivity magnetic field sensors located at the middle and outer end of a 2-m (6-foot) boom. The magnetometer needs to be isolated from any magnetic disturbances on the spacecraft, such as from thruster electromagnets or reaction wheels (if any), etc.

9) a) The Psyche mission involves team members from Germany (DLR) and France (IPGP), but the spacecraft will be built by SSL MDA in the United States. Provide a brief explanation of the purpose of the United States’ International Traffic in Arms Regulations (ITAR) and the Export Administration Regulations (EAR) [Time estimate: 2 min]

ITAR and EAR are two important United States export control laws that affect the manufacturing, sales and distribution of technology.

The legislation seeks to control access to specific types of technology and the associated data. Its goal is to prevent the disclosure or transfer of sensitive information to a foreign national.

ITAR contains a United States Munitions List (USML) of restricted articles and services. EAR contains a Commerce Control List (CCL) of regulated commercial items, including those items that have both commercial and military applications.

b) What are the differences between ITAR and EAR? Why do they apply to space related technology? [Time estimate: 2 min]

ITAR: Covers military items or defense articles. ITAR regulates goods and technology designed to kill or defend against death in a military setting. ITAR includes space related technology because of application to missile technology. ITAR includes technical data related to defense articles and services.

ITAR material is restricted (with some exceptions, such as green card holders) from all non-US persons. EAR material is less restricted, whether a license is required depends on what the item is and the country or nationality of the entity it is being exported to.
EAR: EAR exercises control for certain dual-use items that could have military and civilian applications. Those that have low military potential are lightly controlled, whereas technologies such as those involved in spaceflight are heavily controlled.