Please carefully read the instructions. Not all questions are to be answered. Remember to state all assumptions.

1 Useful Constants and Equations

- $R_E = 6371$ km, radius of Earth
- $G = 6.67408 \times 10^{-11}$ m$^3$ kg$^{-1}$ s$^{-2}$, gravitational constant
- $M_E = 5.972 \times 10^{24}$ kg, mass of Earth
- MIT Coordinates, 42.3601°N, 71.0942°W
- $k_B = 1.38 \times 10^{-23}$ [m$^2$ kg s$^{-2}$ K$^{-1}$] Boltzmann’s constant
- $\rho = \sin^{-1}\left(\frac{R_E}{R_E+h}\right)$ is the angular Earth radius
- $\sin(\eta) = \sin(\rho) \cos(\epsilon)$ where $\eta$ is the angle from nadir and $\epsilon$ is the elevation angle
- $\lambda = \frac{\pi}{2} - \epsilon - \eta$ is the Earth central angle. Please note that $\lambda$ is an overloaded term on this exam as it is elsewhere used for wavelength.
- $E = 7 \times 10^{10}$ N m$^{-2}$ Young’s modulus of aluminum
- $\rho_{Al} = 2700$ kg m$^{-3}$ density of aluminum
- $I_{zz} = \frac{bh^3}{12}$ is the moment of inertia of a rectangular section of a beam, second moment of area in the bending axis [m$^4$]
- $K_n$ is numerical constant associated with each mode of the Euler Bernoulli beam equation for a cantilevered beam, where $n$ is mode. Please use $K_1 = 3.52$, $K_2 = 22.0$, $K_3 = 61.7$, $K_4 = 121$, and $K_5 = 200$. You may treat $K_n$ here as unitless.
- $f_n = \frac{K_n}{2\pi} \left[\frac{EI}{mL^4}\right]^{1/2}$ [Hz], where $A$ is area, $E$ is Young’s modulus, $I$ is second moment of area in the bending axis, $m$ is mass, and $L$ is the length


2 Questions

There are 2 Space Systems tracks. Track 1 students selected Space Systems Engineering. Track 2 students selected Space Systems Architecture and Design.

- **All students should complete questions a) through e).**
- **Space Systems Engineering Students should also complete questions f), g), and h), in addition to a) through e).**
- **Space Systems Architecture and Design Students should complete questions i), j), and k), in addition to a) through e).**

**Problem parameters:** The CubeSat has a ‘3U’ form factor, with dimensions of 10 cm × 10 cm × 30 cm and mass of 5 kg. This particular CubeSat has two deployable panels that have a mirrored coating on the nadir face, and a deployable boom for the imaging system as shown in Figure 1. The CubeSat is deployed in Low Earth Orbit (LEO), from the International Space Station (ISS), which is in a nearly circular orbit with an altitude of ∼ 405 km and an inclination of ∼ 51.6°.

a) All Students: Draw a high level end-to-end system block diagram of a space to ground radio frequency communications link.

b) All Students: Discuss what latitudes are acceptable for the ground station to be located at for a spacecraft in the ISS orbit. Can a ground station be located on MIT campus?

c) All Students: What is the best case and worst case received power at the beginning of the mission at a ground station on Earth for the CubeSat, assuming the following properties: 2 Watts of transmit power at 401 MHz, CubeSat antenna gain of 0 dBi, and a ground station antenna gain of 16 dBi. Use an elevation cutoff of 10°. Assuming clear skies, what are reasonable estimates for atmospheric loss, pointing loss, and transmit and receive implementation loss?

d) All Students: Describe the Doppler shift for the frequency \( f \) of a signal being downlinked from the ISS orbit to ground. How could the Doppler shift affect your ability to communicate, and what could you do to maintain communications despite the shift?

e) All Students: What is the maximum and minimum Doppler shift for the CubeSat downlinking to the MIT campus ground station? You may assume that the minimum elevation cutoff is 0° for this part.

f) Space Systems Engineering Students Only: For the CubeSat, assume the MIT campus ground station has a system noise temperature of 790 K, and that the system must provide \( E_b/N_0 \) of 10 dB in order to be decoded effectively ‘error-free’. The modulation uses one bit per symbol. What data rates are achievable with the minimum and maximum received power from part d?
Figure 1: This figure is only intended to illustrate the deployed structure element, labeled ‘Primary Mirror’ here, for question 2g) and 2h).

g) Space Systems Engineering Students Only: The CubeSat has two deployed panels as shown in Figure 1. Assume the panels are formed from aluminum and rigidly fixed to the bus after deployment. Each panel is 30 cm long, 10 cm wide, and 5 mm thick. What is the mass of each panel? Calculate the first 5 natural frequencies for a deployed panel. What does this mean for the spacecraft attitude determination and control system, assuming the CubeSat uses reaction wheels that can spin between 0 rpm and +/-10,000 rpm?

h) Space Systems Engineering Students Only: Discuss other materials, dimensions, and approaches you might consider when fabricating the deployable panels that are intended for use as a reflective surface for imaging.

i) Systems Architecture and Design Students Only: NASA decides that they want to put the CubeSats in a 800 km circular orbit instead of 405 km for a longer mission lifetime, which means that their original data rate requirement is no longer met (to answer this part of the question, you do not have to calculate the updated data rate). Explain how you might use a Design Structure Matrix to assess the impact of this orbit altitude change on the plan for building the CubeSat and its subsystems.

j) Systems Architecture and Design Students Only: Assume the first CubeSat costs $2M to build, plus an additional $300k launch cost for a single 3U. If you built a 10-satellite constellation for a single orbit plane, discuss how much you think it would cost. What about a 100-satellite constellation for five orbit planes?

k) Systems Architecture and Design Students Only: What ‘ilities’ might come into play when designing for these larger constellations? What tools and approaches could you use to make architecture design decisions?