Air Breathing Propulsion Field Exam Question

In this exam we analyze a turbomachine in a duct (Question I) and then use the configuration as the basis for a propulsion system (Question II). Question I is worth 60% and Question II is worth 40%. There are three pages to the exam.

Question I. An annular duct with a turbomachinery stage and nozzle is shown below, along with station numbers, in Figure 1. The flow far upstream, at station 0, is uniform, steady, axial (i.e., in the x-direction) and parallel. The flow at station 3, at the exit of the turbomachine, is also axial and parallel, but has a non-uniform velocity magnitude, with velocity distribution,

$$u_{x3}(y) = \bar{u}\left[1 + \frac{\sigma y}{\Delta r}\right],$$

where $\bar{u}$ is the average axial velocity and Δr is the half-height (i.e., half radial extent) of the duct, with R the mean radius. The drawing is roughly to scale, with the mean radius constant throughout and Δr constant from nacelle inlet to the start of the nozzle, over an upstream and downstream distance of about six Δr upstream and downstream of the turbomachine .

To avoid algebraic complexity, you can take the flow to be incompressible, inviscid, and two dimensional.

A suggestion about examsmanship: There are a number of parts to this question, covering a range of concepts. If you have difficulty with any aspect, it may be useful to look through all of the questions to see which ones are most useful for you to focus on.

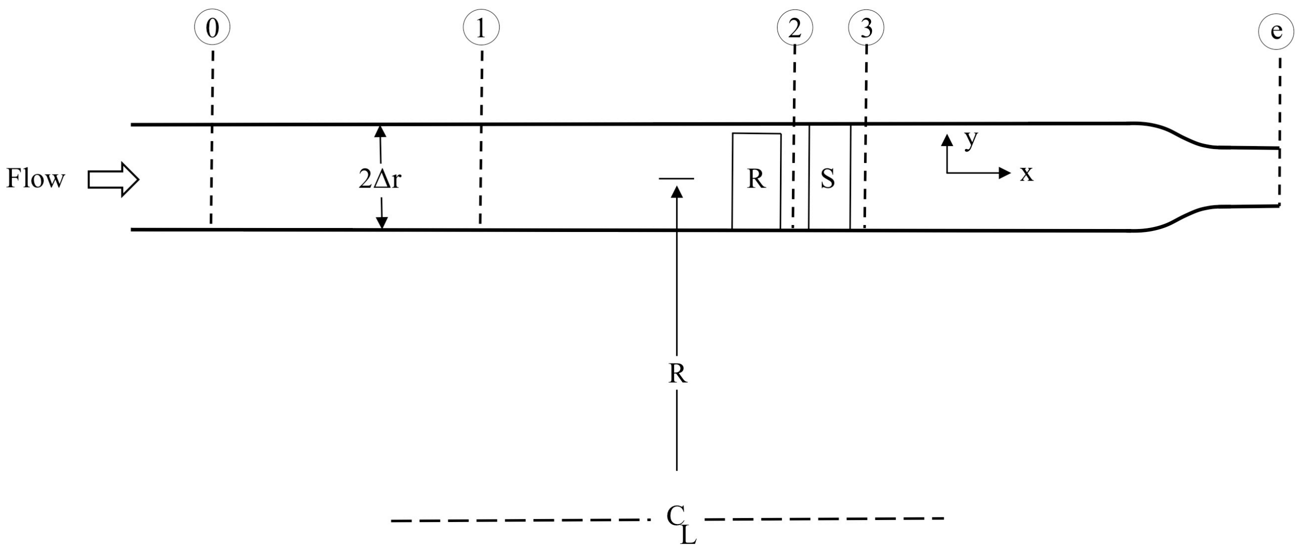

Figure 1. Turbomachinery and nozzle in annular duct with mean radius R

a) Does the difference $p_3 - p_0$, between the *static pressure* at station 3 and at station 0, for a given value of y, vary with y? If so, how?

b) Does the difference between the *stagnation pressure* at station 3 and at station 0, for a given value of y, vary with y? If so, how?

c) Given that the blade speed varies linearly with y from its mean value at y = 0 (where the radius is *R*), and assuming that the fluid particles do not change their radial position across the stator, what is the distribution of the absolute tangential velocity (i.e. the velocity out of the paper, as seen in the stationary coordinate system) at the exit of the *rotor* of the turbomachinery stage? Put your answer in a nondimensional form.

d) Is the rotor exit tangential velocity at the outer radius larger than that at the inner radius? A qualitative answer is looked for (yes, no, it depends), but the answer must be backed up by physical reasoning. Is there a limiting case that you have seen before?

e) Does a streamline at $y = 0$ far upstream move upwards or downwards from station 0 to the exit of the turbomachinery stage? Why?

f) Is there vorticity in the flow downstream of the turbomachine? If so, what is its distribution and which way does the vorticity vector point?

g) Downstream of the stage there is an exit nozzle of contraction area ratio, A_3 / A_e = CR > 1, as shown in Figure 1. What is the velocity difference across the nozzle (top to bottom) compared to that at the exit of the turbomachinery stage?

h) Does a streamline that is at $y = 0$ at station 3 move upwards, downwards, or stay on $y = 0$ as it goes through the nozzle? Why?

i) How would you find the axial force on the turbomachinery stage? (You do not have to find the force, but rather describe the steps in the approach to obtaining this information.)

<u>Question II</u>. We have examined the flow behavior in the context of a turbomachine in a duct. In Question II we wish to extend this to a more general application, namely as a propulsor on a vehicle. A sketch of a possible configuration is given in Figure 2, and a series of conceptual questions are given based on the device illustrated.

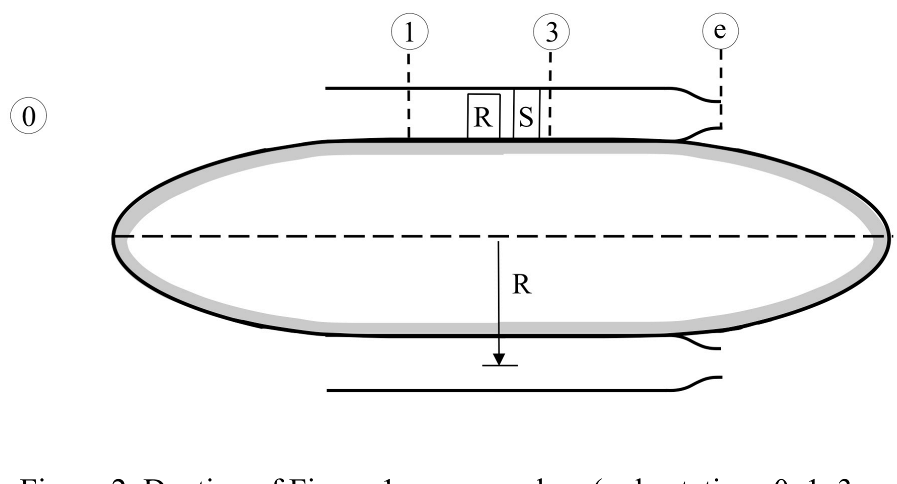

Figure 2. Ducting of Figure 1 as a propulsor (only stations 0, 1, 3, e are indicated)

a) Consider the device operating at near static conditions, say at a device velocity of 1 or 2 % of the turbomachinery wheel speed. Draw the streamlines.

b) Indicate in units of $\rho\bar{u}^2$ the levels of static pressure at the mean radius at stations 0, 1, 3, and e (far upstream, in duct, downstream of rotor, and nozzle exit), where $\bar{u}$ is the average velocity in the annulus upstream of the turbomachinery. Make whatever assumptions you think are relevant, but be prepared to justify them.

c) Consider the device moving at a velocity greater than (say twice) the turbomachinery wheel speed. Draw the streamlines.

d) Indicate in units of $\rho\bar{u}^2$ the levels of static pressure at the mean radius at stations 0, 1, 3, and e (from far upstream to nozzle exit), where $\bar{u}$ is the average velocity in the annulus upstream of the turbomachinery. Make whatever assumptions you think are relevant, but be prepared to justify them.

e) Suppose the turbomachinery has a pressure rise versus flow coefficient that looks as Figure 3, below. The operating point for the near static condition is indicated. Where would the high vehicle velocity operating point described in parts (c) and (d) be in relation to this?

f) Suppose we want the operating point to be the same for both conditions, but the turbomachine can only be operated at a fixed speed. How would we achieve this?

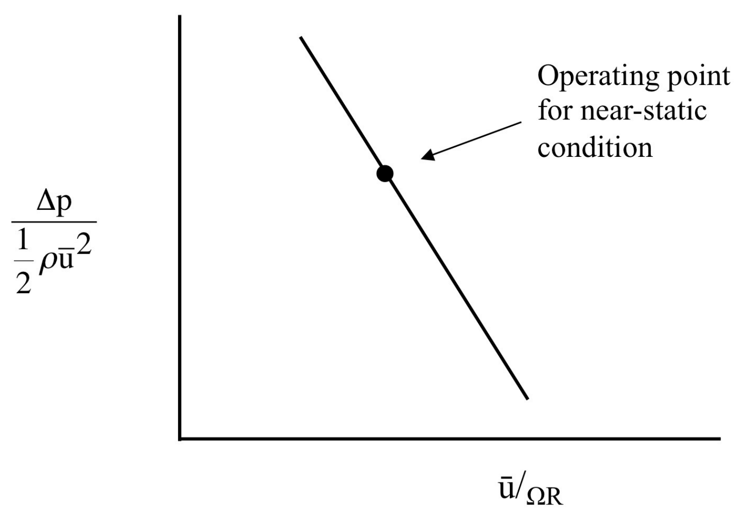

Figure 3. Turbomachinery (compressor) pressure rise versus flow characteristic curve