Field Exam — Autonomy

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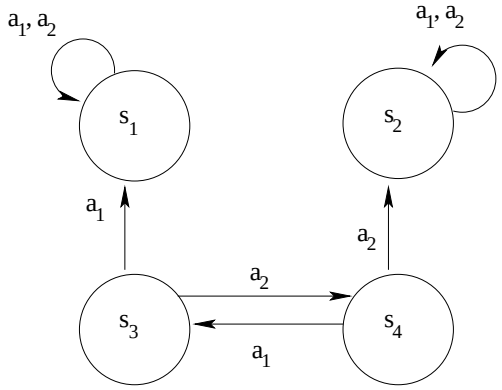
January 2015

All students are expected to answer all parts of both questions during the exam. As a guide to using your time wisely, you should plan to spend approximately 30 minutes of your preparation time on each of the two questions, and 20 minutes of the exam time on each of the two questions.

Question 1

Consider the four state MDP shown here:



Assume that all transitions are deterministic. Assume also that $\gamma = .9$.

1. If the reward for action a_1 is $+1$ everywhere, what is the value of $V^{\pi}(a_1)$?
2. Please construct a reward function such that value iteration converges in exactly three iterations, assuming the value function is initialized to $V(s_i) = 0$.
3. Please give the value function for each iteration.

Question 2

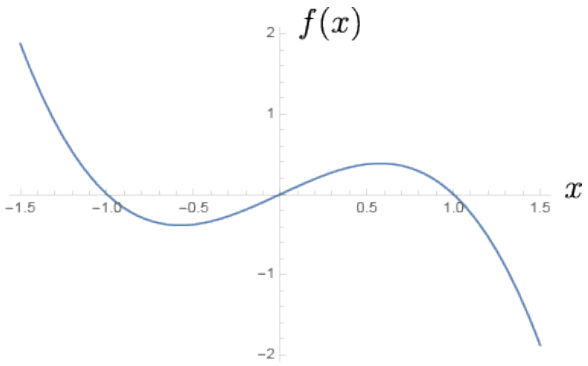
Consider a stochastic first-order system with linear dynamics and nonlinear measurement given by

$$\dot{x} = f(x) + w$$
$$y = x + v$$

where

$$f(x) = x - x^3$$

The process noise w and measurement noise v are independent, zero mean, Gaussian white noise processes with intensities W and R, respectively. A plot of $f(x)$ is below.



Consider three different initial states $x(0) = 0$, $x(0) = -1$, and $x(0) = 1$. Describe how you would develop a state estimator for this problem, and the problems you might encounter. You might consider some limiting cases:

1. $W \ll 1$, $R \ll W$
2. $W \ll 1$, $R \gg W$
3. $W \gg 1$, $R \ll W$
4. $W \gg 1$, $R \gg W$