Field Exam — Controls

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Question 1

Consider a continuous-time, LTI, single-input system described by the following state-space model:

$$\dot{x} = Ax + bu.$$

The matrix A has a single positive eigenvalue λ_1, all other eigenvalues have negative real part.

1. What is the minimum energy necessary to bring the system to rest at the origin, starting from initial condition $x(0) = x_0$? In other words, please compute the value of

   $$J_0^*(x_0) = \min_u \int_0^{+\infty} |u(t)|^2 \, dt$$

   subject to $\lim_{t \to +\infty} x(t) = 0$.

2. It is desired to stabilize the system with a digital computer implementing a sample-and-hold control strategy with sampling time Δ, while keeping the cost of the control system to a minimum. Write down a state-space model describing the evolution of the system between sampling times.

3. Considering that fast computers and strong actuators are expensive, it is desired to minimize a cost functional of the form

   $$J(x, u) = \frac{c J_0^*(x_0)}{\Delta} + \int_0^{+\infty} |u(t)|^2 \, dt.$$

   What is the optimal sampling time Δ, as a function of A, b, and c?

Question 2

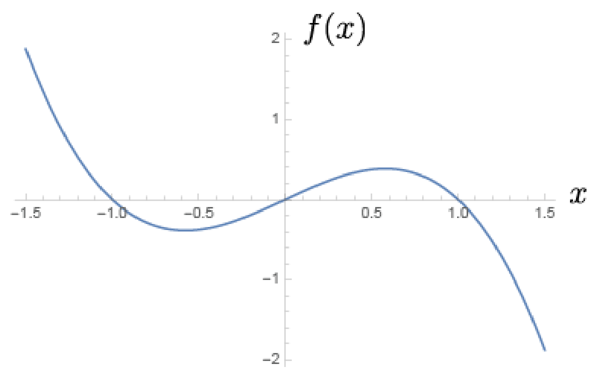
Consider a stochastic first-order system with linear dynamics and nonlinear measurement given by

$$\dot{x} = f(x) + w$$
$$y = x + v$$

where

$$f(x) = x - x^3$$

The process noise w and measurement noise v are independent, zero mean, Gaussian white noise processes with intensities W and R, respectively. A plot of $f(x)$ is below.



Consider three different initial states $x(0) = 0$, $x(0) = -1$, and $x(0) = 1$. Describe how you would develop a state estimator for this problem, and the problems you might encounter. You might consider some limiting cases:

1. $W \ll 1$, $R \ll W$
2. $W \ll 1$, $R \gg W$
3. $W \gg 1$, $R \ll W$
4. $W \gg 1$, $R \gg W$