

Field Exam — Controls

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January 2015

Question 1

Consider a continuous-time, LTI, single-input system described by the following state-space model:

$$\dot{x} = Ax + bu.$$

The matrix A has a single positive eigenvalue λ_1 , all other eigenvalues have negative real part.

1. What is the minimum energy necessary to bring the system to rest at the origin, starting from initial condition $x(0) = x_0$? In other words, please compute the value of

$$J_0^*(x_0) = \min_u \int_0^{+\infty} |u(t)|^2 dt$$

subject to $\lim_{t \rightarrow +\infty} x(t) = 0$.

2. It is desired to stabilize the system with a digital computer implementing a sample-and-hold control strategy with sampling time Δ , while keeping the cost of the control system to a minimum. Write down a state-space model describing the evolution of the system between sampling times.
3. Considering that fast computers and strong actuators are expensive, it is desired to minimize a cost functional of the form

$$J(x, u) = \frac{cJ_0^*(x_0)}{\Delta} + \int_0^{+\infty} |u(t)|^2 dt.$$

What is the optimal sampling time Δ , as a function of A , b , and c ?

Question 2

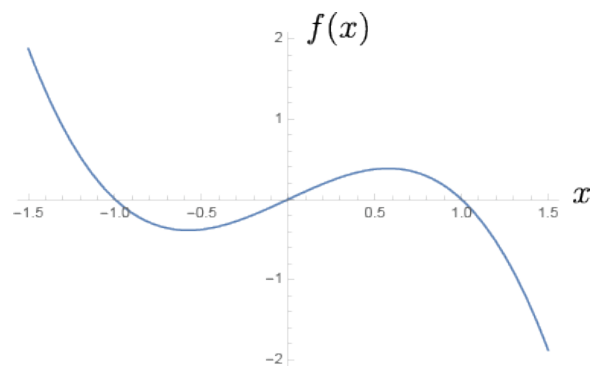
Consider a stochastic first-order system with linear dynamics and nonlinear measurement given by

$$\begin{aligned}\dot{x} &= f(x) + w \\ y &= x + v\end{aligned}$$

where

$$f(x) = x - x^3$$

The process noise w and measurement noise v are independent, zero mean, Gaussian white noise processes with intensities W and R , respectively. A plot of $f(x)$ is below.



Consider three different initial states $x(0) = 0$, $x(0) = -1$, and $x(0) = 1$. Describe how you would develop a state estimator for this problem, and the problems you might encounter. You might consider some limiting cases:

1. $W \ll 1, R \ll W$
2. $W \ll 1, R \gg W$
3. $W \gg 1, R \ll W$
4. $W \gg 1, R \gg W$