

Field Exam - Communications and Networks

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Part 1: Probability and Statistics

Let $X_1, X_2, \dots, X_n, X_{n+1}, X_{n+2}, \dots, X_{n+m}$ be i.i.d. random variables with the PDF:

$$f_{X_1}(x) = \begin{cases} 1, & 0 \leq x \leq 1, \\ 0, & \text{otherwise.} \end{cases}$$

Let $Y_1 = \sum_{i=1}^n -2 \ln X_i$ and $Y_2 = Y_1 / (-2 \sum_{i=1}^{n+m} \ln X_i)$.

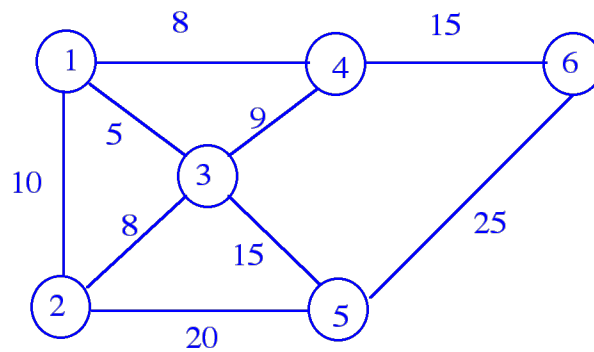
- (a) Find the PDF of Y_1 and Y_2 (either provide the expression of the PDF, or determine which probability distribution family they belong to and specify the parameters).
- (b) Show that Y_1 and Y_2 are independent.

Part 2: Networks

Variation on Bellman-Ford Algorithm

Suppose that the weights on the links in a network represent the link's capacity. We would like to devise an algorithm that finds paths with greatest end-to-end capacity where the capacity of the path is limited to the minimum capacity link along the path.

- A) How would you change the Bellman-Ford algorithm so that it can find the maximum capacity path from the source node to all other nodes? Clearly describe your new version of the algorithm including the initialization stage and the update equations¹.
- B) The graph below shows a network where the numbers associated with each edge represent the capacity of that edge. Use your algorithm to find the maximum capacity path from node 1 to all other nodes for this graph. At each iteration of your algorithm, show the path to each node and the respective capacity.



- C) What is the computation complexity of your algorithm?

¹ Reminder: Bellman-Ford shortest path algorithm: $d_{ij} = \infty$ if (i,j) is not an arc; $d_{ij} = 0$.
Let $D_i(h)$ be the shortest distance from 1 to i using at most h arcs. $D_i(1) = d_{1i}$; $i \neq 1$; $D_1(1) = 0$
For $h=1$ to $n-1$: $D_i(h+1) = \min_j [D_j(h) + d_{ji}]$; for all $i \neq 1$.