MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Department of Aeronautics and Astronautics

Field Exam in Space Propulsion

January - 2015

- There are two problems in this exam
- Read carefully each problem before writing your solution
- Make sure to state and be consistent with your assumptions
- Identify clearly your line of thought in your solutions
- Manage your time with care

Problem 1

A partially ionized gas from a plasma thruster is in contact with a non-emitting and catalytic wall. Assume vanishingly small Hall parameter, $\beta_e \ll 1$, steady state and collisional plasma. If no net charge is collected on the wall surface, then ions and electrons have to move with the same velocity towards the wall. This is called "ambipolar" diffusion.

(a) Given the large difference in ion and electron mobilities ($\mu_e \gg \mu_i$), how can both species be moving with the same velocity?

(b) In light of your explanation above, start from the momentum equations for ions and electrons and write expressions for their diffusion velocities (hint: $D/\mu = kT/e$, Einstein relation).

(c) Since diffusion velocities are the same for both charged species, prove that the ambipolar diffusion coefficient is $D_a \approx D_i$ away from the wall sheath (both charged species are seen as if diffusing like neutrals following concentration gradients). D_i is the **ion** diffusion coefficient.

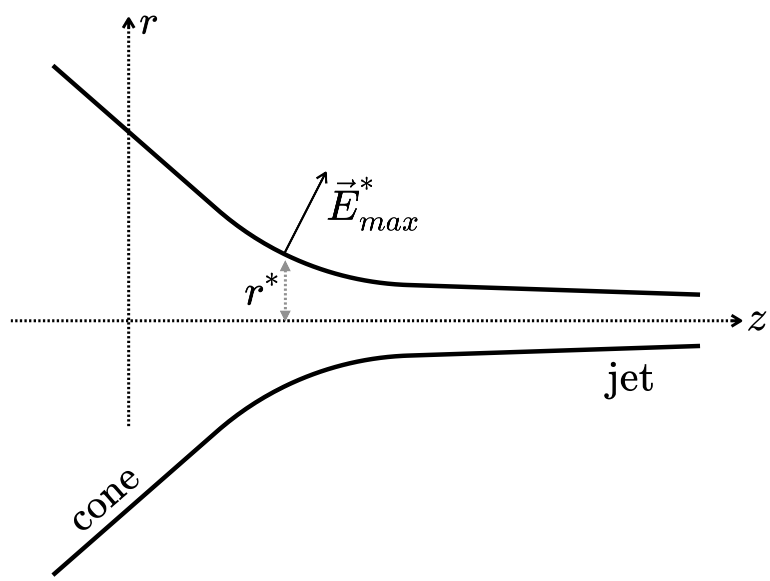
(d) Can you use your results above to derive an expression for the number density of electrons, n_e for the particular case of uniform electron temperature T_e?

Problem 2

Electrospray thrusters are capable of operating in different regimes where the specific impulse can change from very high (pure ion mode) to relatively low (pure droplet mode). Assume you have an electrospray source in which a Taylor cone transitions into a thin liquid jet carrying a current,

$$I \approx \sqrt{\gamma K Q}$$

where γ, K, Q are respectively the liquid surface tension, electric conductivity and volumetric flow rate. Assume the jet breaks up into droplets far away ($z \gg r^*$) from the Taylor cone-to-jet transition region.



(a) Write an expression for the characteristic dimension of the cone-to-jet transition region r^* in terms of the volumetric flow rate Q.

(b) Prove that the normal component of the electric field on the liquid surface has a maximum near this transition region and that it scales as,

$$E^*_{max} \approx \frac{\gamma^{1/2}}{\epsilon_0^{2/3}} \left(\frac{K}{Q}\right)^{1/6}$$

(c) Is it possible for the field on the transition region to be strong enough to induce ion evaporation? Elaborate with an example.