Minimizing Actuator-Induced Residual Error in Active Space Telescope Primary Mirrors

Aero/Astro Research Evaluation
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Advisor:
Prof. David W. Miller
• **Background**: Large telescopes

• **Motivation and Research Question**: Lightweight error-free mirrors

• **Literature Review**

• **Approach**: Parametric FE modeling

• **Results**: Rib shaping

• **Conclusion**
• Trend toward large aperture space telescopes
  – Higher resolving power
  – Increased light-gathering ability
  – Launch constraints drive new technology for segmented mirrors and *low areal density*

**Previous generation:**
Hubble Space Telescope (HST)  
- Material: Ultra low expansion (ULE) glass  
- Areal density: 180 kg/m$^2$  
- Diameter: 2.4 m  
- Shape control: passive  
- Actuators: none

**Current generation:**
James Webb Space Telescope (JWST)  
- Material: Beryllium  
- Areal density: ~30 kg/m$^2$  
- Diameter: 6.5 m (1.3 m segments)  
- Shape control: active (7 DOF)  
- Actuators: cryogenic stepper motors

**Next generation:**
“Highly Integrated” Mirror Design  
- Material: Silicon Carbide (SiC)  
- Areal density: <15 kg/m$^2$  
- Diameter: 1 m segments (baseline)  
- Shape control: active (100+ DOF)  
- Actuators: piezoelectric stacks
Motivation and Research Objective

- Low areal density requires active shape control
  - Correct quasi-static disturbances (thermal)
  - Correct manufacturing errors (SiC casting process)
- Problem: actuator-induced residual error
  - Symptom of discrete actuators commanding low-order shapes
  - Degrades image sharpness

Research Objective
To reduce actuator-induced high spatial frequency residual error by taking advantage of changes in mirror geometry using a parametric finite element mirror model while keeping areal density (mass) and number of actuators (power, complexity) constant.
**Relationship with Prior Work**

**Finite Element Mirror Modeling**
- Jordan [10] Temperature sensors

**Rib-stiffened Space Telescope Mirrors**
- Stahl [2,6] JWST segment development
- Ealey [3,4] Surface-parallel actuation, agile manufacturing
- Bikkannavar, et al. [16] Phase retrieval for deformable mirrors

**Shape Optimization to Decrease High Frequency Residual Error**
- Non-optimized mirror design: 225 nm RMS
- Optimized mirror design: 30.6 nm RMS

**SPOT**
Research Objective
To reduce actuator-induced high spatial frequency residual error by taking advantage of changes in mirror geometry using a parametric finite element mirror model while keeping areal density (mass) and number of actuators (power, complexity) constant.

- Model of primary mirror
  - Parameterization allows for rapid iteration

Parameter file
- Areal density [kg/m²]
- Number of actuators
- Rib shaping function
- Diameter [m]
- ...

Figure of merit
Residual error [nm RMS]

Finite Element Mirror Model

Final design

Iterate
Vary parameters of interest
• Mirror model details
  – Varying the rib shaping function

**Approach (cont.)**

**Input**
- Parametric rib shaping function

**Auto-construct FE model**

**Generate influence functions: **$H$

**Solve for commands**

$$Hu - z = 0$$

$$u = (H^T H)^{-1} H^T z$$

$z$: desired shape
$u$: commands

**MSC.Nastran**

**MATLAB**

**Figure of Merit**

Residual error [nm RMS]
• **Sinusoidal rib shaping**
  – Idea: spread the load/influence of a localized actuator further along a rib
  – Parameterize shaping function by amplitude $a$ ($0 \text{ mm} \leq a \leq 20 \text{ mm}$)
  – Areal density stays constant
Results

- Sinusoidal shaping function
  - $a = 12.5$ mm minimizes residual error (27% reduction compared to the baseline)
• Sinusoidal shaping function
  – Reason for improvement: “broader” influence function
  – Actuator becomes less “discrete”
Concluding

• Sinusoidal rib shaping
  – Rib shaping can reduce residual error
  – Mass is constant
  – Number of actuators is constant
  – $a = 12.5$ mm (best case) reduces residual by 27%
  – Broadens influence functions and “spreads” the effect of discrete actuators

Research Objective
To reduce actuator-induced high spatial frequency residual error by taking advantage of changes in mirror geometry using a parametric finite element mirror model while keeping areal density (mass) and number of actuators (power, complexity) constant.
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• Path ahead
  – Rib shaping: Explore additional “basis functions” for ribs and optimize for best combination (e.g. series of cosines)
  – Rib blending: Blend the ribs smoothly into the facesheet, instead of the current 90° junction
  – General shape optimization: Using parameterized rib shaping and blending functions, optimize to find mirror shape that best reduces actuator-induced residual, while meeting stiffness requirements
Conclusion (cont.)

Research Objective
To reduce actuator-induced high spatial frequency residual error by taking advantage of changes in mirror geometry using a parametric finite element mirror model while keeping areal density (mass) and number of actuators (power, complexity) constant.

• Expected contributions
  – Set of optimized mirror shapes that reduce actuator-induced residual error (constant mass and constant number of actuators)
  – New approach to mirror design: start with actuator specification, then design mirror
  – Compatible with existing manufacturing techniques
References

[1] National Aeronautics and Space Administration, image archive.
Backup
Model Validation

- Convergence
  - Sensitivity of model outputs to FE mesh density
  - Does the model go to one value as mesh density changes?

![Modal Frequency vs. Mesh Density](chart1)

![Residual vs. Mesh Density](chart2)
• Benchmarking against empirical data
  – Does that one value match real life?

<table>
<thead>
<tr>
<th>Model output</th>
<th>Error compared to empirical data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stiffness (fundamental frequency)</td>
<td>2%</td>
</tr>
<tr>
<td>Mirror stress due to launch vibration</td>
<td>8%</td>
</tr>
<tr>
<td>Residual error for 1 mm ΔROC</td>
<td>7%</td>
</tr>
</tbody>
</table>
Model Validation (cont.)

- Feature-level validation
  - Validate individual features of the model for which there is no “global” test data
  - Compare with first-principles models, published test results, published analysis
  - E.g.: modeling and validation of piezoelectric actuators in [7,9]
  - E.g.: actuator length validation via analytical beam model (below)

\[
M_i = -k_i \theta = -k_i \frac{dy}{dx}
\]

\[
y(x) = \frac{M_i}{2EI} x^2 - \frac{M}{2EI} \left( x - \frac{L - l_{act}}{2} \right)^2 + \frac{M}{2EI} \left( x - \frac{L + l_{act}}{2} \right)^2 + \frac{M l_{act} - M_i L}{2EI} x
\]

\[
M_i = \frac{M l_{act}}{L + EI/k_i}
\]

\[
\langle x - x_0 \rangle^2 = \begin{cases} 
0 & x < x_0 \\
(x - x_0)^2 & x \geq x_0 
\end{cases}
\]
**Baseline Mirror**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter (flat-flat)</td>
<td>1.0 m</td>
</tr>
<tr>
<td>Areal density (SiC only)</td>
<td>8 kg/m²</td>
</tr>
<tr>
<td>Rib rings</td>
<td>4</td>
</tr>
<tr>
<td>Primary rib height</td>
<td>25.4 mm</td>
</tr>
<tr>
<td>Primary rib thickness</td>
<td>1 mm</td>
</tr>
<tr>
<td>Face sheet thickness</td>
<td>1.8 mm</td>
</tr>
<tr>
<td>Actuator length</td>
<td>7.2 cm</td>
</tr>
<tr>
<td>Actuator length/cell</td>
<td>50%</td>
</tr>
<tr>
<td>Number of actuators</td>
<td>156</td>
</tr>
<tr>
<td>Radius of curvature</td>
<td>6 m</td>
</tr>
</tbody>
</table>

![Diagram of Baseline Mirror with details such as quadrilateral rib elements and triangular facesheet elements.](image-url)
Wavefront Sensing Methods

- **Shack-Hartmann Wavefront Sensor**
  - Detects the slope of the wavefront by measuring spot displacements
  - Measures the wavefront at discrete locations on the pupil
  - Ignores non-common path errors

- **Modified Gerchberg-Saxton (MGS)**
  - Computes wavefront by inverse transforming a defocused point image
  - Uses the focal plane array sampling (more of a continuous map)
  - FT and IFT between image and pupil, enforcing pupil plane image constraints
  - Known defocus removes ambiguity in phase map
  - Captures non-common path errors
  - Baseline method for fine phasing in JWST
Finite Element Method

- **Static Analysis**
  - **Goal**: Compute displacements from known forces and constraints
  - Relate forces to displacements via a global stiffness matrix $K$
  - Build the stiffness matrix from kinematic relationships, Hooke's Law, and force balance

\[ \varepsilon = Au \]
\[ \sigma = E\varepsilon \]
\[ f = A^T \sigma \]

Horizontal and vertical displacements of node $i$
Horizontal and vertical forces acting at node $i$
Elastic modulus of element $j$

Elongations (strain) from displacements
Internal force (stress) from elongation (strain)
Force balance (internal and external)

Force from displacements (combining above equations)

\[ f = A^T EAu \]
\[ f = Ku \]  

(1)

Finite Element Method (cont.)

• Normal Modes Analysis
  – **Goal**: Compute the normal frequencies and normal modes of a system
  – Transform into an eigenvalue problem

\[ M \ddot{u} + K u = 0 \]
\[ u = \phi \sin \omega t \]
\[ (K - \omega^2 M) \phi = 0 \quad (2) \]

Last line is an eigenvalue equation of the form \( A\lambda = \lambda x \)

- Solve for normal frequencies (eigenvalues) and normal modes (eigenvectors)

Imaging Consequences

- Impact of residual error on imaging
  - Residual error causes wavefront error \((WFE = 2 \times \text{residual error})\)
  - This adds an additional phase term to the pupil function
  - Result: degraded image quality via reduced peak energy and image artifacts

\[
\phi \text{ [rad]} = -2 \times \frac{2\pi}{\lambda} \times f(x,y) \text{ [nm]}
\]
Previous Geometry Results

• Effect of geometric parameters on residual error
  – Number of actuators (rib rings)
  – Actuator length
  – Areal density
  – Rib height
• Changing actuator length
  – Longer actuators give better performance
  – Increasing actuator length increases the distance between the applied moments, smoothing the commanded shape
• Changing actuator length
  – Longer actuators give better performance (3.6x improvement over the baseline)
  – Increasing actuator length increases the distance between the applied moments, smoothing the commanded shape
• Beam model
  – Single rib cell modeled; actuator modeled as a moment couple
  – As space between couple increases, more of the beam experiences parabolic deflection
  – Influence function becomes less localized

\[
y(x) = \frac{M_0}{2EI} \left[ x - \frac{a + l_{act}}{2} \right]^2 - \frac{M_0}{2EI} \left[ x - \frac{a - l_{act}}{2} \right]^2 + \frac{M_0 l_{act}}{EI} \left( 1 - \frac{a}{2L} \right) x
\]

\[
\langle x - x_0 \rangle^2 = \begin{cases} 
0 & x < x_0 \\
(x - x_0)^2 & x \geq x_0 
\end{cases}
\]
Initial Blending Results

- Rib-to-facesheet blending
  - Idea: smooth the transition from rib to facesheet spread the influence of a given actuator
  - Initial implementation: Use 2D shell elements but vary the thickness to approximate 3D blending

- Implementation details:
Initial Blending Results (cont.)

- **Rib-to-facesheet blending**
  - Idea: smooth the transition from rib to facesheet spread the influence of a given actuator
  - Initial implementation: Use 2D shell elements but vary the thickness to approximate 3D blending

- **Results:**
  - Increases the “vertical” extent of the actuator’s influence function
• 2D patch actuator
  – Idea: use a thin piezoelectric patch and take advantage of the $d_{31}$ behavior
  – Place within the cells to add another set of influence functions
  – Initial implementation: Use 2D shell elements attached at nodes

• Results
  – Well-defined influence function within the rib cell
• **Spherical Primary Optical Telescope**
  – NASA Goddard
  – Single segment test bed

<table>
<thead>
<tr>
<th>Figure shape</th>
<th>sphere</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter</td>
<td>0.876 m</td>
</tr>
<tr>
<td>RoC</td>
<td>5 m (nominal)</td>
</tr>
<tr>
<td>Max surface error for 400 µm ΔRoC</td>
<td>15 nm RMS</td>
</tr>
<tr>
<td>Material</td>
<td>Pyrex</td>
</tr>
<tr>
<td>Minimum areal density (10 mm thickness throughout)</td>
<td>22.3 kg/m² (Pyrex only)</td>
</tr>
</tbody>
</table>

Select basis functions
\[ f(r) = a_1 r + a_2 r^2 + a_3 r^3 + \ldots \]
\[ g(\theta) = b_1 \sin(6\theta) + b_2 \sin(12\theta) \]
Optimize to find coefficients

Residual error:
225 nm RMS

Residual in non-optimized mirror

Residual error:
225 nm RMS

Residual error:
30.6 nm RMS

Optimized shape

• Past MOST results
  – Parametric space telescope model
    • Primary mirror and kinematic mounts
    • Secondary mirror and support tower
    • Solar arrays and bus
  – Allows rapid tradespace exploration
  – Compare designs and identify driving parameters