Partnership for Air Transportation
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Assessment of the effects of operational procedures and derated thrust on American Airlines B777 emissions from London's Heathrow and Gatwick airports

prepared by
Mr. Daniel King
Professor Ian A. Waitz

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PARTNER
37-311, Massachusetts Institute of Technology, 77 Massachusetts Ave., Cambridge, MA 02139 USA
http://www.partner.aero • info@partner.aero • 01-617-253-4929

1. Summary

We were asked by American Airlines (AA) to assess the effects of operational procedures and derated or reduced thrust (collectively called thrust derate in this report)[1] on emissions of oxides of nitrogen (NOx), carbon monoxide (CO), and unburned hydrocarbons (HC) for Boeing 777 (B777) departures from London's Heathrow Airport (LHR) and London's Gatwick Airport (LGW). To enable us to perform this assessment, American Airlines provided computerized flight data recorder (CFDR) information for 36 B777 flights from LHR and LGW with various levels of derate, aircraft weight, and atmospheric conditions. The aircraft all employed Rolls-Royce Trent 892 engines.

International Civil Aviation Organization (ICAO) emissions certification data, including emissions indices (EI), times-in-mode, throttle settings and fuel flow are frequently used to estimate aircraft emissions relevant for local air quality. Such ICAO-based emission calculations are independent of pilot operational procedures, such as thrust derate, noise flight profiles, weight of the aircraft and atmospheric conditions. In particular, the methods for computing the emissions assume full-power takeoffs and fixed times spent at different throttle settings. American Airlines B777 aircraft departing LHR and LGW employ an average derate of approximately 20% and also spend less time below 3000 feet than assumed in the ICAO certification methods. The takeoff-NOx we calculated using the flight recorder data is *50.7%* less than that computed using the ICAO Emissions Indices (EI), times-in-mode, and fuel flow for Trent 892 engines.

In addition to comparing emissions estimates derived from CFDR times-in-mode and throttle settings to ICAO-based estimates, we also performed an analysis to determine the relationship between thrust derate and emissions of NOx, CO, CO_2 and HC. We did this by simulating the emissions that would have been produced if the aircraft had been flown at full power and comparing this to the emissions estimated for derated take-offs. We found that thrust derate accounts for an average *14.5%* NOx reduction from a full-power takeoff for the distribution of derate percentages employed by the AA B777's departing LHR and LGW. For each 1% of derate approximately 0.7% reduction in NOx below 3000 ft is estimated. Also, for each 1% of derate, fuel burn below 3000 ft (and hence CO_2) increases by 0.6%. Small changes in CO and HC are also estimated in the report. However, the changes are negligible due to the small contribution of take-off and climb emissions to the overall landing-take-off cycle (LTO) emissions of CO and UHC.

The distribution of thrust derates, aircraft weight and atmospheric conditions in the data sample we analyzed are typical of AA B777 operations from LHR and LGW. Therefore, the NOx emissions calculated from the flight data are expected to be representative of those that would be obtained if a larger sample were considered. These results are specific to AA B777 operations from LHR and LGW and should not be generalized to other aircraft types, airports or airlines.

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[1] Derated and reduced thrust are both certified thrust ratings that are less than 100% takeoff thrust, or full power. Although defined differently, they both result in a reduction in thrust for takeoff. For the B777, derated thrust can be up to 15% and reduced thrust can be an additional 25% less than 100% takeoff thrust.

2. ICAO Emissions Calculations

ICAO emissions calculations are based on ICAO emissions certification data that are available for all aircraft engines rated greater than 26 kN. Emissions calculations are intended to cover the landing-takeoff (LTO) cycle, which includes operations below the mixing height, generally assumed to be 3,000 ft altitude above ground level (AGL), although the true mixing height varies from airport to airport and seasonally. NOx, HC, CO, and fuel flow are reported for takeoff (TO), climb-out (C/O), approach (AP), and taxi (TX) engine throttle settings. For calculating total emissions, the ICAO method assumes times-in-mode and engine settings for each segment as shown in Table 1. Test data are corrected to standard sea level static conditions. Engine settings are assumed to represent actual flight setting, irrespective of the aircraft type, pilot procedures, or atmospheric conditions.

Table 1: ICAO LTO Cycle

Segment	Throttle Setting	Time-In-Mode (min)
Takeoff	100%	0.7
Climb	85%	2.2
Approach	30%	4
Taxi	7%	26

ICAO times-in-mode were set in the 1980s based on flight data from the 1970s. While the test data can be assumed to be accurate at the thrust levels specified, the times-in-mode do not reflect typical flight profiles as observed in CFDR data for many aircraft. Likewise, the throttle settings do not always reflect those actually used. This is particularly important because a derated thrust, rather than 100% takeoff thrust, is most commonly used. A study of LTO cycle emissions by Unique and Swiss Flight Data Monitoring, based on thousands of flights into and out of Zurich, found that times-in-mode and thrust settings are significantly different from the ICAO assumptions [2].

Therefore, while ICAO standard emissions calculations are useful as a consistent and long-term certification benchmark for engines performance, they are not accurate for calculating emissions from aircraft in operation. More accurate methods for calculating emissions, such as Boeing Method 2, employ the ICAO certification data, but correcting for atmospheric and flight conditions. ICAO emissions and fuel flow data for the Trent 892 engine are shown in Table 2 below.

Table 2: Trent 892 ICAO Certification Data

Segment	TO	C/O	AP	TX
EI NOx (g/kg fuel)	45.70	33.30	11.58	5.33
EI HC (g/kg fuel)	0.01	0.00	0.00	0.70
EI CO (g/kg fuel)	0.28	0.20	0.57	13.07
Fuel Flow (kg/s)	3.91	3.10	1.00	0.30
Time-in-mode (min)	0.70	2.20	4.00	26.00

3. Boeing Method 2

Boeing Method 2 (BM2), or the ”Boeing curve fitting method,” calculates emissions indices based on fuel flow and ICAO certification data. ICAO data at the four certified power settings at sea-level static (SLS) conditions are used to compute resulting emissions of the full range of power settings while correcting for atmospheric conditions. BM2 is accepted and widely used for calculating flight emissions. ICAO's Committee on Aviation Environmental Protection (CAEP) found that the Boeing curve fitting method is acceptable for calculating emissions [5]. For this analysis the following standard BM2 calculation procedures were followed as detailed by Boeing [1]:

1. Calculate ICAO fuel flow corrected for engine bleed and installation effects.
2. Plot ICAO EI at each power setting versus corrected fuel flow on a log-log scale.
3. For NOx, fit 3 lines between 7% and 30%, 30% and 85%, 85% and 100% power setting points on the EI versus corrected fuel flow plot. For CO and HC, fit two lines between the 7% and 30%, and 85% and 100%, and extrapolate those lines to where they meet for a bilinear fit.
4. Correct actual fuel flow at the condition of interest for pressure, temperature, density, Mach number, and humidity to find fuel flow in the reference SLS condition.
5. Calculate EI at the reference condition using the reference fuel flow from step 4 and the curve fits from step 3.
6. Correct the reference condition EI to the flight condition of interest using pressure, temperature, density, humidity, and Mach number.
7. Total emissions = actual fuel flow times EI from step 6 times # engines times time-in-mode.

For the NOx, CO, and HC curve fits between EI and corrected fuel flow, linear fits of the data plotted on a log-log scale were used as recommended by Boeing [1]. The NOx fit is also consistent with the method used by CAEP 6 in Reference [5] (References [5] and [1] do not agree on the CO and HC fitting methods, though [5] references [1] as the source of its method. We have confirmed our methods in conversations with Doug DuBois of Boeing. ICAO fuel flows values were corrected for engine bleeds and installation effects by multiplication by the correction factors used in SAGE, the FAA System for Assessing Aviation's Global Emissions. Fuel flow correction factors are 1.1, 1.02, 1.013, and 1.01 for Takeoff, Climb, Approach, and Taxi, respectively [3]. 60% relative humidity was assumed.

4. BADA Models

While CFDR recorded fuel flow was used for the ICAO emissions comparison, the Base of Aircraft Data (BADA) was used for the aerodynamic and fuel flow models for the derate versus full-power analysis. This provided a consistent basis for comparing the two modes of flight. BADA was developed by Eurocontrol primarily for air traffic control (ATC) simulations. The BADA drag model gives drag as a function of airspeed and C_L, and is used to calculate thrust used in the derated flight, and subsequently the full-power thrust.

BADA v3.5 data exist for 87 aircraft types giving operational and performance coefficients for each. BADA v3.5 provides a single set of data for the B777 and does not distinguish between the performance of the three engine types. In addition to the fuel flow and drag models used for this analysis, a thrust model, operational speeds, and nominal flight performance data are also available [4]. The BADA drag model for a B777 in takeoff mode with the gear up is shown in Equation 1. C_L is obtained from the weight and climb angle by assuming steady flight ($L = W \cos \gamma$).

$$C_D = C_{Do} + kC_L^2; \quad C_{Do} = 0.0215; \quad k = 0.0453$$

$$D = qS\left(C_{Do} + kC_L^2\right) \tag{1}$$

The BADA fuel flow model calculates the specific fuel consumption (SFC) as a function of flight speed and altitude, and returns fuel flow as a function of SFC and thrust in kg/s. The fuel flow model for altitudes below 7,500 feet was used for this analysis and is shown in Equation 2.

$$CFL1 = 0.75416; \quad CFL2 = 3972.3$$

$$ff = \frac{CFL1\left(1 + 1.9438\dfrac{V}{CFL2}\right)}{60 \cdot 1000} F \tag{2}$$

5. Flight Data Analyzed

Table 3 shows a full listing of the flights we analyzed. These were selected to provide a range of derate levels, aircraft weights (GTOW) and atmospheric conditions. The GTOW average in Table 3 is 236,190 kg, which is 19.6% less than the Maximum Allowed GTOW of 293,930 kg. This is significant since GTOW is less than Maximum Allowed even when derate thrust is almost zero. Because of the reduced weight, the aircraft climb faster at a given thrust leading to a reduced time-in-mode during climb to 3000 ft. NOx emissions will be less than the ICAO value since time-in-mode will be less at the lower weight. Although not presented in this report, we expect a strong correlation between GTOW and emissions produced.

With a statistically small sample of 36 flights it is important to assess whether the flights we analyzed are representative of typical AA B777 operations from LHR and LGW. Since the sample of flights we analyzed employs a similar distribution of derates and was obtained for a range of take-off weights and atmospheric conditions we have some confidence that this is the case. In Figure 1, we compare the distribution of derates within the sample to those for 3000 flights for LHR and 600 flights for LGW during a 7 month period. The distribution of the sample falls between the distributions for LHR and LGW. The means and standard deviations are similar as reported on the chart. Table 4 shows tallies of monthly average derate employed by AA

B777's departing LHR and LGW. From this it is apparent that there is little variation from month-to-month at either LHR or LGW.

The samples span a range of conditions that we expect is representative of the conditions for yearly AA operations of B777 aircraft at these two airports. Thus the results are expected to reflect typical average emissions for this aircraft type at these two airports.

Table 3: Flights Analyzed

Month	Dep. Sta.	Actual EPR	Max EPR	Derate %	GTOW (kg)	Pressure Alt. (ft)	T (K)	Winds aloft (m/s)
2	LGW	1.497	1.502	0.5	245888	-174	278	15
3	LGW	1.381	1.484	12.6	251113	-412	277	4
3	LGW	1.333	1.49	20.09	241678	-398	278	4
5	LGW	1.362	1.497	16.42	239646	495	283	8
5	LGW	1.357	1.499	17.3	238630	508	283	9
5	LGW	1.345	1.499	19.04	233404	501	285	6
5	LGW	1.33	1.496	21.03	236743	341	288	5
5	LGW	1.303	1.498	25.39	224840	-225	292	2
6	LGW	1.374	1.493	14.39	246468	-30	292	9
6	LGW	1.367	1.49	15.1	242839	-168	290	13
7	LGW	1.488	1.492	0.42	249807	211	288	13
7	LGW	1.435	1.49	6.18	243855	298	289	13
7	LGW	1.388	1.494	12.56	251113	138	292	11
1	LHR	1.363	1.495	16.1	248500	682	281	17
1	LHR	1.363	1.501	16.61	245307	651	279	10
1	LHR	1.311	1.509	24.95	223244	122	275	12
2	LHR	1.327	1.489	20.92	237033	-413	279	10
2	LHR	1.298	1.493	25.82	226873	-268	279	13
2	LHR	1.277	1.493	29.29	214099	-391	284	6
3	LHR	1.337	1.496	19.98	242404	42	282	17
4	LHR	1.333	1.498	20.74	234421	-69	280	7
4	LHR	1.29	1.503	27.86	219615	180	289	7
5	LHR	1.373	1.496	14.79	257209	-119	288	9
5	LHR	1.37	1.496	15.2	248645	1037	281	13
5	LHR	1.333	1.493	20.34	241968	-181	285	5
5	LHR	1.309	1.505	24.97	220196	958	283	9
5	LHR	1.298	1.493	25.82	225566	-174	295	6
5	LHR	1.281	1.517	30.28	205971	377	286	8
5	LHR	1.259	1.502	32.98	202923	-172	295	4
6	LHR	1.486	1.492	0.62	232969	625	289	20
6	LHR	1.339	1.491	19.27	246178	-42	293	10
7	LHR	1.348	1.503	18.92	244001	70	294	8
7	LHR	1.32	1.503	19.17	245017	46	287	5
7	LHR	1.33	1.499	21.27	234856	-44	287	7
7	LHR	1.32	1.503	23.11	230211	129	288	8
7	LHR	1.31	1.5	24.44	229631	45	294	12

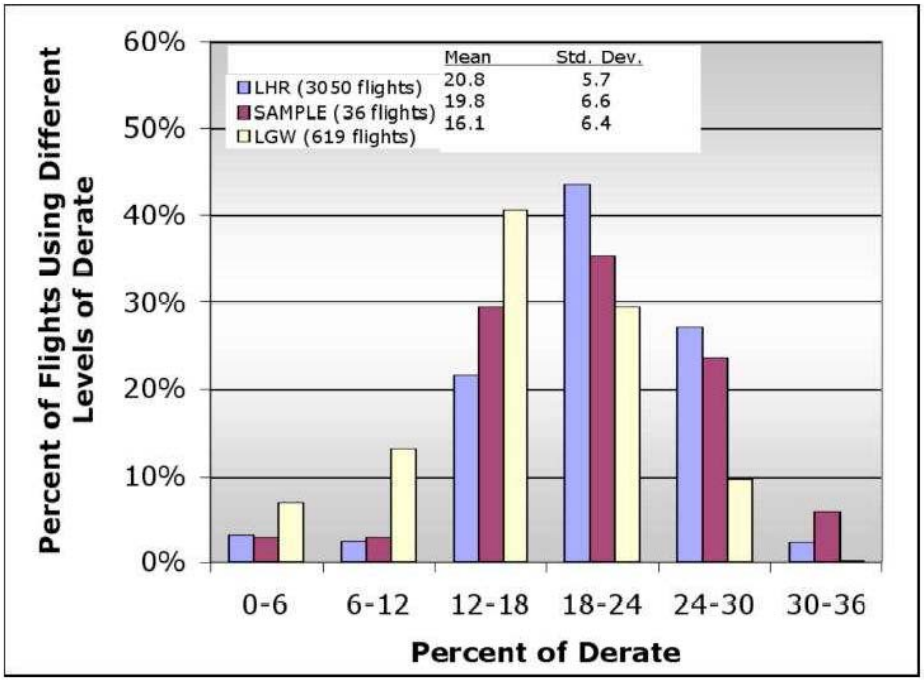

Figure 1: Comparison of derate distribution for sample flights and all B777 flights from LHR and LGW for January-July 2004.

Table 4: LHR and LGW B777 Average Takeoff Derated Thrust Reduction

LHR

Month	Act. EPR	Max EPR	Derate %
JAN	1.338	1.499	20.466
FEB	1.325	1.495	22.099
MAR	1.339	1.494	19.868
APR	1.340	1.498	19.834
MAY	1.327	1.497	21.782
JUN	1.329	1.497	21.512
JUL	1.340	1.497	19.891
TOTAL	1.334	1.497	20.773

Table 4, continued…

LGW

Month	Act. EPR	Max EPR	Derate %
JAN	1.365	1.499	16.550
FEB	1.352	1.495	18.009
MAR	1.369	1.494	15.569
APR	1.368	1.498	15.933
MAY	1.353	1.498	17.972
JUN	1.371	1.496	15.215
JUL	1.380	1.495	13.953
TOTAL	1.366	1.496	16.124

6. ICAO Comparison Analysis Approach

Fuel flow, time, altitude, ambient condition, and flight speed for all flights were used as inputs to BM2 to compute the emissions for each flight's takeoff and climb-out. EI's for NOx, HC, and CO were corrected for flight speed and atmospheric conditions based on the fuel flow reported in the data.

Takeoff and climb-out emissions were calculated using all data points below 3,000 ft, from the first point available in the data. The start of the takeoff roll is not clearly defined for some flights where it appears that the aircraft transitioned from taxi to takeoff without stopping. Reviewing the data, it appears that when 15 knots of ground speed is reached or an Engine Pressure Ratio of 1.2 is reached, the aircraft is clearly in takeoff mode. At these points, engine throttle is near the maximum reached for each flight, and acceleration has reached takeoff acceleration. These points are reached by an average of 16 to 17 seconds after the start of the data. This means that there is an average of 16-17 seconds of pre-takeoff, or transition time included in the analysis.

ICAO emissions were calculated for the entire LTO cycle and separately for takeoff and climb-out only. For each LTO segment: takeoff (TO), climb-out (C/O), approach (AP), and taxi (TX); ICAO certification data includes fuel flow (ff), time-in-mode (t), and emissions indices (EI) for each emission. LTO NOx is calculated as follows:

$$LTO\_NO_x = \sum_{m=TO,C/O,AP,TX} EI_{NO_{x,m}} \times ff_m \times t_m \qquad (3)$$

HC and CO were calculated in the same manner with their respective EIs for each segment. Because the data were only for takeoff and climb-out, ICAO emissions for TO and C/O were calculated separately from those for approach and taxi. Takeoff and climb-out ICAO emissions were compared directly to emissions calculated from the data.

7. ICAO Comparison Results

As shown by the first bar in Figure 2, ICAO NOx for the full LTO profile was 53 kg, and 42 kg for only the takeoff and climb-out segments (second bar). NOx was calculated for 33 flights where derated thrust was used (3 flights with less than 1% derate were considered to be full power). The average NOx (third bar) calculated for takeoff and climb-out below 3,000 ft was 20.8 kg for the 33 derated thrust data sets, which is *50.7%* less than the ICAO calculated value for takeoff and climb-out.

CO and HC increase at lower thrust levels. This can be seen in the ICAO emissions indices for the B777 which are negligible for takeoff and climb-out compared to approach and taxi for CO, and only significant in taxi for HC. ICAO CO was 13 kg for the full LTO cycle, and 0.26 kg for only takeoff and climb-out. ICAO HC was 0.66 kg for the full LTO cycle and 0.003 kg for only takeoff and climb-out. Takeoff and climb-out CO and HC from the data remain small for derated thrust takeoffs with an average value of 0.19 kg of CO and 0.003 kg of HC.

Figures 3 and 4 shows similar plots for CO and HC respectively. ICAO CO and HC for the full LTO cycle are again the first bars, ICAO takeoff and climb-out only, the second bar, and the average of all flights, the third bar. The remaining bars are again the totals produced for each derated flight analyzed. The CO and HC totals produced for each flight are very small compared to LTO cycle totals reflecting that takeoff has very little impact on total CO and HC emissions near airports.

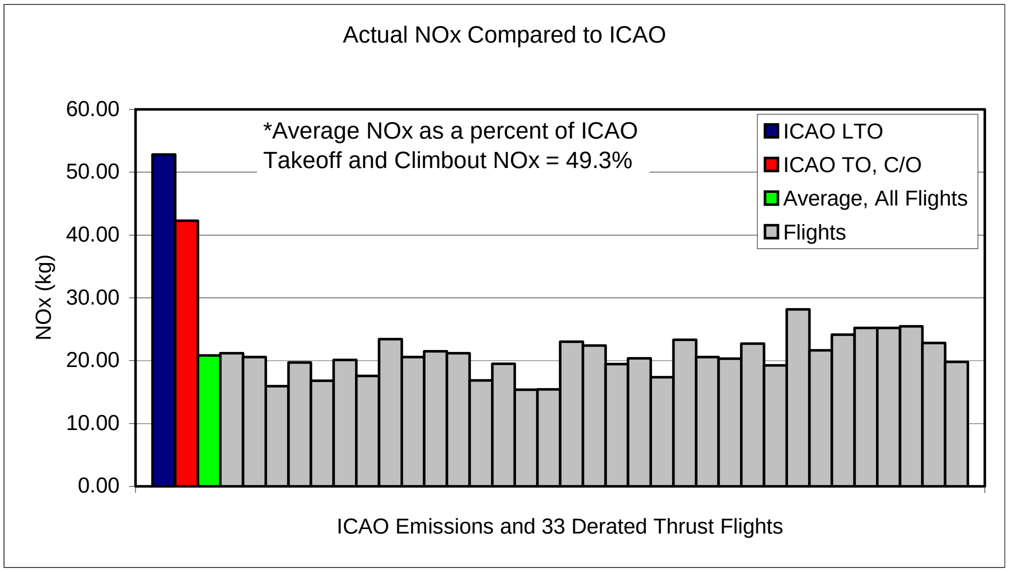

Figure 2: ICAO LTO cycle and ICAO takeoff and climb-out NOx versus actual takeoff and climb-out emissions for 33 derated thrust takeoffs.

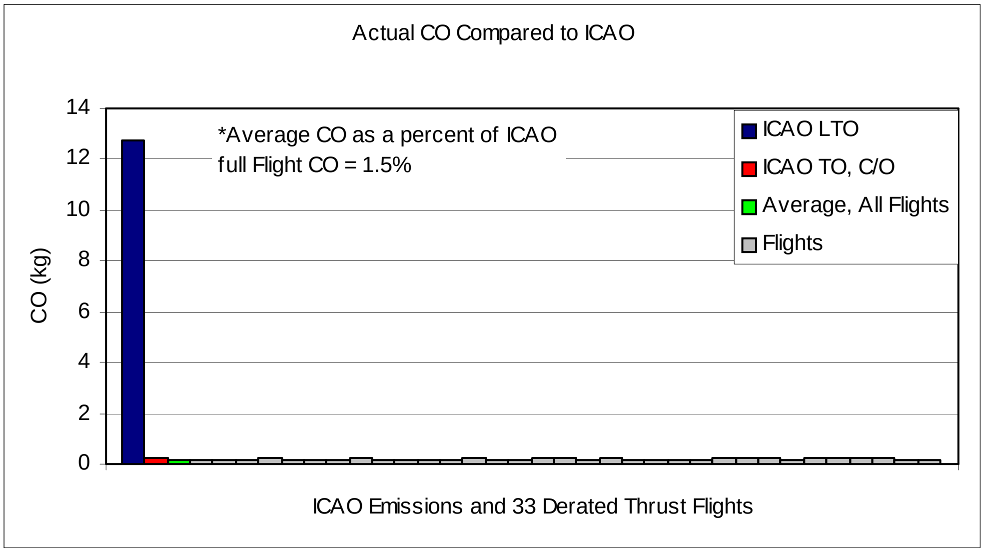

Figure 3: ICAO LTO cycle and ICAO takeoff and climb-out CO versus actual takeoff and climb-out emissions for derated thrust takeoffs.

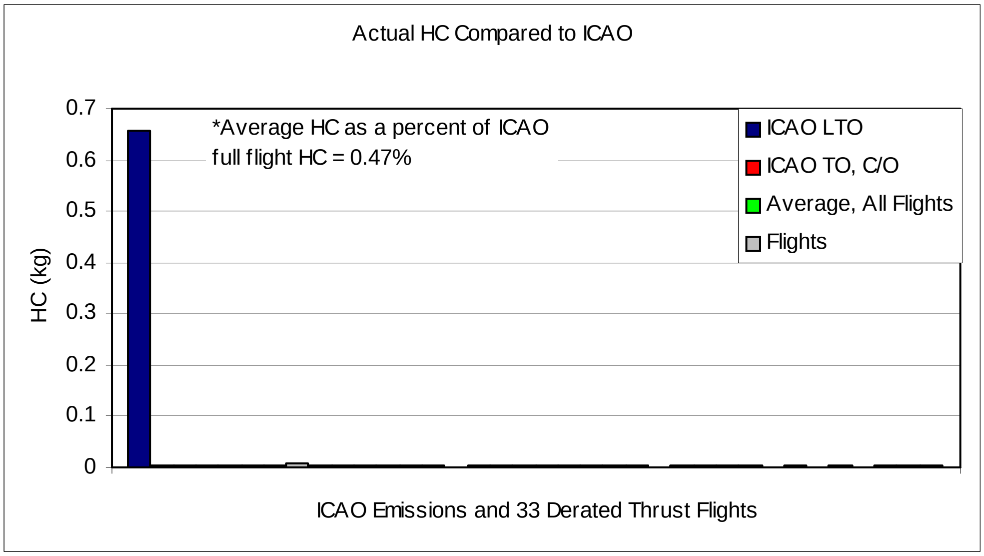

Figure 4: ICAO LTO cycle and ICAO takeoff and climb-out HC versus actual takeoff and climb-out emissions for derated thrust takeoffs.

8. Derate-Power Versus Full-Power Analysis

Flight data from 36 B777 flights were used to analyze the change in NOx, HC, and CO emissions resulting from derated-power settings on takeoff. The analysis compares the modeled emissions of the actual flight to the modeled emissions of the flight if full-power had been used. This section describes the methods used. Section 9 provides the results of this analysis.

A program for modeling the derated flight and simulating a full-power flight was created in Matlab. The program used BADA performance coefficients of the B777 for the aerodynamic and fuel flow models and Boeing Method 2 for emissions. A climb model was developed for this project for simulating the flight profile. The results compare emissions produced by the aircraft in the CFDR data to the emissions that would have been produced by that same aircraft *if it had flown with full-power on that same day through the same atmospheric conditions.*

The analysis was limited to the part of each flight in climb between 100 and 3,000 feet altitude above ground level (AGL). The landing gear is not modeled and it is assumed that the gear is up by the time the airplane reaches 100 ft, although it may still be retracting at that point. It is assumed that the flight procedures in terms of airspeed as a function of altitude would be the same for a full-power and derated takeoff. This is consistent with the flight procedures at the two airports. Vertical wind shear is ignored, and only the variation in horizontal wind speed with respect to altitude is considered. The same wind field is used for the full-power and derated flights. A final defining assumption is that lift is always equal to W cos γ, so that flight path angle changes to balance the forces. Variation in C_L due to changes in angle-of-attack from wind is not modeled.

8.1. Profile Calculation

Climb calculations are fairly straightforward in textbooks, however the unique set of flight data available for this problem makes the calculation more complicated. Data points are reported in 1-second intervals. Segment variables are calculated between points. The following point data is used in the calculation:

- Ground speed (V_g), inertial speed from GPS.
- True airspeed (V_t), computed from calibrated airspeed.
- Altitude (h).
- Ambient Temperature (Temp, to distinguish from T, thrust).
- Ambient pressure (P).

For some flights, only total pressure is available so P is calculated from total pressure, V_t, and Temp. Additional segment variables used in the derivation include:

- Rate of climb (ROC or $\dot{h}$), forward differenced derivative of h.
- Flight path angle (γ).
- Change in horizontal wind speed ($\frac{dV_w}{dh} or \frac{dV_w}{dt}$).
- Inertial horizontal acceleration ($\ddot{x}$).
- Drag (D), calculated from BADA drag model.

Averages of point values are other segment values used for calculations. Angle γ is the angle between the flight vector and the ground as shown in Figure (5), and is computed as arcsin($\dot{h}/V_{t,avg}$). Derivatives, including $\dot{h}$ and others, are calculated by forward differencing.

Vertical winds are assumed to be negligible and are ignored. Change in horizontal wind with respect to t or h is required but V_w itself is not. dV_w/dh and dV_w/dt are calculated by subtracting forward differenced ΔV_g from $\Delta V_t \cos\gamma$ and dividing by Δh or Δt.

V_t is converted from CAS at each data point using standard atmosphere for h_p at that point. Differencing amplifies noise data, so V_t, V_g, and h are smoothed with a five point moving average before they are differenced to obtain $\frac{dh}{dt}, \frac{dV_t}{dt}, \frac{dV_t}{dh}, \frac{dV_g}{dt}, and \frac{dV_w}{dh}$. The order of operations for each flight analyzed is as follows:

1. Import and process data; perform smoothing; convert CAS to V_t.
2. Calculate the derated thrust from velocity, altitude, and time data and the BADA drag model for the B777.
3. Markup derated thrust to full power using derate percentage given for each flight.
4. Calculate flight profile for the full-power flight using the same V vs. h profile as the derated flight.
5. Concurrently with profile calculation, calculate fuel flow based on BADA fuel flow model and use to find ΔW along the profile.
6. Calculate emissions with Boeing Method 2.

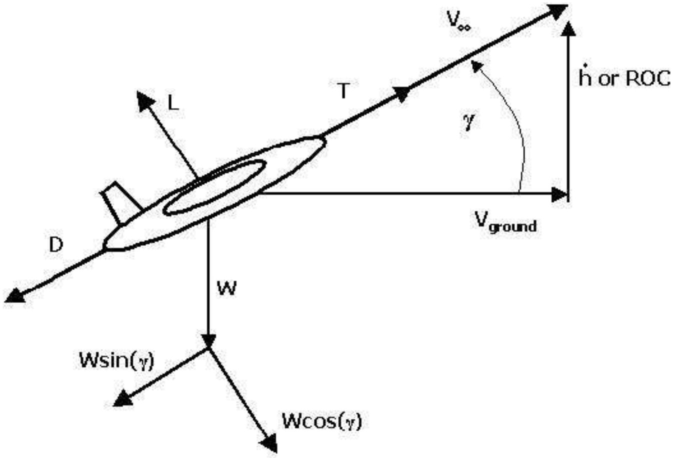

Figure 5: Climbing Flight

From Figure 5, the basic equation from which the derivation follows is Equation 4 below. From this equation, flight data and the BADA drag model can be used to calculate thrust given climb rate, or climb rate can be calculated for a given thrust.

$$\sum F_x = m\ddot{x} = T\cos\gamma - D\cos\gamma - L\sin\gamma$$

$$L = W\cos\gamma; \quad \sin\gamma = \frac{\dot{h}}{V_{t,avg}}; \quad \cos\gamma = \frac{\sqrt{V_{t,avg}^2 - \dot{h}^2}}{V_{t,avg}} \tag{4}$$

$$\ddot{x} = \frac{dV_g}{dt}$$

The resulting thrust equation calculates thrust based on drag given by the drag model and the flight conditions as a function of time.

$$T = \frac{W}{g\cos\gamma}\left(\frac{dV_g}{dt}\right) + W\sin\gamma + D \tag{5}$$

$$T_{full-power} = \frac{T_{derate}}{1 - \%Derate} \tag{6}$$

Full-power thrust is calculated by “uprating” the derated thrust by the derate percent. The resulting flight path is then calculated for the full-power and derated flights. The derated flight is calculated because the BADA fuel flow model used on the profile calculation has significant error. Consistency is maintained by calculating the profile fuel flow, weight, and emissions for the derated and full-power flights, giving an “apples to apples” comparison.

Ambient atmospheric conditions, true airspeeds, changes in wind speed, and aircraft takeoff weight are held fixed between derated and full-power flights. Except for takeoff weight, these values are held fixed with respect to altitude (not time). This ensures that the two simulations are flown in exactly the same conditions with the only difference being thrust. V_g, time-to-climb, total fuel burn, and total emissions depend on $\dot{h}$ which must be calculated based on the available data and thrust. The first step in solving for $\dot{h}$ is to write V_g as a function of the known parameters.

$$\begin{aligned}\ddot{x} &= \frac{V_t}{dt}\cos\gamma - \frac{dV_w}{dt} \\ &= \left(\frac{V_t}{dt}\right)\frac{\sqrt{V_{t,avg}^2 - \dot{h}^2}}{V_{t,avg}} - \frac{dV_w}{dt}\end{aligned} \tag{7}$$

At this point $\dot{h}$ and drag are the only unknowns. Drag can be broken down as a function of weight, $\dot{h}$, and airspeed leaving $\dot{h}$ as the only variable. C_D is calculated using a BADA drag model for the B777 with takeoff flaps and the gear up. Equation 1 shows the drag model that is inserted into the calculation.

Climb rate is solved for each segment in the flight giving a time between altitude points. Emissions and fuel burn are found for each segment. The final equation that can be solved for $\dot{h}$ is:

$$\begin{aligned} & m\left(\frac{dV_t}{dh}+\frac{W}{V_{t,avg}}\right)-m\left(\frac{dV_w}{dh}\right)\left(\frac{\dot{h}V_{t,avg}}{\sqrt{V_{t,avg}^2-\dot{h}^2}}\right) \\ & -\frac{kW^2}{qS}\left(\frac{\dot{h}^2}{V_{t,avg}^2}\right)+qSC_{Do}+\frac{kW^2}{qS}-T=0 \end{aligned} \tag{8}$$

Solving Equation 8 for $\dot{h}$ solves the flight path. From $\dot{h}$ the time-to-climb, ground speed, and flight path angle can all be determined. With time, altitude, airspeed, and fuel flow emissions can be calculated. Equation 8 is solved in Matlab using the *fzero* function to find the root. The full-power flight was flown through the simulation to calculate flight time, fuel burn, and emissions. The derated flight was also flown through the simulator to ensure that the results were consistent.

The BADA fuel flow model was used at each segment to calculate fuel flow. The fuel flow is used to calculate segment fuel burn. Fuel burn at each segment, and the flight conditions at that segment are used in Boeing Method 2 to calculate segment emissions indices and corresponding segment emissions. The sum of segment emissions gives total emissions for a flight. The fuel flow model does not match the fuel flows recorded in the data. The BADA fuel flow model is not specific to a single airframe-engine combination and is intended to represent all B777's, regardless of engine type, which could account for the over prediction of fuel seen on most flights. Figure 6 shows the percentage error in calculated total fuel burn for each flight compared to the reported fuel burn in the data, with an average over-prediction of 8.1%. Figure 7 shows the total fuel burn calculated for each flight and the total reported in the CFDR data (actual total).

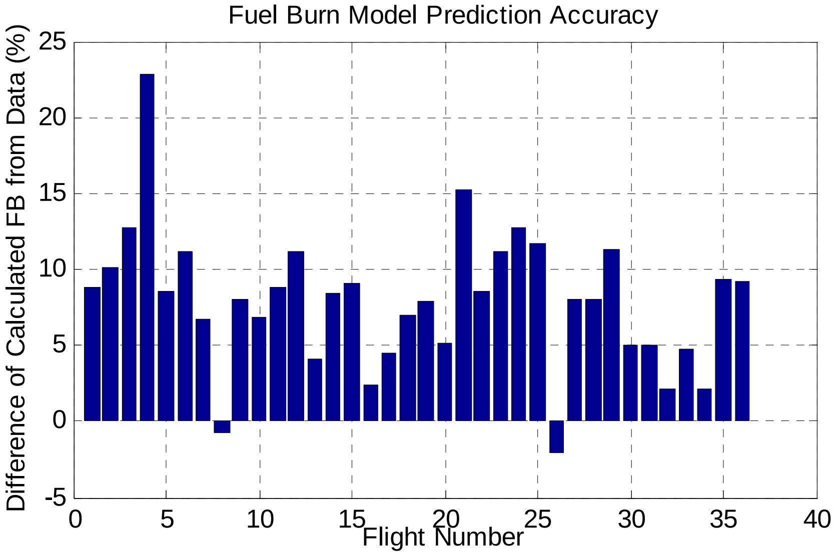
Fuel Burn Model Prediction Accuracy

Figure 6: Fuel burn prediction error compared to flight data

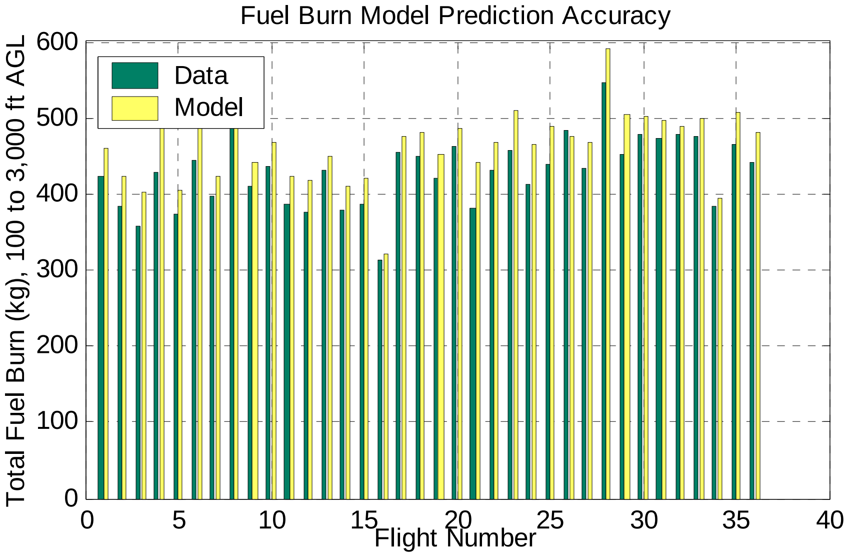

Figure 7: Fuel burn prediction compared to flight data.

9. Results of Derate-Power Versus Full-Power Analysis Approach

For the thirty-three derated takeoffs, the results show an average NOx reduction of *14.5%* compared to a full-power takeoff flown on the same day with the same airplane, between 100 and 3,000 feet AGL. Fuel burn increases by an average of by 12%. Changes in HC and CO from full-power to derate at minus 16.6 grams and minus 19.5 grams, respectively, are negligible relative to the emissions for the whole LTO cycle. Table 5 summarizes these results.

Table 5:Summary of Results, Averages for All Derate-Thrust Flights

Segment	Fuel Burn (kg)	NOx (kg)	HC (kg)	CO (kg)
Full-Power Total	416	17.0	0.017	0.098
Derate Total	467	14.6	8E-5	0.079
Difference	51	-2.5	-0.017	-0.020
Percent Change From Full to Derate	12.3%	-14.5%	-99%	-20%

Figures 8 and 9 show the correlation between percentage derate used, and the change in NOx and fuel burn. A first order trend line fit of the data is shown in the figures. Variance of the error calculated for the first order fit was 2.36 for NOx and 0.79 for fuel burn. A second order fit is only slightly better at a variance of 2.10 and 0.69 respectively. The equation used for variance of the error in this analysis is included below in Equation 9, where the data are a set of points (x_i, y_i), where i = 1...n, and $f(x_i)$ is the y-value calculated by the fit line at x_i.

$$Var = \frac{\sum_{i=1\ldots n}\left(y_i - f(x_i)\right)^2}{n-1} \qquad (9)$$

Figures 8 and 9 show that the percentage of fuel burn increase is about 0.6 times the percentage derate, and the percentage of NOx reduction is about 0.72 times the derate percentage. Variations away from the trend line show that change in fuel and emissions are caused by more than just changes in thrust. Wind variation, takeoff weight, and ambient atmospheric conditions are all modeled effects that likely contribute to the deviation. Un-modeled effects that may contribute to deviations may include late gear retractions, variations in flap settings, and variations in humidity.

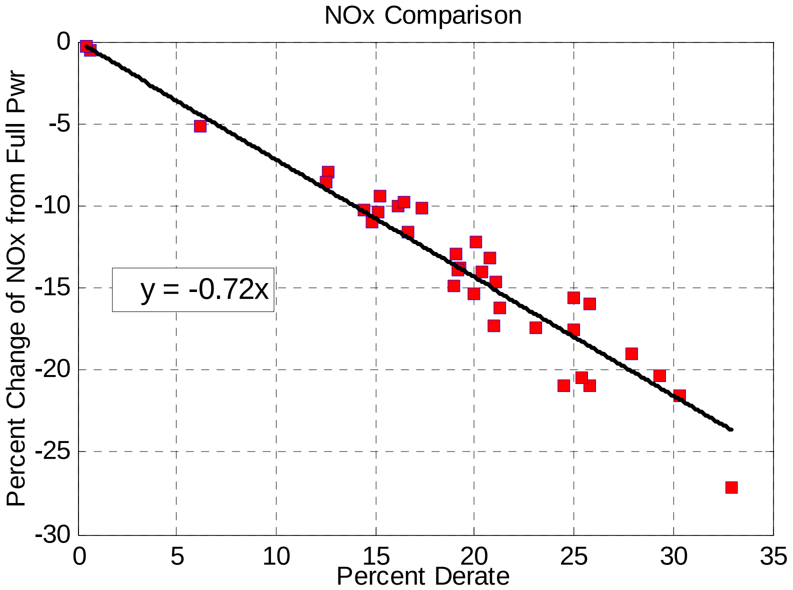

Figure 8: NOx reduction versus percent derate.

SOx, CO_2, and H_2O emissions are directly proportional to total fuel burn. The percentage change in these emissions between derate and full-power are the same as the change in fuel burn. Corresponding constant emissions indices are $EICO_2 = 3.155$, $EIH_2O = 1.237$, and $EISO_2 = .8$. Average emissions are therefore 1.5, 0.6, and 0.4 kilograms of CO_2, H_2O, and SO_2 respectively for derate; and 1.3, 0.5, 0.3 kilograms for full-power takeoffs.

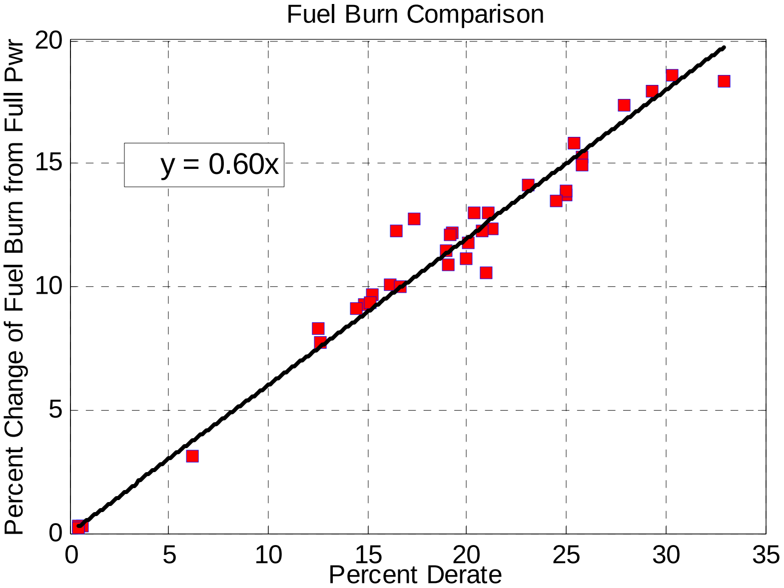

Figure 9: Fuel burn increase versus percent derate.

Figures 10 and 11 show the total quantities of CO and HC produced. The figures also note the total LTO and takeoff-only ICAO emissions for comparison. The CO and HC produced in the flight segment analyzed comprise only 2.5% or less of the total LTO emissions expected by an ICAO analysis. The insignificance can be expected by observing the ICAO emissions indices shown in Table 2, and noting that taxi EI's are much higher than all other segments. EIHC is zero between 30% and 85% throttle settings, explaining the presence of many zeros in Figure 10 for derate.

Figure 12 shows the true airspeed versus altitude profile for all 36 flights. These TAS profiles were used for the derated and full-power calculations. Using the same values of TAS versus altitude for both calculations ensures that that same procedures would be used for each flight, as if the same pilot had flown the same airplane on the same day. Figure 13 shows altitude profiles for all 33 derated thrust flights, plus the 3 full-power flights, along with their respective full-power profiles. The profiles represent the results of the $\dot{h}$ calculation of Equation 8. This figure shows that one basic result of derated throttle is increased time-to-climb. Reduction in fuel flow rate is overcome by increased time-to-climb causing increased fuel burn, and oppositely, increased fuel burn is overcome by reduction in EINOx resulting in reduced NOx at derated thrust.

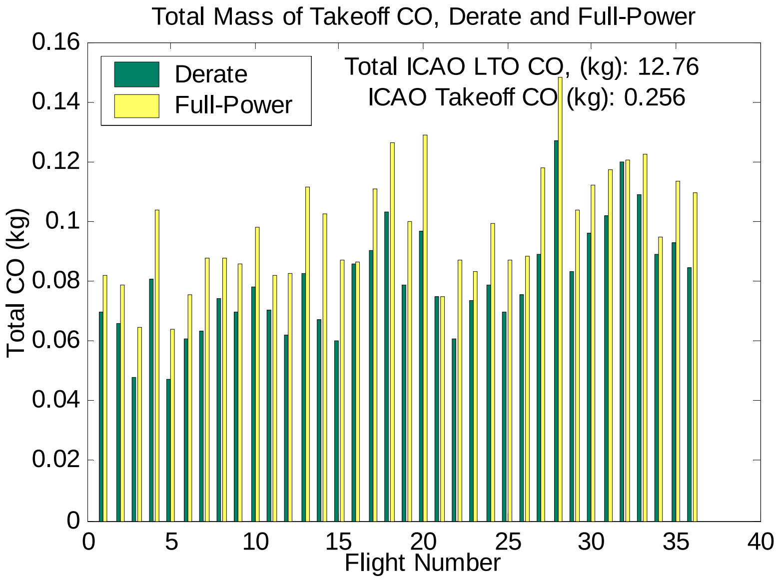

Figure 10: Total CO emissions, derate and full-power.

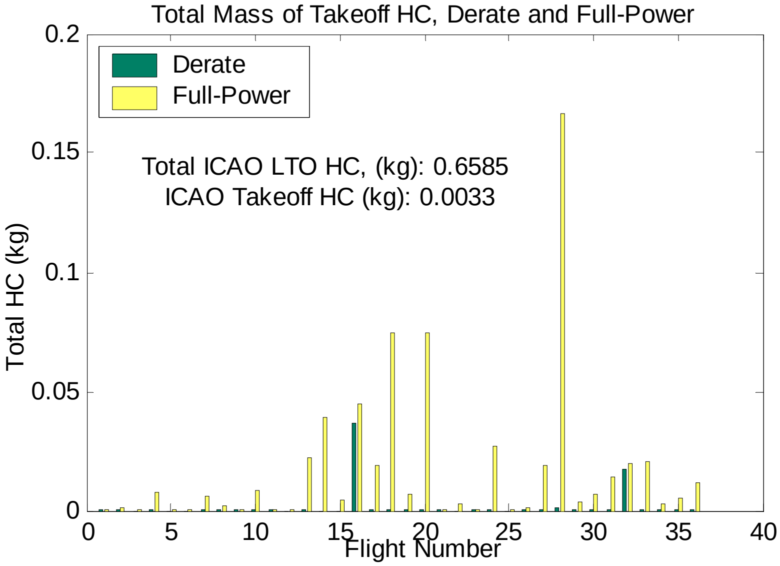

Figure 11: Total HC emissions, derate and full-power.

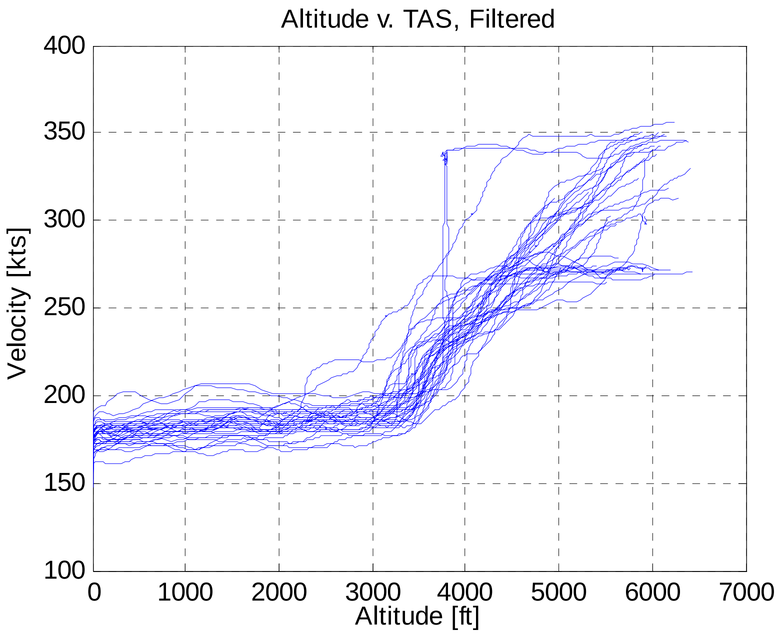

Figure 12: True airspeed (TAS) profile, all flights.

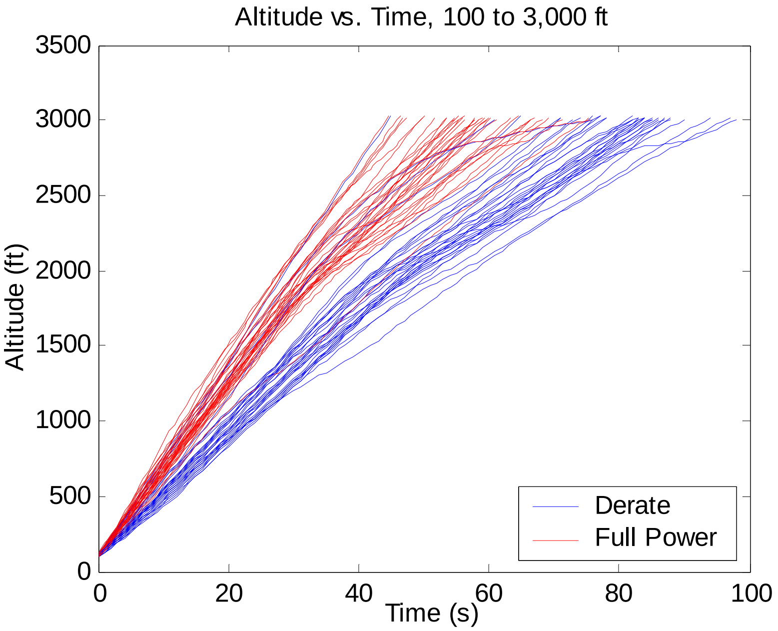

Figure 13: Altitude profile results of the h-dot equation.

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