Assessing Delay Propagation in Airline Plans: An Update

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Review

- Our goal is to better understand the relationship between planned and actual operations
- How can changes in the plan improve operational performance?
- Two stages of project:
  - Analyze potential for delay propagation
  - Decrease potential for delay propagation
Review of Analysis

- Build *propagation trees* to evaluate how an individual *root delay* might propagate through the network
- Construct trees for each flight, each delay interval
- Summarize metrics
Propagation Tree: Example

Nodes: flights  
Arcs: connections due to transfer of resources
Defining Metrics

- **Propagation magnitude**
  - Total minutes of delay propagated in the flight network divided by the original delay
  
  \[
  \text{propagation magnitude} = \frac{\text{total propagated delays}}{\text{original delay}}
  \]

- **Propagation severity**
  - Total number of disrupted flights
Severity Across All Delay Lengths

Cohn, Belobaba, Ahmadbeygi, and Guan, 2007
What’s New in Analysis

- For first carrier, evaluated two more data sets
  - Both after sizeable change in fleet composition
  - Two different dates (different demand levels, schedules, weather patterns)
Severity Across All Delay Lengths

Severity - June 2007 data set - before optimization

Severity - May 2006 data set - before optimization

Severity - July 2007 data set - before optimization
What’s Next in Analysis

☐ Cabin crews
☐ Critical passenger itineraries
☐ Recovery
Review of Optimization

- Can we improve robustness by changing flight times slightly, in order to better utilize the slack?
  - Don’t change crew assignments, fleeting, or routing
  - Only changes are to re-allocate slack where it is most needed
- Does not capture all the opportunities to improve robustness
- A starting point that does not require explicit assignment of costs or values to delay
Linear Programming Formulation I

- Minimize the expected value of \textbf{one-layer} delay propagation while keeping the connections feasible.

\[ \text{Min} \quad \sum_{m \in \mathcal{M}(f_1,f_2)} \sum_{f \in \mathcal{F}} p^m_{f_1} d^m_{f_1,f_2} \]

\[ y_{f_1,f_2} = s_{f_1,f_2} + x_{f_1} + x_{f_2} \quad \forall (f_1,f_2) \in \mathcal{A} \]

\[ \begin{align*}
    d^m_{f_1,f_2} & \geq m - y_{f_1,f_2} \quad \forall (f_1,f_2) \in \mathcal{A} \quad \forall m \in \mathcal{M} \\
    d^m_{f_1,f_2} & \geq 0 \quad \forall (f_1,f_2) \in \mathcal{F} \quad \forall m \in \mathcal{M} \\
    d^m_{f_1,f_2} & = \max \{0, m - y_{f_1,f_2}\} \\
    k^-_f & \leq x_f \leq k^+_f \quad \forall f \in \mathcal{F} \quad y_{f_1,f_2} \geq 0 \quad \forall (f_1,f_2) \in \mathcal{A} 
\end{align*} \]
What’s New in Optimization

☐ First approach only looked at one layer of delay

☐ New approach allows delay to propagate until fully absorb

☐ Little change on performance (run time)

☐ Still a linear program

☐ Some difference in outcome
Linear Programming Formulation II

- Minimize the expected value of **all-layers** delay propagation while keeping the connections feasible

\[
\min \sum_{m \in M} \sum_{f_0 \in F} \sum_{f_i \in T_{f_0}^m} p_{f_0}^m d_{f_0, f_i}^m
\]

\[
y_{f_1, f_2} = s_{f_1, f_2} + x_{f_1} + x_{f_2} \quad \forall (f_1, f_2) \in A
\]

\[
d_{f_0, f_i}^m \geq m - y_{f_0, f_i} \quad \forall f_i \in T_{f_0}^m \text{ s.t. } r_{f_0}^m(f_i) = f_0 \quad \forall m \in M
\]

\[
d_{f_0, f_i}^m \geq d_{f_0, r_{f_0}^m(f_i)}^m - y_{r_{f_0}^m(f_i), f_i} \quad \forall f_i \in T_{f_0}^m \text{ s.t. } r_{f_0}^m(f_i) \neq f_0 \quad \forall m \in M
\]

\[
d_{f_0, f_i}^m \geq 0 \quad \forall f_0 \in F, f_i \in T_{f_0}^m \quad \forall m \in M
\]

\[
k_f^- \leq x_f \leq k_f^+ \quad \forall f \in F \quad y_{f_1, f_2} \geq 0 \quad \forall (f_1, f_2) \in A
\]
Implementation

- Implemented the model using CPLEX10.0/C++

- Used historical data in order to compute the probability of departure delays ($p_f^m$)

- Assumptions:
  - Equal time windows $k_f^+ = k_f^- = 15$
  - For the flights that start a duty period $k_f^+ = 15, \quad k_f^- = 0$
  - For the flights that end a duty period $k_f^+ = 0, \quad k_f^- = 15$
Results

Severity - May 2006 data set - before optimization

Severity - May 2006 data set - after optimization (all layers)
Results

Severity - June 2007 data set - before optimization

Severity - June 2007 data set - after optimization (all layers)
Results

Severity - July 2007 data set - before optimization

Severity - July 2007 data set - after optimization (all layers)
## Results, cont.

**May 2006**

<table>
<thead>
<tr>
<th>Model I: One-layer propagation model</th>
<th>Model II: All-layers propagation model</th>
</tr>
</thead>
<tbody>
<tr>
<td>obj. fun. value before opt = 789.22</td>
<td>obj. fun. value before opt = 1187.76</td>
</tr>
<tr>
<td>obj. fun. value after opt = 519.103</td>
<td>obj. fun. value after opt = 768.949</td>
</tr>
<tr>
<td>reduction% = 34.2%</td>
<td>reduction% = 35.3%</td>
</tr>
<tr>
<td>running time = 1 sec</td>
<td>running time = 1 sec</td>
</tr>
<tr>
<td>obj. fun. based on model I = 787.989</td>
<td>obj. fun. based on model I = 787.989</td>
</tr>
<tr>
<td>reduction% = 33.6%</td>
<td>reduction% = 33.6%</td>
</tr>
</tbody>
</table>
### Results, cont.

June 2007

<table>
<thead>
<tr>
<th>Model I: One-layer propagation model</th>
<th>Model II: All-layers propagation model</th>
</tr>
</thead>
<tbody>
<tr>
<td>obj. fun. value before opt = 543.895</td>
<td>obj. fun. value before opt = 637.878</td>
</tr>
<tr>
<td>obj. fun. value after opt = 441.049</td>
<td>obj. fun. value after opt = 519.523</td>
</tr>
<tr>
<td>reduction% = 18.9%</td>
<td>reduction% = 18.5%</td>
</tr>
<tr>
<td>running time = 1 sec</td>
<td>running time = 2 sec</td>
</tr>
<tr>
<td>obj. fun. based on model I = 526.812</td>
<td>reduction% = 17.4%</td>
</tr>
</tbody>
</table>
## Results, cont.

**July 2007**

<table>
<thead>
<tr>
<th>Model I : One-layer propagation model</th>
<th>Model II : All-layers propagation model</th>
</tr>
</thead>
<tbody>
<tr>
<td>obj. fun. value before opt = 652.408</td>
<td>obj. fun. value before opt = 758.635</td>
</tr>
<tr>
<td>obj. fun. value after opt = 551.009</td>
<td>obj. fun. value after opt = 636.817</td>
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<tr>
<td>reduction% = 15.5%</td>
<td>reduction% = 16.05%</td>
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<tr>
<td>running time = 1 sec</td>
<td>running time = 3 sec</td>
</tr>
<tr>
<td>obj. fun. based on model I = 641.107</td>
<td>obj. fun. based on model I = 641.107</td>
</tr>
<tr>
<td>reduction% = 15.4%</td>
<td>reduction% = 15.4%</td>
</tr>
</tbody>
</table>
What’s Next in Optimization

- Implementing a simulation to evaluate our surrogate objective function
- In the future, need to better incorporate recovery decisions
Conclusions

- Standard plea for data
- Standard plea for feedback
- Special plea for guidance about modeling recovery