Competitive Frequency Analysis
and Impacts on Congestion

Vikrant Vaze
Prof. Cynthia Barnhart

Department of Civil and Environmental Engineering
Operations Research Center
Massachusetts Institute of Technology
Delays and Over-scheduling

- Total aircraft delay in 2007: 134M minutes \(^1\) (cost = $8.1B\(^1\))
- Total passenger delay in 2007: 17B minutes \(^2\) (cost = $8.5B\(^2\))
- 92.5% of National Aviation System (NAS) delays attributed to scheduling more than the realized capacity

Causes of National Aviation System Delays

\(^1\)Source: Air Transport Association, 2008; \(^2\)Source: U.S. Airline Passenger Trip Delay Report, 2008; \(^3\)Source: Bureau of Transportation Statistics, 2009
Decreasing Aircraft Sizes

- Airlines prefer to fly many small planes rather than few big planes

[Source: Bonnefoy and Hansman, 2008]
An Example of Over-scheduling

LGA-BOS:
40 direct flights per day

<table>
<thead>
<tr>
<th>Carrier</th>
<th>Flight No.</th>
<th>Dep. Time</th>
<th>Arr. Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>DL 1906</td>
<td>6:00</td>
<td>7:00</td>
<td></td>
</tr>
<tr>
<td>US 2114</td>
<td>6:00</td>
<td>7:00</td>
<td></td>
</tr>
<tr>
<td>DL 1908</td>
<td>6:30</td>
<td>7:34</td>
<td></td>
</tr>
<tr>
<td>MQ 4803</td>
<td>7:00</td>
<td>8:15</td>
<td></td>
</tr>
<tr>
<td>US 2116</td>
<td>7:00</td>
<td>8:12</td>
<td></td>
</tr>
<tr>
<td>DL 1910</td>
<td>7:30</td>
<td>8:37</td>
<td></td>
</tr>
<tr>
<td>US 2118</td>
<td>8:00</td>
<td>9:12</td>
<td></td>
</tr>
<tr>
<td>MQ 4802</td>
<td>8:20</td>
<td>9:30</td>
<td></td>
</tr>
<tr>
<td>DL 1912</td>
<td>8:30</td>
<td>9:40</td>
<td></td>
</tr>
<tr>
<td>US 2120</td>
<td>9:00</td>
<td>10:16</td>
<td></td>
</tr>
<tr>
<td>DL 1914</td>
<td>9:30</td>
<td>10:46</td>
<td></td>
</tr>
<tr>
<td>US 2122</td>
<td>10:00</td>
<td>11:15</td>
<td></td>
</tr>
<tr>
<td>DL 1916</td>
<td>10:30</td>
<td>11:47</td>
<td></td>
</tr>
<tr>
<td>MQ 4805</td>
<td>10:50</td>
<td>12:05</td>
<td></td>
</tr>
<tr>
<td>US 2124</td>
<td>11:00</td>
<td>12:15</td>
<td></td>
</tr>
<tr>
<td>DL 1918</td>
<td>11:30</td>
<td>12:46</td>
<td></td>
</tr>
<tr>
<td>US 2126</td>
<td>12:00</td>
<td>13:10</td>
<td></td>
</tr>
<tr>
<td>DL 1920</td>
<td>12:30</td>
<td>13:39</td>
<td></td>
</tr>
<tr>
<td>US 2128</td>
<td>13:00</td>
<td>14:11</td>
<td></td>
</tr>
<tr>
<td>DL 1922</td>
<td>13:30</td>
<td>14:39</td>
<td></td>
</tr>
</tbody>
</table>
Frequency Competition

- S-curve relationship between market share and frequency share
- Higher frequency shares associated with disproportionately higher market shares
Computation of a Lower Bound on Airport Congestion

• Problem Statement:
  – Design a schedule to minimize airport congestion while satisfying all the demand and maintaining the same level-of-service
  – Carry as many passengers as being carried currently for each market for each time of the day
  – Provide a daily frequency equal to the maximum daily frequency provided currently in that market

• Results:
  – No more than 92% of bad-weather capacity (IFR) is required
  – Substantial reduction in airport congestion can be achieved with existing capacity
Multi-Agent Model

• A system of profit maximizing agents
• Optimal frequency decision \( (f_a) \) for an airline \( a \) depends on actions by other airlines \( (f_{-a}) \)
• Nash Equilibrium:
  A frequency profile \( f \) is a Nash Equilibrium if for every airline \( a \), \( f_a \) is the best response to \( f_{-a} \)

• Solution Methodology: “Myopic Best Response”
  While there exists a carrier whose current decision is not optimal in relation to others’ decisions, re-optimize for that carrier
• Optimization problem solved using dynamic programming

Results fit reality reasonably well: 7% error in frequency estimates
**Optimization Sub-Model**

Maximize: \[ \sum_{s \in S} (P_{a,s} \cdot Q_{a,s} - C_{a,s} \cdot f_{a,s}) \]

Maximize total profit = fare revenue – operating cost

Subject to:

\[ Q_{a,s} \leq \frac{f_{a,s} \cdot \alpha}{\sum_{a' \in A} f_{a',s} \cdot \alpha} \cdot M_s \quad \forall s \in S \]

S-curve relationship between market share and frequency share

\[ Q_{a,s} \leq Seats_{a,s} \cdot f_{a,s} \quad \forall s \in S \]

Seating capacity constraint

\[ \sum_{s \in S} f_{a,s} \leq MAX\_SLOTS_a \]

Maximum number of available slots

\[ \sum_{s \in S} f_{a,s} \geq MIN\_SLOTS_a \]

Minimum number of slots that must be utilized (Use-It-Or-Lose-It)

\[ f_{a,s} \in \mathbb{Z}^+ \quad \forall s \in S \]
Solution using Dynamic Programming

- Nonlinear constraints together with integrality constraints
- But the structure is suitable for dynamic programming since:
  - Slot restrictions are the only coupling constraints across different segments
  - Objective function is additive across segments
- No. of stages = No. of segments
- No. of states per stage = Maximum no. of slots

\[
\begin{align*}
\text{Profit}(s, n) &= \text{Segment } s \text{ profit due to exactly } n \text{ flights per day} \\
R(0, 0) &= 0, \quad R(0, n) = -\infty \text{ for } n \geq 1 \\
R(s, n) &= \max_{0 \leq n' \leq n} (R(s - 1, n') + \text{Profit}(s, n - n')) \\
\text{Optimal total profit} &= \max_{\text{MIN}_SLOTS_a \leq n \leq \text{MAX}_SLOTS_a} R(|S|, n)
\end{align*}
\]
Slot Reduction Schemes

1) Proportionate slot reduction
   - Number of slots available to each carrier reduced by same proportion

2) Reward based slot reduction
   - Slot reduction for each carrier proportional to inverse of passengers/slot
   - Idea is to reward those who are using their slots efficiently

Note: In this experiment we assume that the aircraft sizes remain unchanged
## Overall Impact

<table>
<thead>
<tr>
<th>Scheme</th>
<th>Do Nothing</th>
<th>20% Reduction (Proportionate)</th>
<th>20% Reduction (Reward-based)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Operating Profit</td>
<td>$ 1,252,362</td>
<td>$ 1,568,814 (25.27%)</td>
<td>$ 1,565,490 (25.00%)</td>
</tr>
<tr>
<td>Passengers Carried</td>
<td>22,260</td>
<td>21,291 (-4.35%)</td>
<td>21,464 (-3.58%)</td>
</tr>
<tr>
<td>NAS Delay per Flight</td>
<td>12.74 min</td>
<td>7.52 min (-40.97%)</td>
<td>7.52 min (-40.97%)</td>
</tr>
</tbody>
</table>
Proportionate Slot Reduction Scheme

Increase in Profit Vs. Slot Reduction Percentage
Reward Based Slot Reduction Scheme

Increase in Profit Vs. Slot Reduction Percentage

![Graph showing the relationship between percentage increase in profit and percentage slot reduction. The graph indicates an increasing trend from 0% to 50% slot reduction, with a significant increase in profit as slot reduction increases.](image-url)
Proportionate Slot Reduction Scheme

Decrease in Number of Passengers Vs. Slot Reduction Percentage
Reward Based Slot Reduction Scheme

Decrease in Number of Passengers Vs. Slot Reduction Percentage

Percentage Decrease in Passengers

Percentage Slot Reduction
## Impact on Individual Airlines

<table>
<thead>
<tr>
<th>Slots</th>
<th>100%</th>
<th>80%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scheme</td>
<td>Proportionate</td>
<td>Reward Based</td>
</tr>
<tr>
<td>Carrier</td>
<td>Profit</td>
<td>Profit</td>
</tr>
<tr>
<td>AA</td>
<td>365,582</td>
<td>447,897</td>
</tr>
<tr>
<td>CO</td>
<td>66,450</td>
<td>73,205</td>
</tr>
<tr>
<td>DL</td>
<td>188,352</td>
<td>285,531</td>
</tr>
<tr>
<td>FL</td>
<td>36,908</td>
<td>52,891</td>
</tr>
<tr>
<td>MQ</td>
<td>33,630</td>
<td>43,579</td>
</tr>
<tr>
<td>NW</td>
<td>107,006</td>
<td>107,920</td>
</tr>
<tr>
<td>OH</td>
<td>34,638</td>
<td>54,144</td>
</tr>
<tr>
<td>UA</td>
<td>200,796</td>
<td>233,188</td>
</tr>
<tr>
<td>US</td>
<td>170,939</td>
<td>225,209</td>
</tr>
<tr>
<td>Total</td>
<td>1,252,362</td>
<td>1,568,814</td>
</tr>
</tbody>
</table>
Impact of Aircraft Upgauges

Decrease in Number of Passengers Vs. Upgauge Percentage
(20% proportionate reduction)
Conclusions

- Significant part of airport congestion and delays can be attributed to inefficient utilization of airport capacity.
- Frequency competition can be depicted as a Nash equilibrium model where frequency decisions of each airline depend on those of its competitors.
- Simple slot control schemes have the potential to reduce the congestion and become attractive to all the stakeholders.
- Next steps:
  - integrate aircraft size decisions with frequency planning
  - introduce time-of-the-day element in slot allocation
Thank You!

vikrantv@mit.edu