One Dimensional Gravitational Collapse

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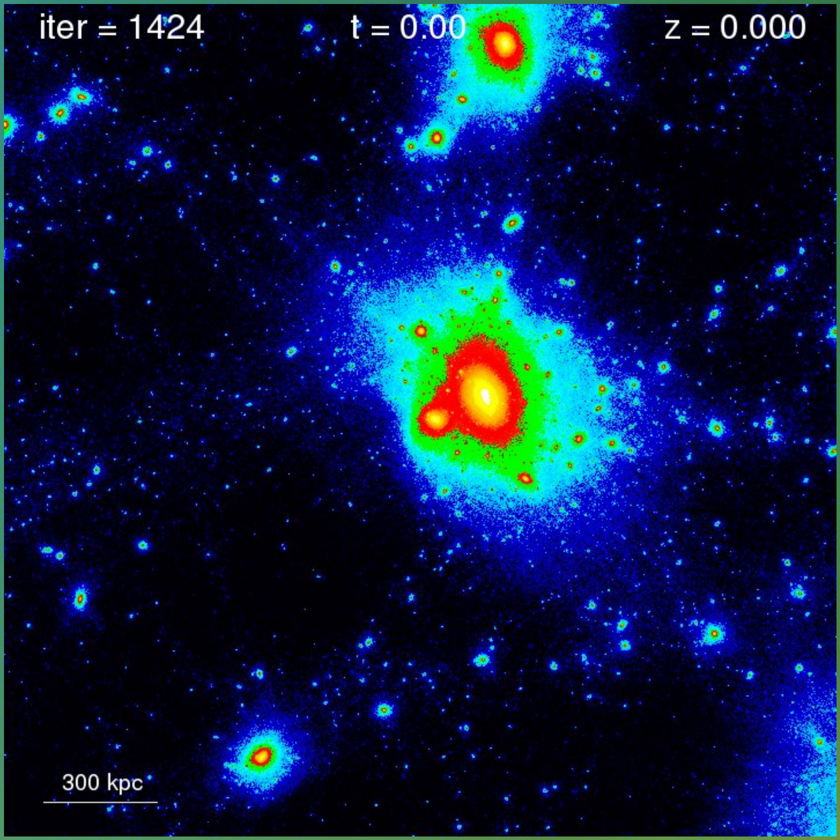
Overview

- Background and context
- The dynamical system
- The algorithm
- Numerical results
    - Phase mixing
    - Violent relaxation
    - The density profile and phase space density
    - The evolution of the system
    - The origin of the density scaling relation
    - Changing the initial density profile
- Conclusions

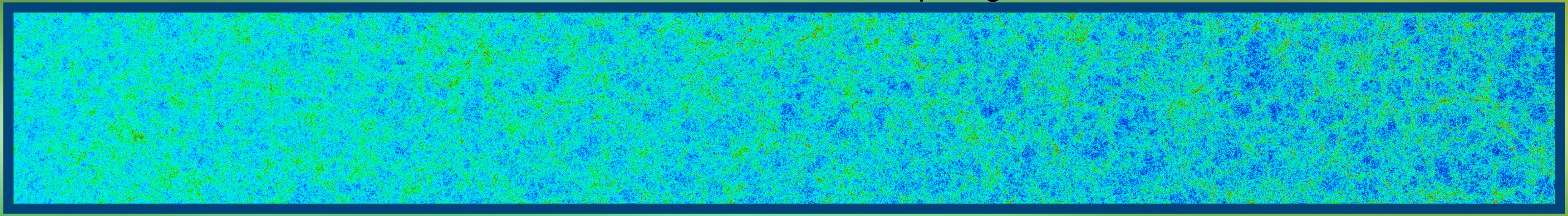
Background and Context

- Cosmologists rely on simulations to make predictions about
  - The clustering of dark matter
  - The mass functions of dark matter structures and substructures
  - The velocity structure of dark matter
  - Many (non-physics) factors influence how accurately the simulations represent dark matter properties
    - Particle masses
    - Particle number
    - Gravitational softening
    - Time step resolution
    - Number of modes used to generate the initial conditions
    - Redshift at which initial conditions are generated



Simulations by Mike Warren

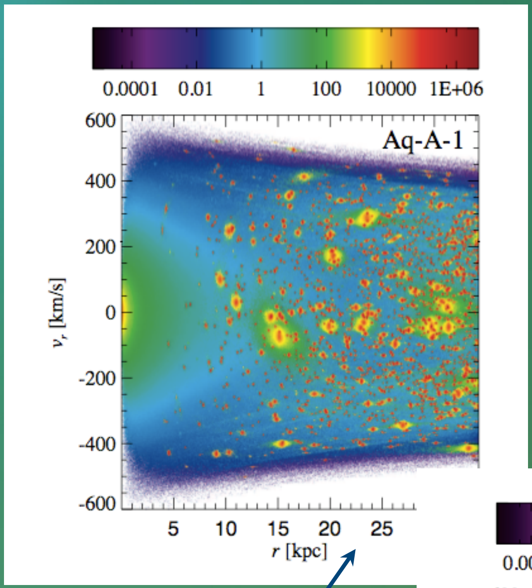
4.4 Gpc Lightcone



Phase Space Density from Simulations

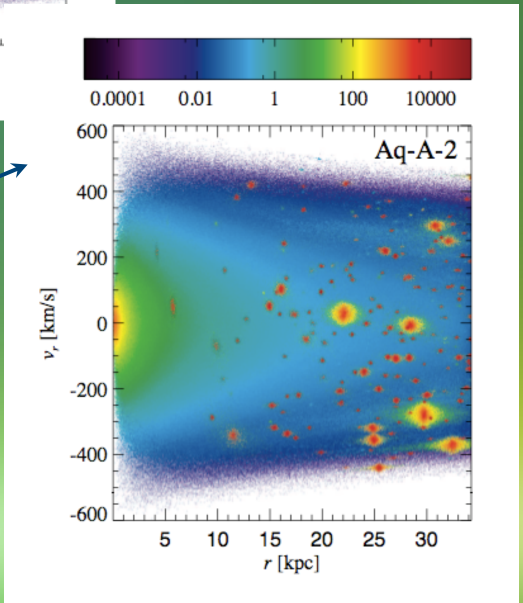
- Evolution of the phase space density influences observable properties of dark matter. Notably
  - Halo profile shapes
  - Velocity distributions
  - Substructure
    - Number
    - Profiles
    - Velocities
- The peak phase space density in a simulation may significantly depart from that of a true system because of discreetness effects
- Discreetness effects can influence structure formation at any point in collapse (before and after virialization and relaxation)

Peak Phase Space Density at 2 resolutions



Resimulations of the SAME halo!

Maciejewski et al. arXiv:1010.2491



Aquarius Simulations

Phase Space Density from Simulations

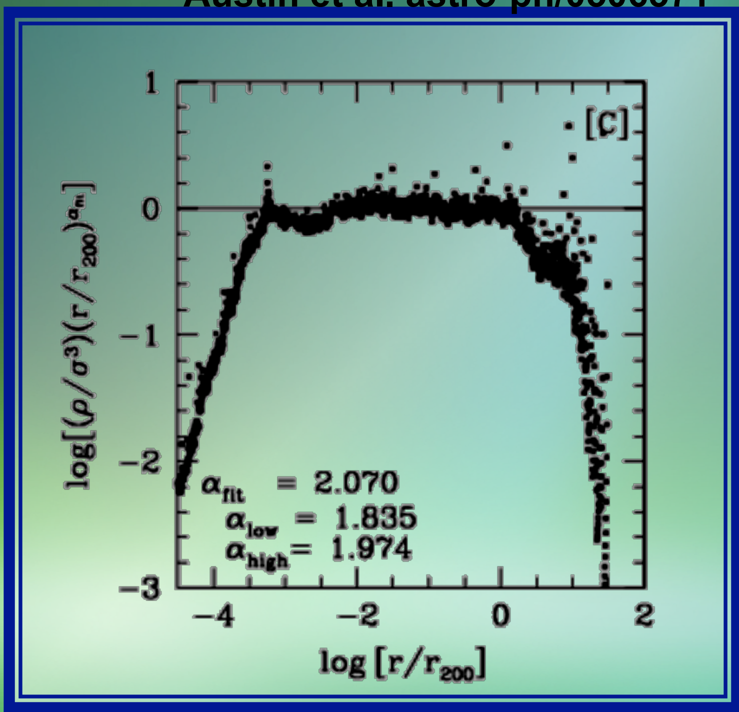
- Numerical studies show a power law scaling in the phase space density indicator (or pseudo-phase space density)

$$Q = \frac{\rho}{\sigma^3}$$

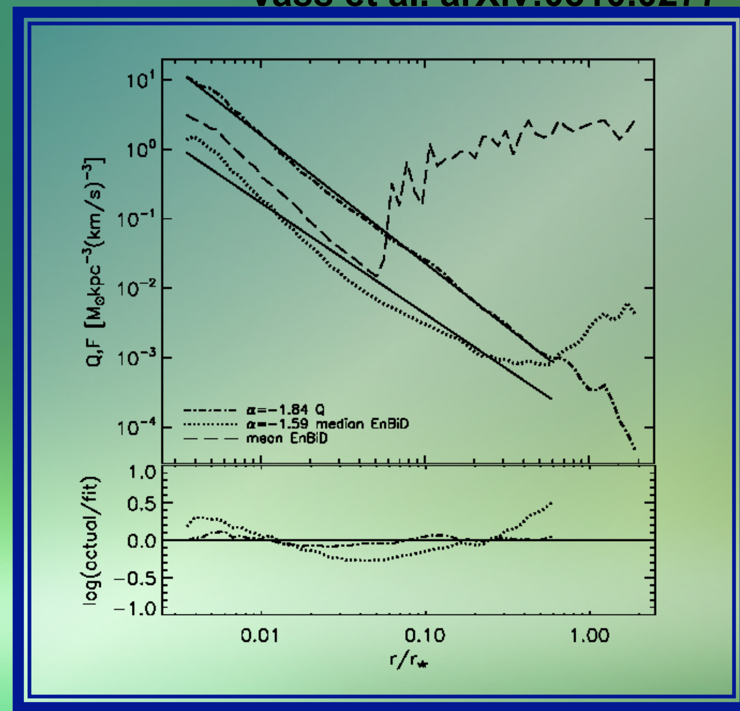
- Q is not actually phase space density since density and velocity averages are computed separately in each volume (hard to interpret)

- The origin of this scaling is unknown

Austin et al. astro-ph/0506571



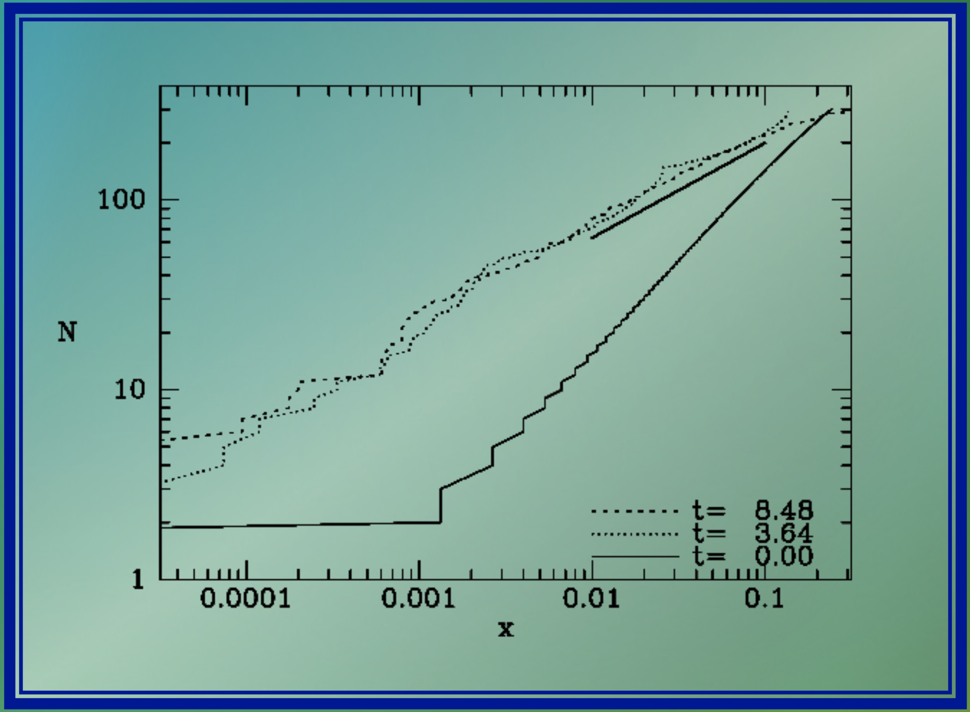
Vass et al. arXiv:0810.0277



Phase Space Density from Simulations

- Binney [MNRAS 350 (2004) 939] used one dimensional gravitating systems to suggest
  - Resolution effects during sheet collapse influence the final phase space density of a halo
  - The early stage discreetness may limit central halo densities more than late time processes such as two body relaxation
- In one dimension, the profile of the central density scales as a power law
- This implies that sheet collapse in the early universe will have singular central densities

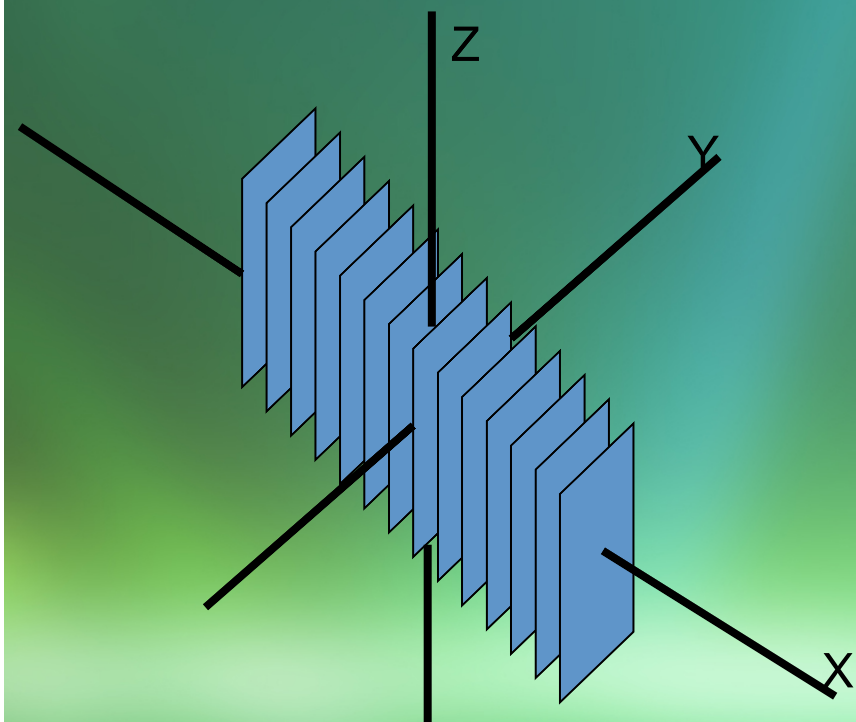
$$\rho \propto x^{-1/2}$$



Binney astro-ph/0311155

Defining the system

- ✦ A finite number of sheets collapsing along the x axis
- ✦ Sheets are infinite in the y and z directions
- ✦ Force on any sheet does not depend on separation



- Let the surface density be (Σ/N) so that the total "mass" is independent of N
- Index each "particle" by a rank ***k***, indicating its order from the central sheet

$$F_k = -4\pi G\Sigma\left(k/N\right)$$

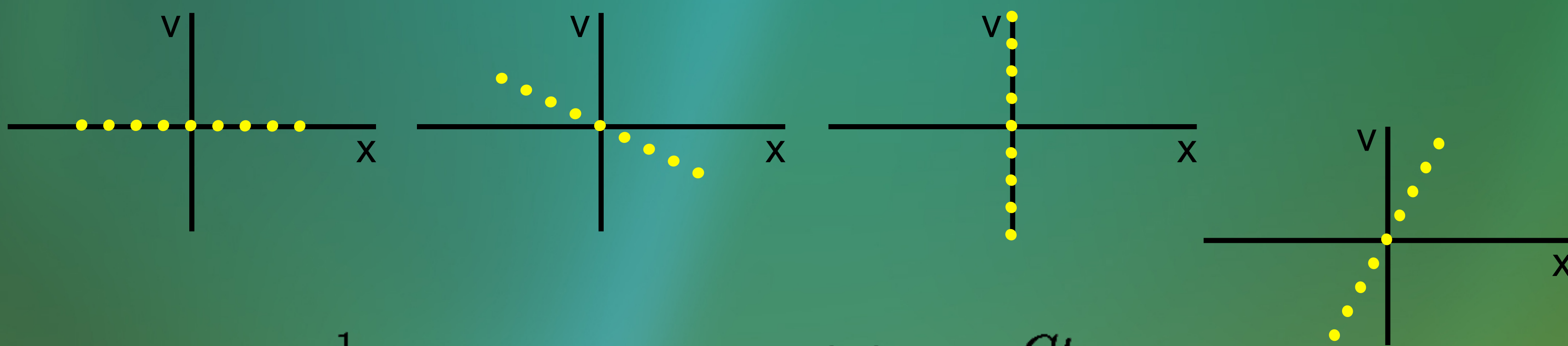
- This force changes only when the **collisionless** particles pass one another, exchanging ranks
- Between crossing the particle orbits are quadratic in time

The Algorithm

- Particle positions are analytically computed between particle crossings
- The "obvious" approach
  - Compute crossing times between each particle and its nearest neighbors
  - Sort through and find the earliest crossing time
  - Update all the particle positions to that time
  - Exchange the forces (and trajectories) on active particles
  - Go to step one and recurse
- An improved method (Implemented by Walter Dehnen)
  - The non-active particles' ranks are unchanged at crossing
  - Therefore their forces and trajectories are unchanged
  - It is only necessary to update the active particles' positions
  - Only the active particles crossing times need be recomputed
  - Particles in the simulation exist at different times, in the correct rank order
  - Periodically, all particles are updated to the same time and dumped
- The improved algorithm significantly improves the scaling

The zero velocity case

- If the system begins with equally spaced particles and zero initial velocity, the orbits remain in phase



$$x = x_0 + \frac{1}{2}at^2$$

$$v = at$$

$$a = Cx_0$$

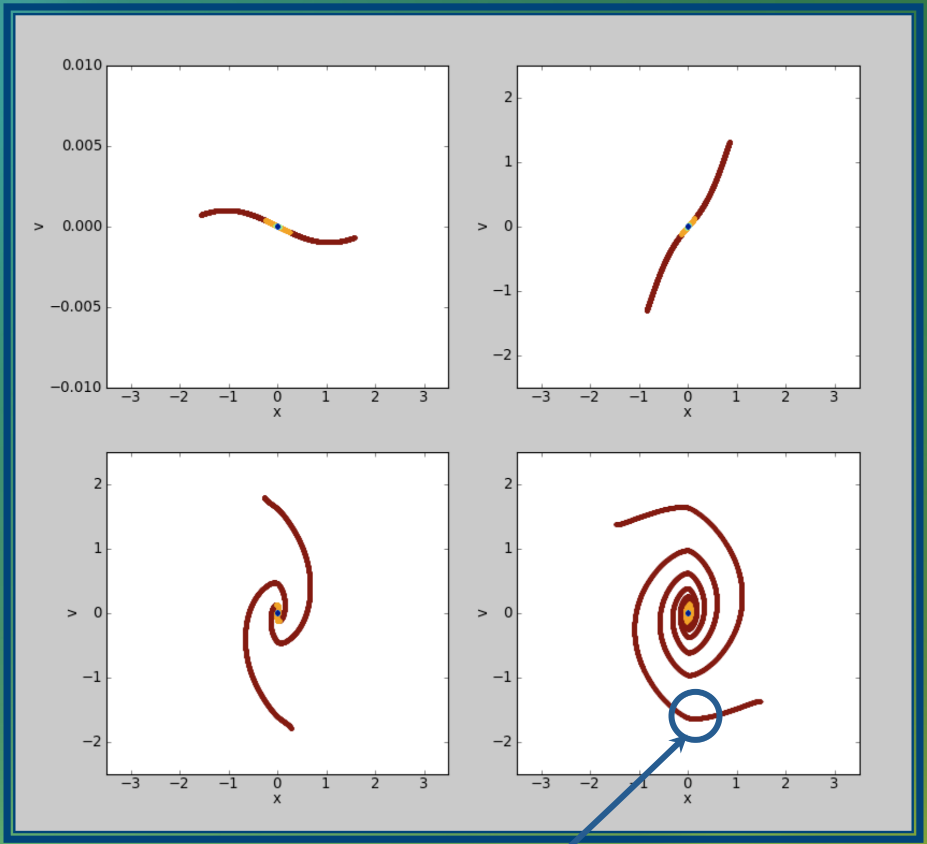
$$\left(\frac{v}{x}\right) = \frac{Ct}{1 + Ct^2}$$

Independent of initial position

- All particles have the same period, and they all cross only at x=0

Initial velocity causes phase mixing

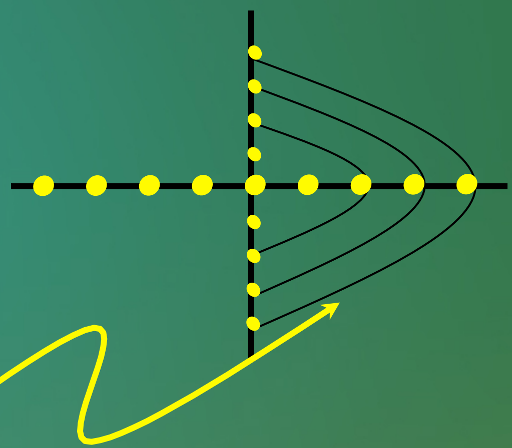
- Example: A small amplitude sinusoid for initial velocities
  - If $v_0(x_0)$ is not proportional to x_0, particles at different initial positions have different periods
  - The period is shorter if the mean density interior to the particle is higher
  - Since the structure is collapsing the particles closer to the center undergo faster oscillations through the center.
  - Points in phase space wind into an ever tighter spiral



Whence the kink?

The kink and the phase space orbits

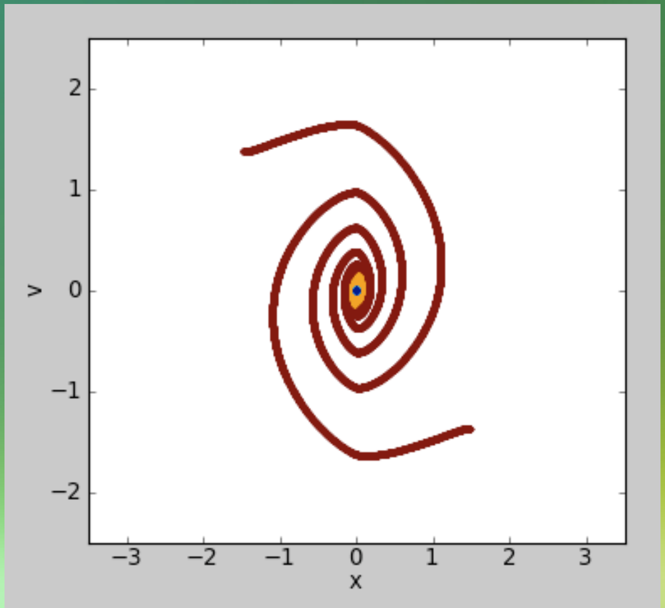
- Parametric form of orbit in phase space
  - Solve velocity eqn for t
  - Plug into position equation



$$t = \frac{v - v_0}{C x_0}$$

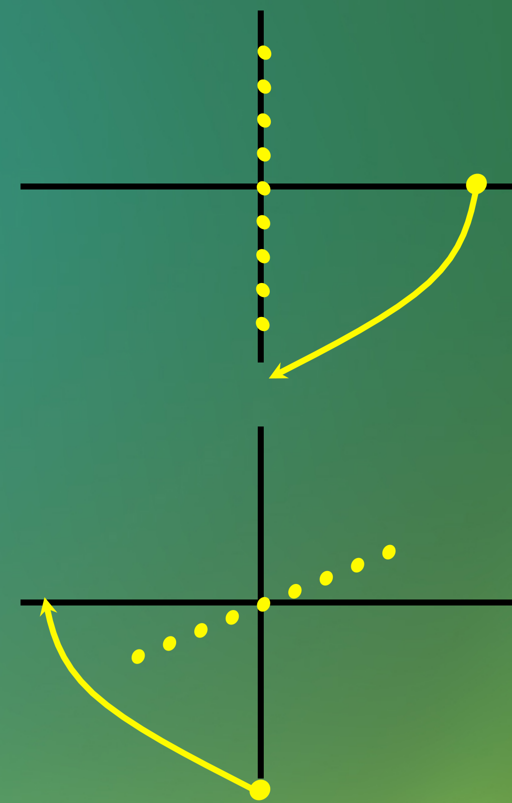
$$x = (\text{Const.}) + \frac{v^2}{2 C x_0}$$

  - If no particles cross, orbits are parabolic in phase space
  - Since the system is symmetric, at x=0 a particle crosses with its partner -- the kink is a meeting of two parabolic shapes
  - Other particle crossings prior to x=0 modify this picture quantitatively



The outward transfer of energy

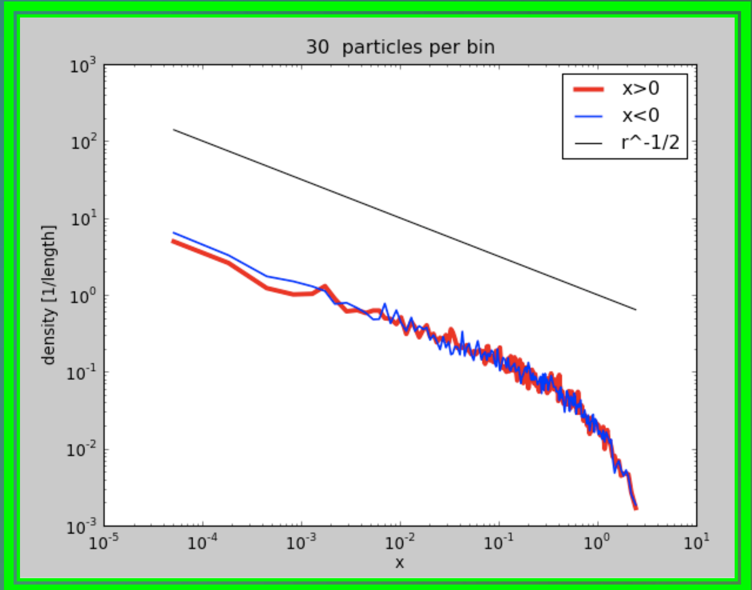
- Phase mixing generates a process "violent relaxation"
  - Evolving gravitational potential create an effect similar to the ISW effect
  - able to transfer energy from the inner to the outer particles
- The innermost particles behave much like the "rod"
- The acceleration of outer particles on the way toward the center far exceeds the deceleration on their way back out
- The outer particles get a "kick" and the inner particles lose energy and their orbits contract inward
- When the phase lag is close to $\pi/4$ the energy transfer is most efficient
- There is a scale that separates inward from outward moving orbits

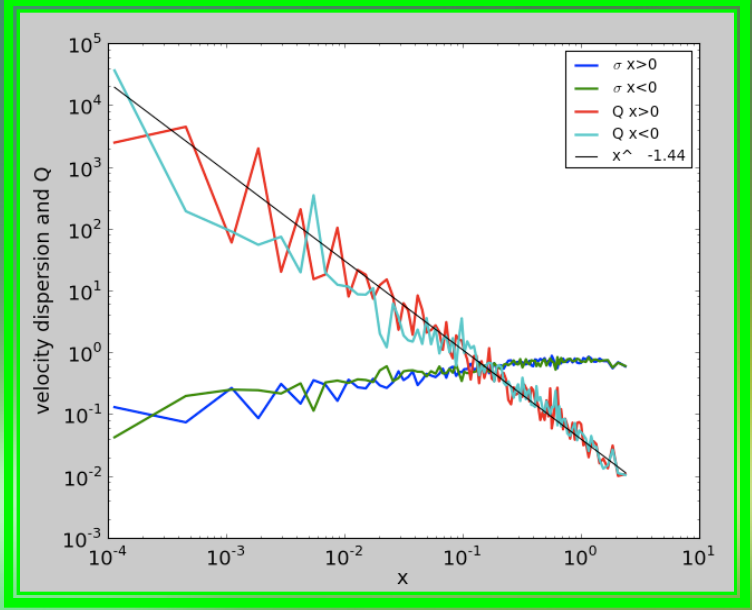


The density profile

- The system eventually reaches an equilibrium state, that satisfies the virial condition
- In this state, the winds in phase space have been wound so tightly that visually there is no longer a coherent phase structure
- The density profile obeys a power law scaling behavior over many decades in x
- The velocity dispersion σ also displays a power law behavior
- Thus, in agreement with previous findings, the phase space density indicator Q also scales as a power law

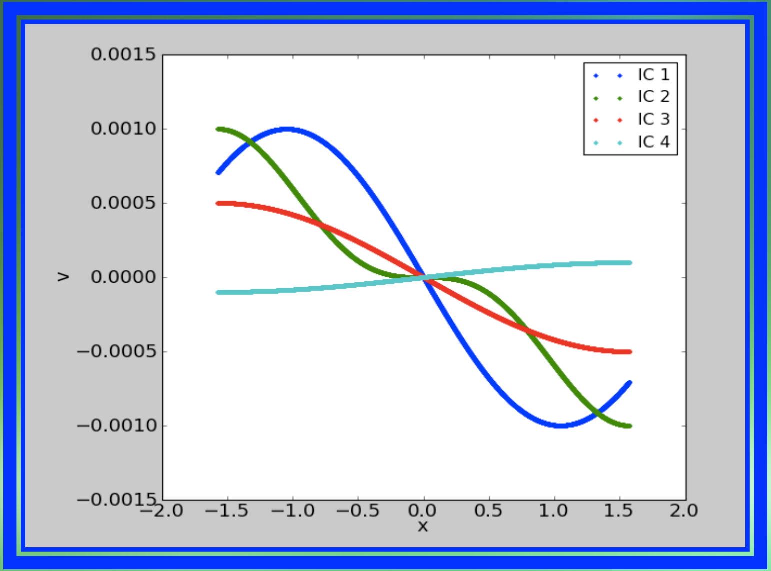
$$Q = \frac{\rho}{\sigma^3}$$

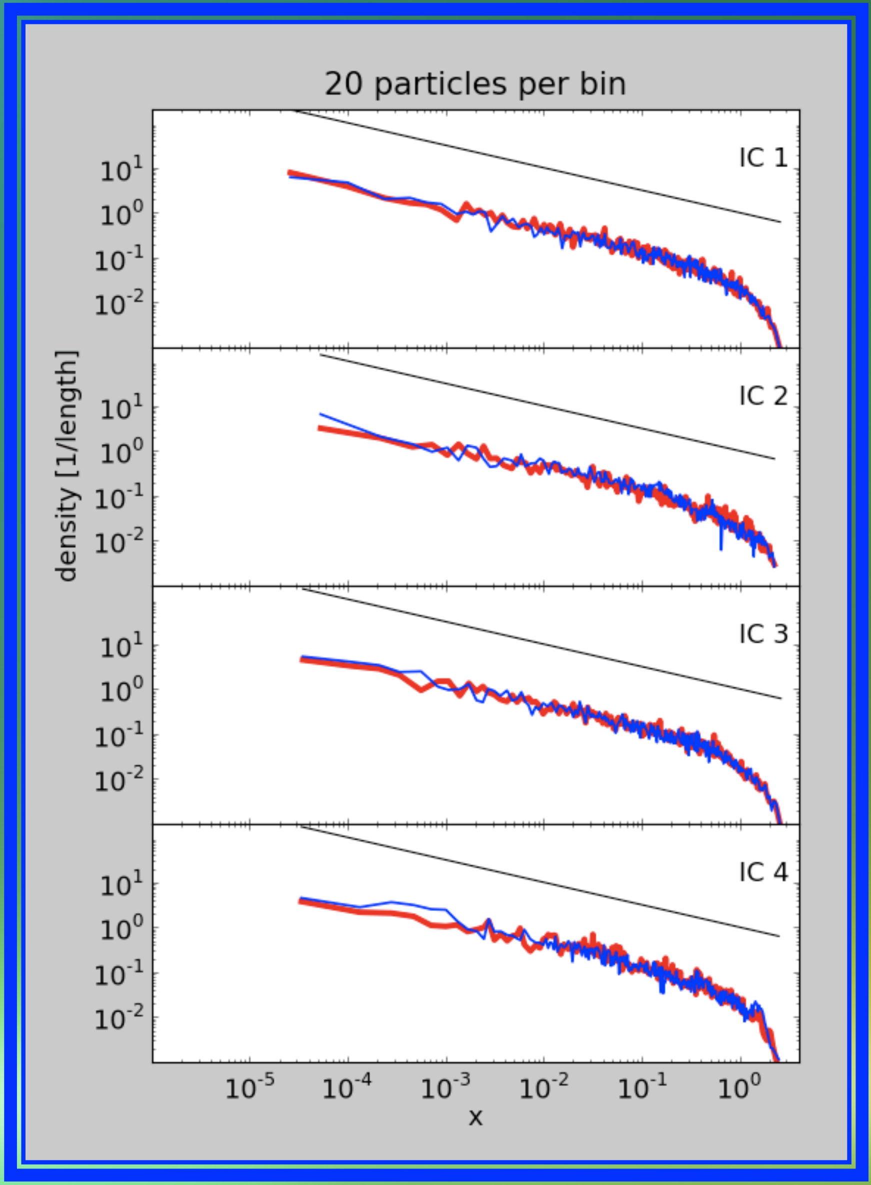




Changing the initial conditions

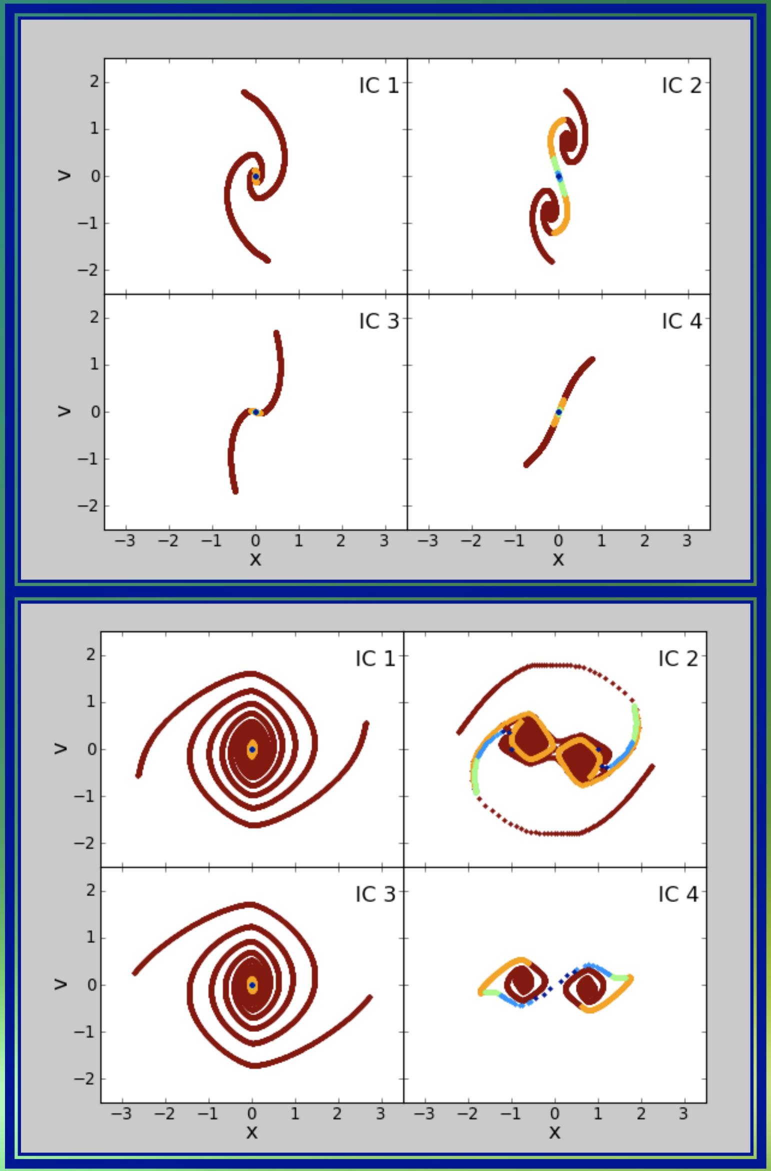
- The scaling law of the density profile is robust to changes in the initial conditions
- Four variations:
  - Slower tail in outer particles
  - Central inflection
  - Smaller amplitude
  - Coherently Expanding





Different paths to the same destination

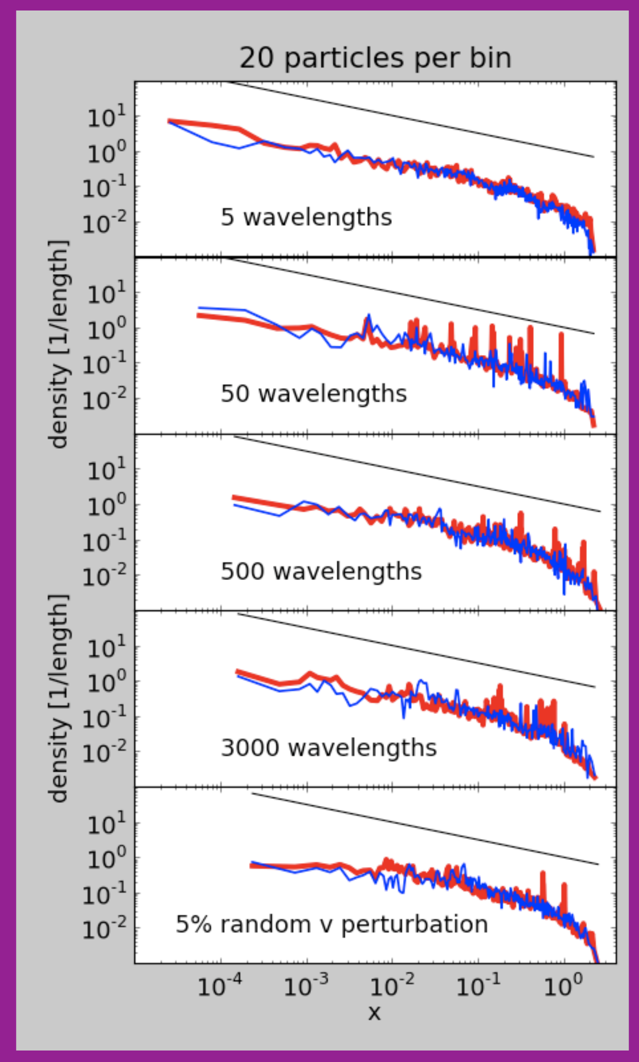
- Intermediate snapshots show very different formation mechanisms for the final object
- Color coding indicates initial particle position (binned logarithmically)
- IC 1 and IC 3 proceed similarly in a monolithic collapse
- IC 1 proceeds faster because it had a larger velocity amplitude
- IC 2 and IC 4 each collapse into two distinct objects, that later merge to form the final product.
- The cores of IC 2 and IC 4 are both composed of particles that started in the exterior
- The history of the inner particles is different in these two cases, but they ultimately wind up on the outer layer of the final product

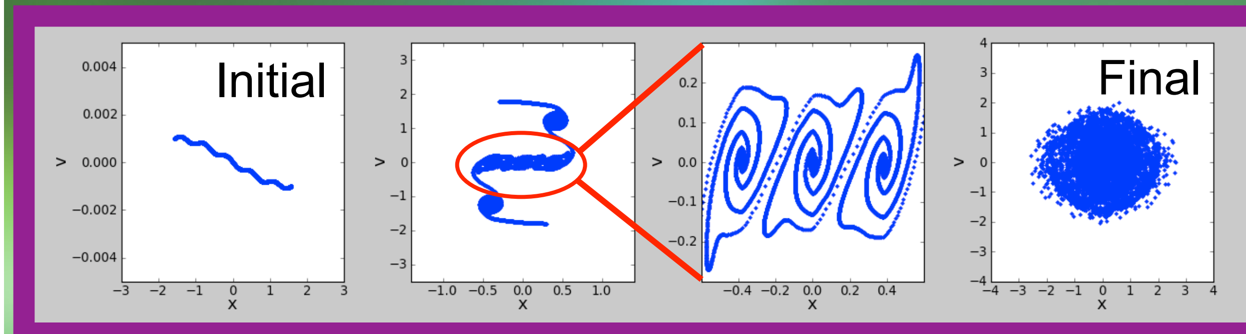


What will break the scaling?

- The maximum phase space density depends on the resolution (number of particles) forming the object
- A high frequency component in the initial velocities results in a large number of small objects forming initially
- If sufficiently numerous, these larger objects undergo their own violent relaxation
- The product of the mergers cannot have a higher phase space density than the progenitors
- Final maximum phase space density depends both on number of particles and on resolving small scale power in the initial conditions

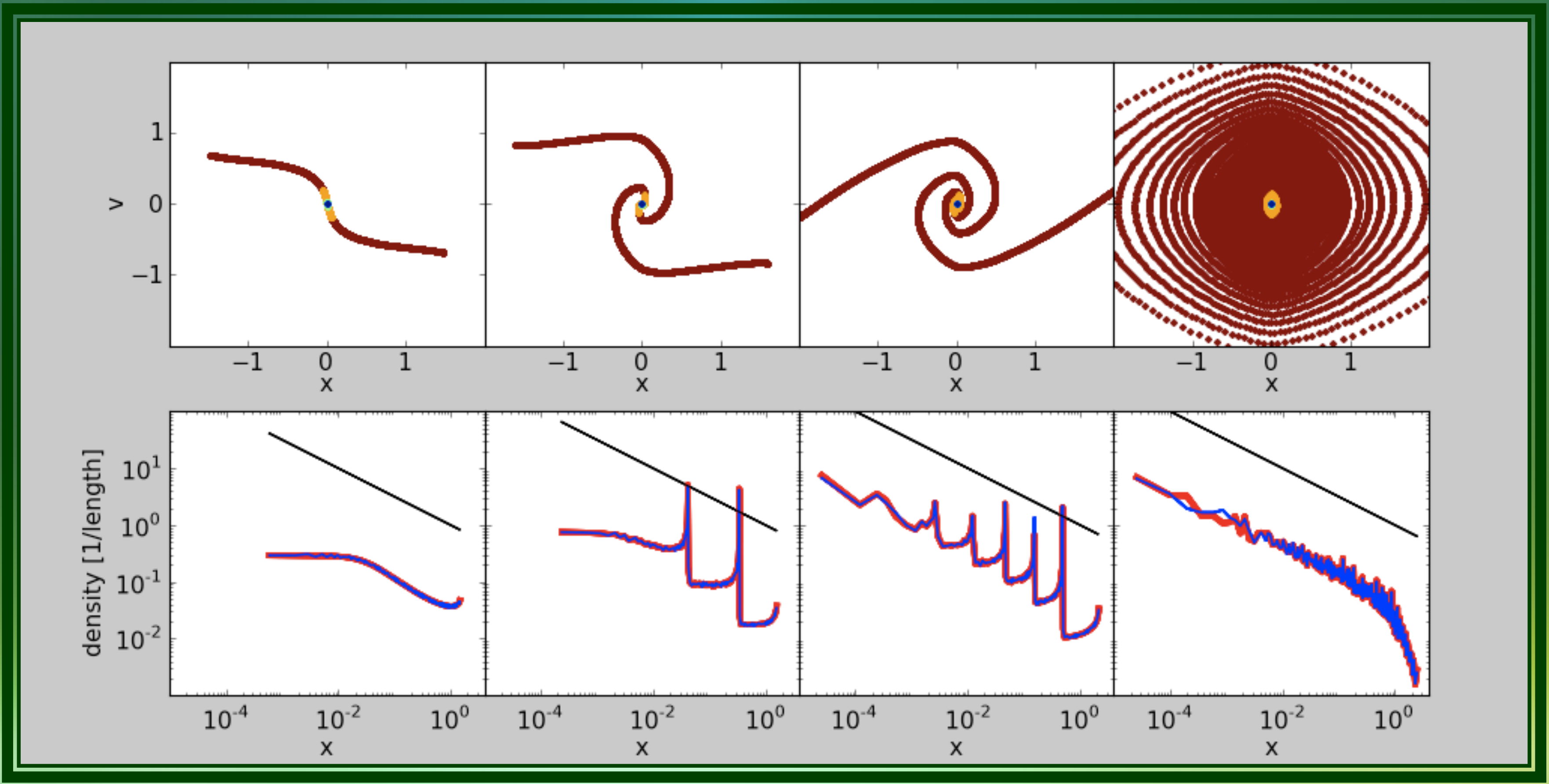
10000 particles





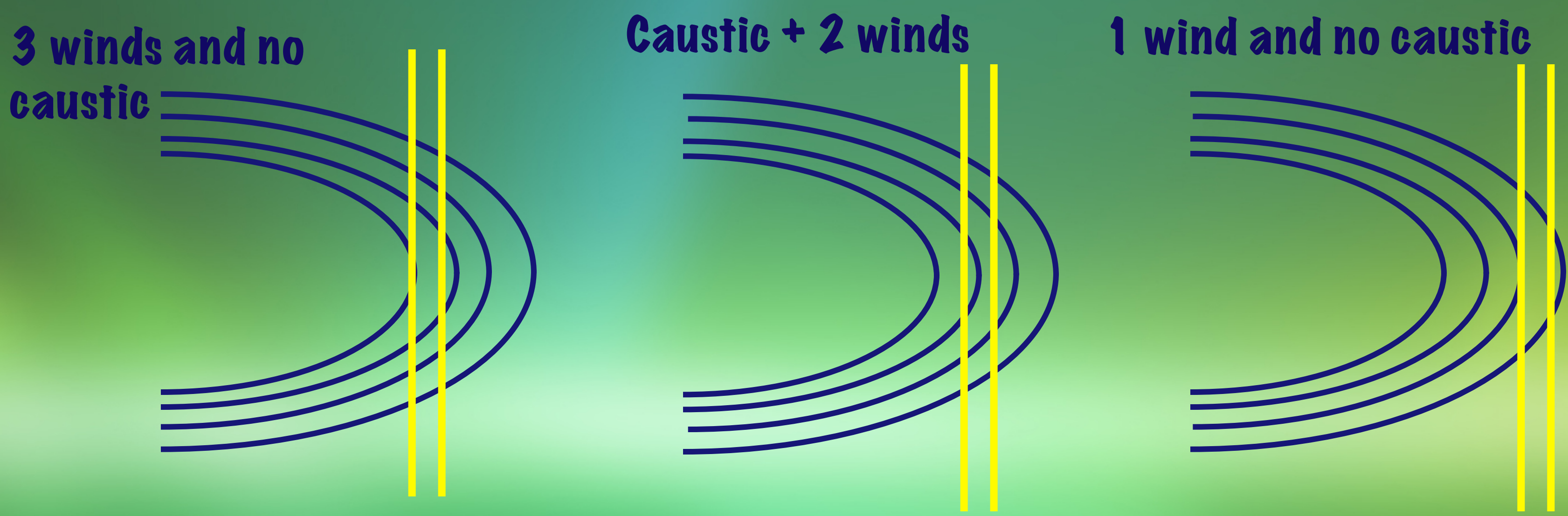
The build-up of the density profile

- The r$^{-1/2}$ scaling behavior is **not** a mean field effect once the coherence of the winds has been lost due to finite sampling

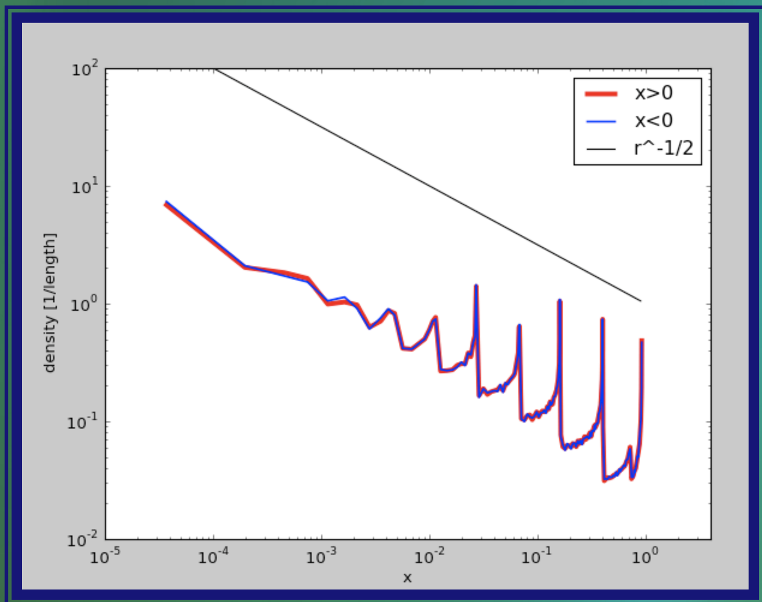


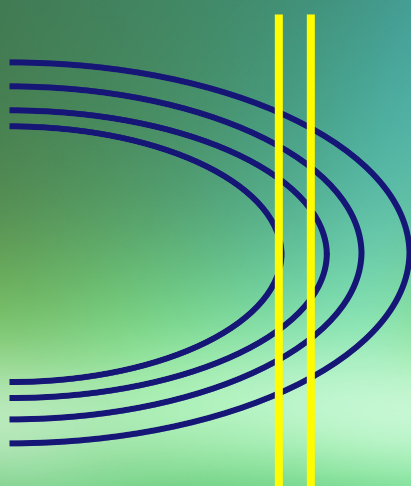
The build up of the density profile

- The formation of the power law density profile is a consequence of the phase mixing and violent relaxation
- Density is a projection through the phase space
- Two properties influence the density at any given radius
  - Geometric effects
    - The “slope” of the winds as they pass through the bin
    - The number of winds contributing to a bin
  - Sampling effects
    - how many particles are sampling a particular segment of the wind



The build up of the density profile





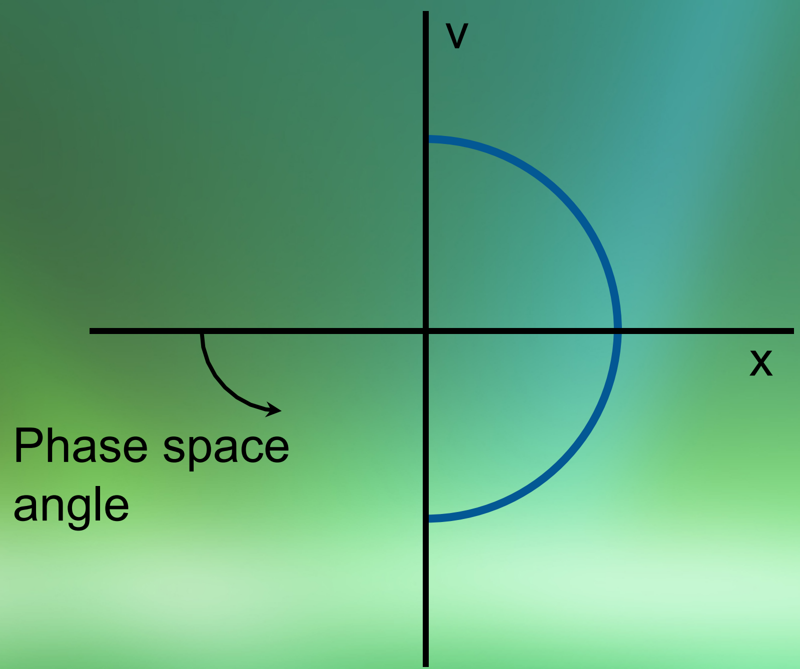
- Caustic heights irrelevant
  - Depend on bin width and placement relative to the exact position of the caustic
- Density directly outside caustic is highly relevant
  - Each "gap" is built of one fewer wind than the one just interior
  - These scale with the same slope as the finial virialized object
  - More caustics appear over time
    - Heights decrease
    - Spacing decreases
- Eventually the "gaps" form a continuum with the same scaling behavior

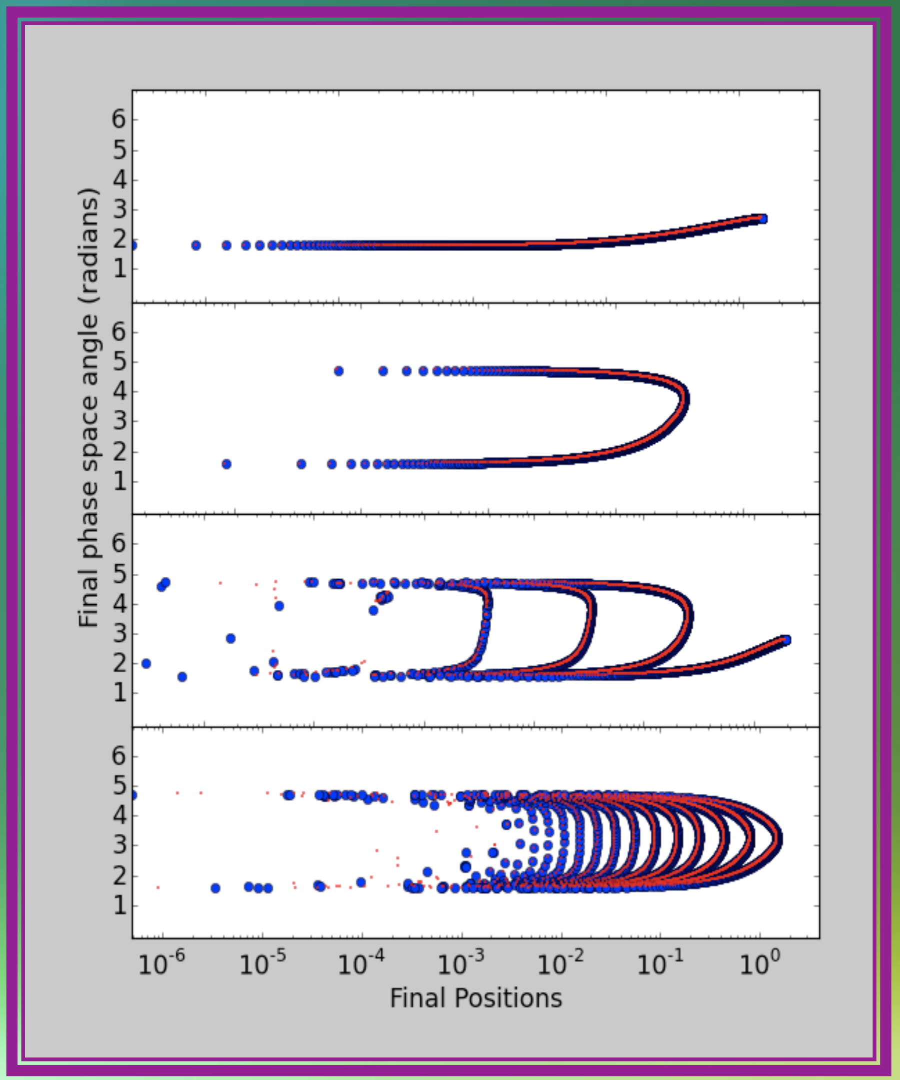
The build up of the density profile

$$\rho = \frac{1}{L_{\text{bin}}} \sum_{w}^{n_w} N_w$$

N_w = Number of particles per wind

n_w = Number of winds



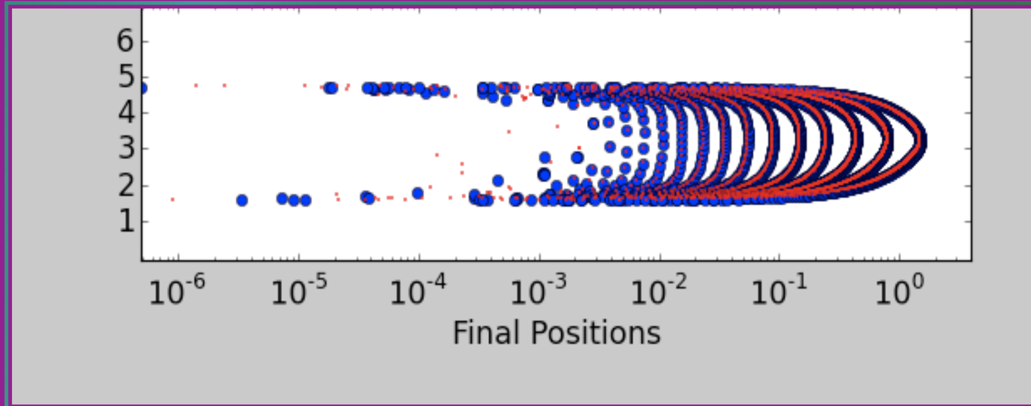


The build up of the density profile

- Number of winds n_w is close to equally spaced in log x -- power law
- Spacing decreases as a function of time

$$n_w \propto x^{a(t)}$$

- Number of particles per wind N_w is more complicated
  - Phase angles are not equally sampled
  - Interior winds more poorly sampled than exterior
  - Hard to model



$$\rho = \frac{1}{L_{\text{bin}}} \sum_{w}^{n_w} N_w$$

- To give us $\rho \propto x^{-1/2}$ N_w must either conspire to somehow be of the form

$$N_w \propto f(w) x^{b(t)}$$

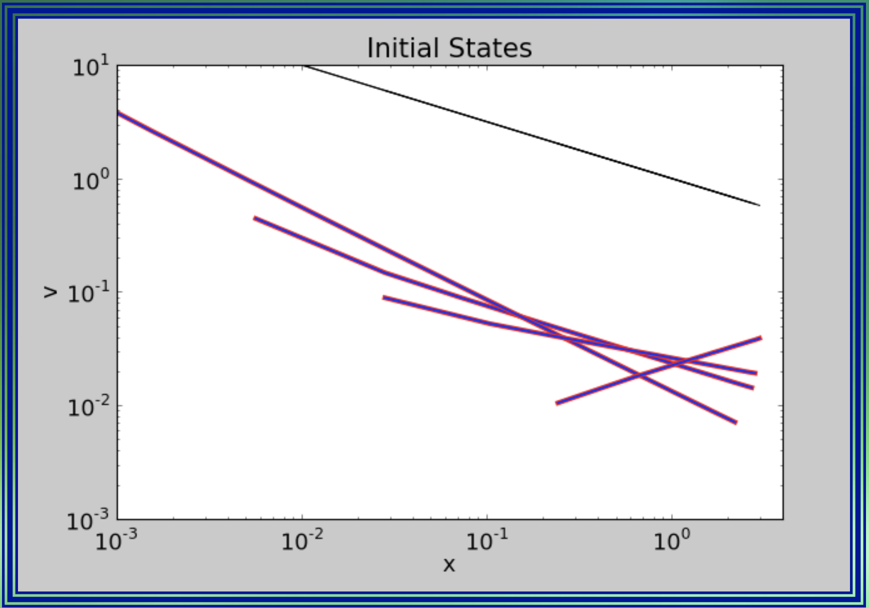
Weird. $\rightarrow$ $b(t) = -(a(t) + 1/2)$

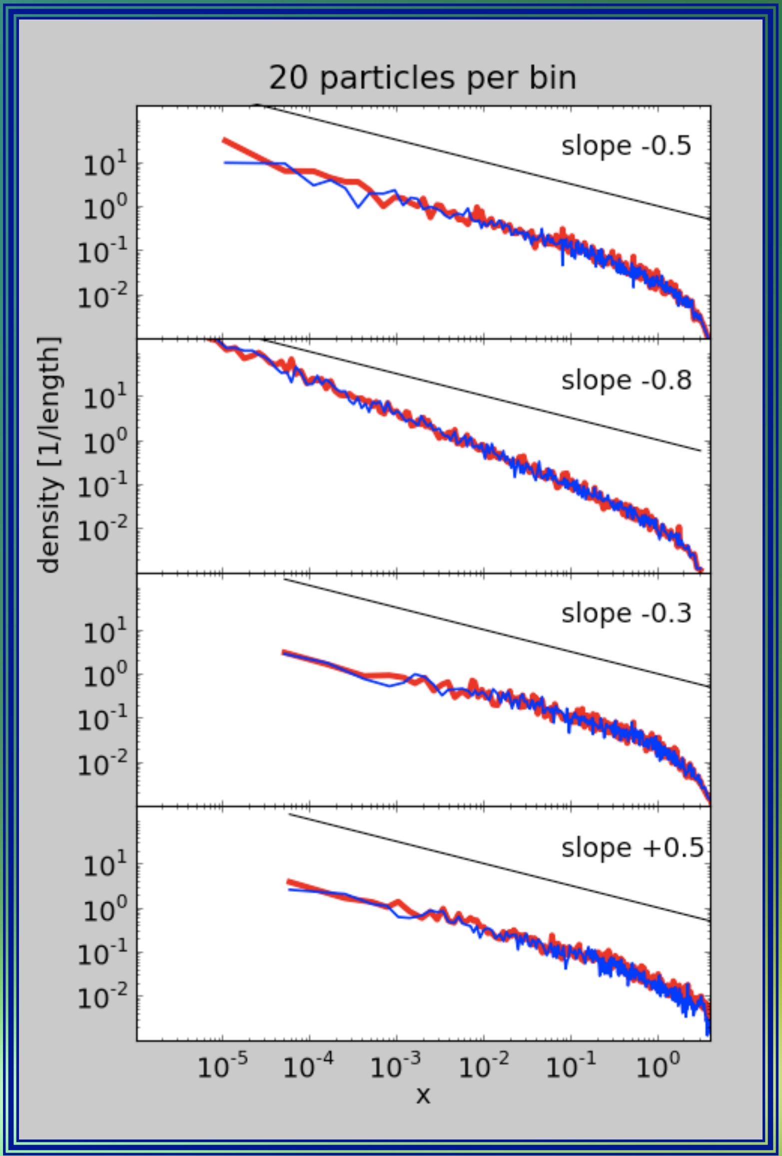
- Or more likely:

$$N_w \propto x^{b(w,t)}$$

Non-uniform initial conditions

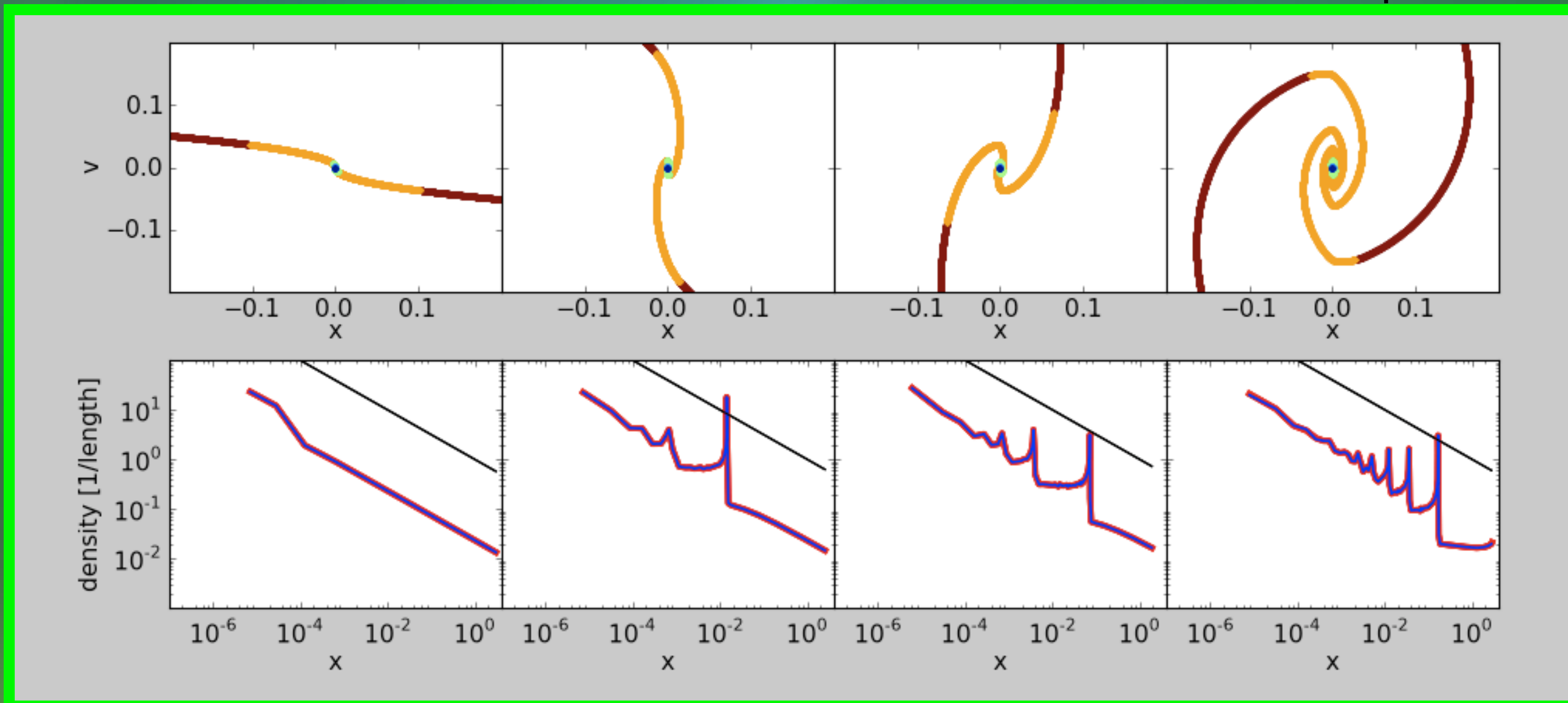
- Another class of cold initial conditions has particles arranged in an initial density profile
- This class of solution can be steady state if self-gravity is turned off





Non-uniform initial conditions

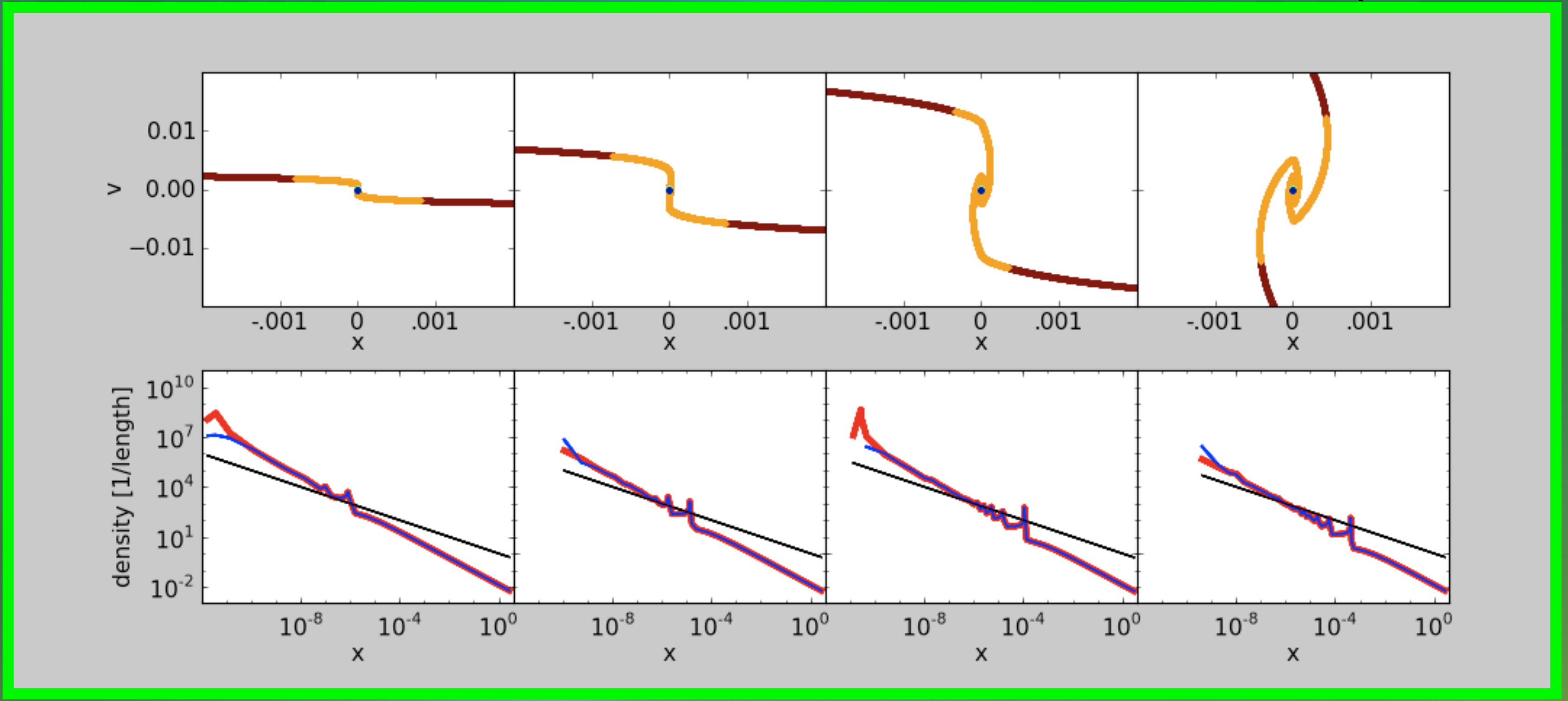
Initial slope -0.5



- Scaling before and after caustics maintains initial -1/2 slope
- Region in between caustics are strangely flat (as yet unexplained)
- Profile maintains slope as more and more caustics accumulate

Non-uniform initial conditions

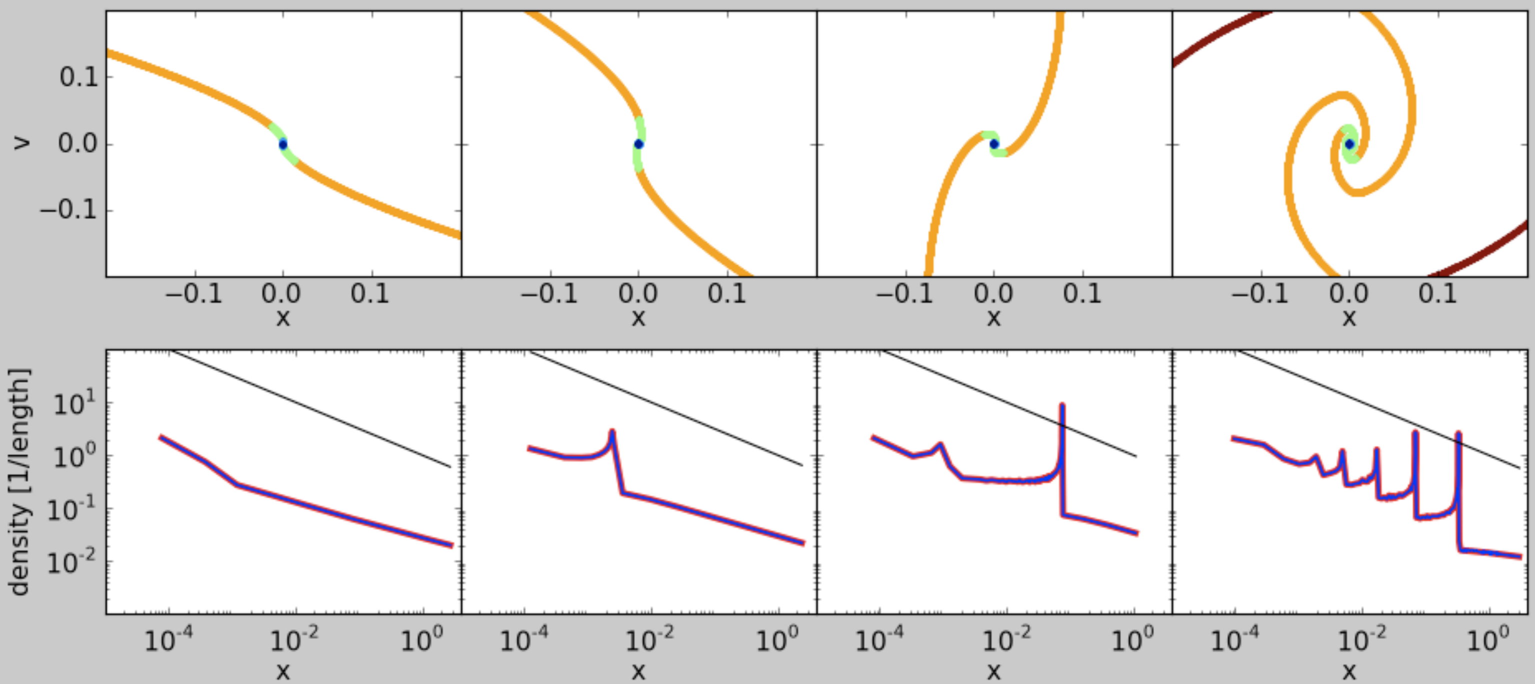
Initial slope -0.8



- Center winds up very quickly while the outside region has barely moved
- Caustics are very short because interior winds are poorly sampled
- This result is qualitatively similar to initial slopes -0.7 and -0.9
- No results for initial slope -1 or steeper.
- Unclear why this does not go to the attractor solution, but possibly it is because phase mixing happens so rapidly between interior and exterior that energy transfer is inhibited.

Non-uniform initial conditions

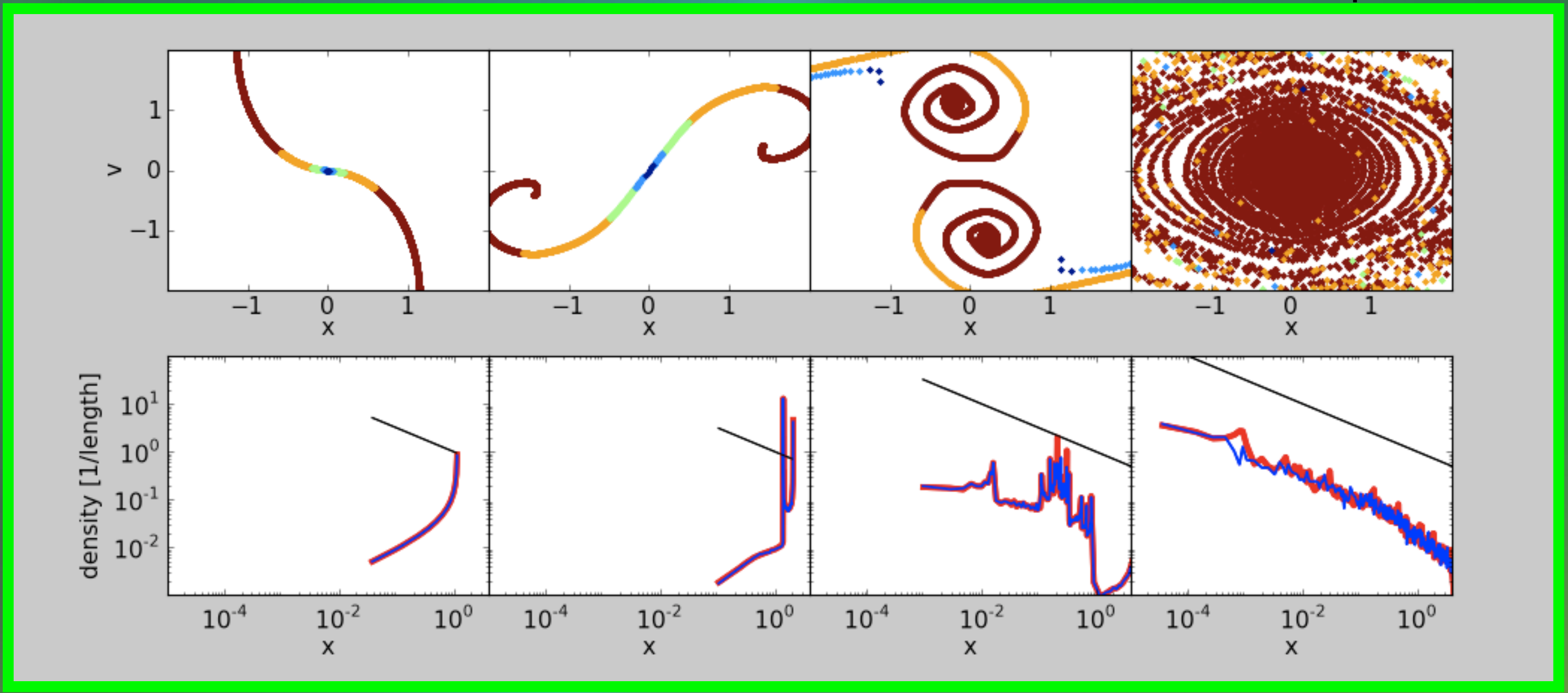
Initial slope -0.3



- ✦ In early stages scaling before and after caustics maintains initial -0.3 slope
- ✦ Region in between caustics are strangely flat (as yet unexplained)
- ✦ Caustic "gaps" seem to scale as $r^{-1/2}$, so the profile ultimately reaches that scaleing as it gradually becomes all "gaps"
- ✦ Caveat: All of these observations are "visual"

Non-uniform initial conditions

Initial slope +0.5



- ✦ This initial condition “silliness” shows that the $r^{-1/2}$ scaling is seriously robust to changes in power law density with slopes > -0.5
- ✦ Caustic seem to start at the outside rather than the inside
- ✦ This is a merger of two “one tailed” objects, both of which presumably have $r^{-1/2}$ scaling -- therefore one tailed and two tailed objects scale the same (obvious

Relevance to Dark Matter Detection Experiments

- Dark matter interaction rates depend on density and velocity of dark matter particles

- Direct Detections (Local dark matter interactions here on Earth)
  - For some specific class of models, this is sensitive only to high velocity tail of dark matter distribution
  - Rate predictions assume smooth NFW halo
  - A sizable fraction of mass may be tied up in dense phase space clumps
  - Local density and velocity may differ substantially from NFW-like predictions

- Indirect detection (annihilation signal from galactic center)
  - Shape of the inner profile is highly relevant
  - Changes or errors in phase space density may affect predicted boost (which depends on higher densities in substructure) and Sommerfeld enhancement (which depends on kinematic properties of substructure) from simulations
  - Velocity structure of the center will also affect signal

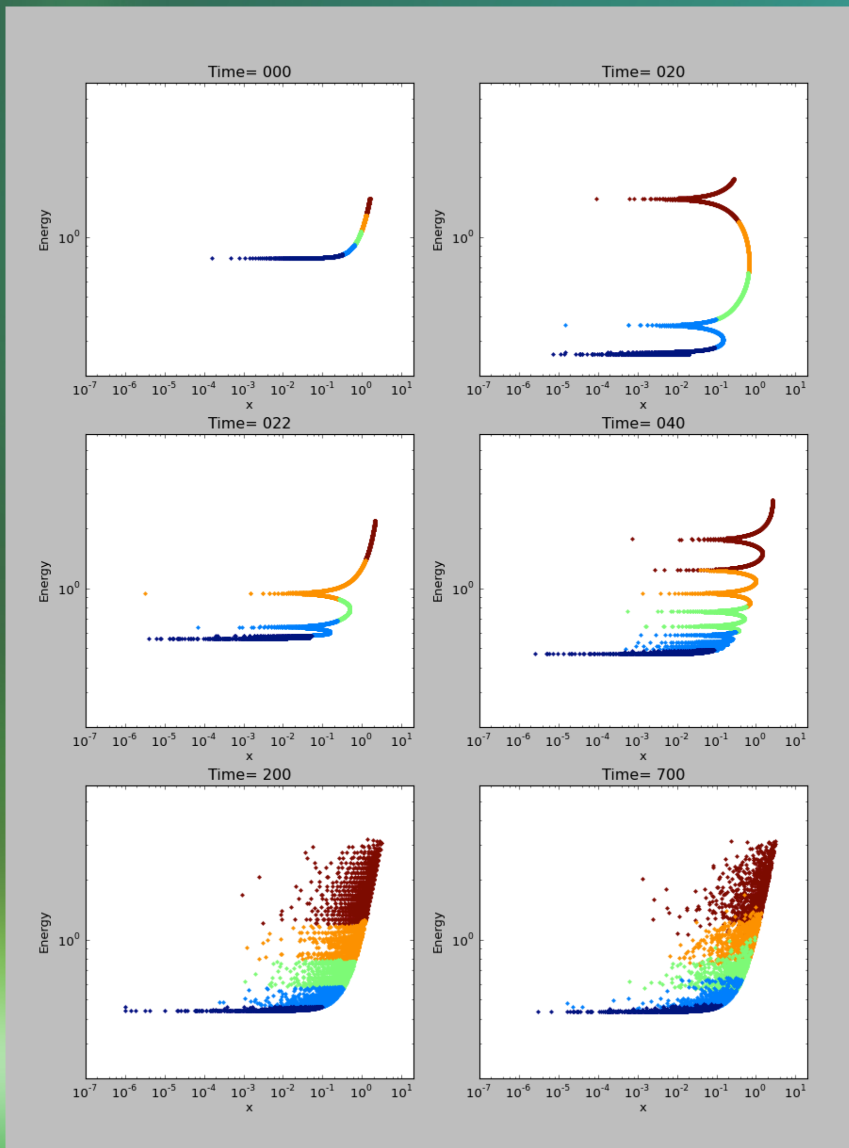
Conclusions

- One dimensional systems are cosmologically relevant because they model an early phase of sheet collapse
- The density profiles of cold 1D systems are attracted to a solution scaling as $r^{-1/2}$
- The final scaling law is quite robust to changes in both the initial velocity profile and the initial density profile
- The scaling law is insensitive whether there was monolithic collapse or mergers
- The scaling becomes more centrally cored if the initial conditions are "warm"
- High frequency perturbations of initial velocities decrease the maximum phase space density
- Initial conditions whose density follows a power law
  - Go to the attractor solution if the initial slope is >-0.5
  - Maintain the initial slope if initial slope is -0.5 < slope < -1

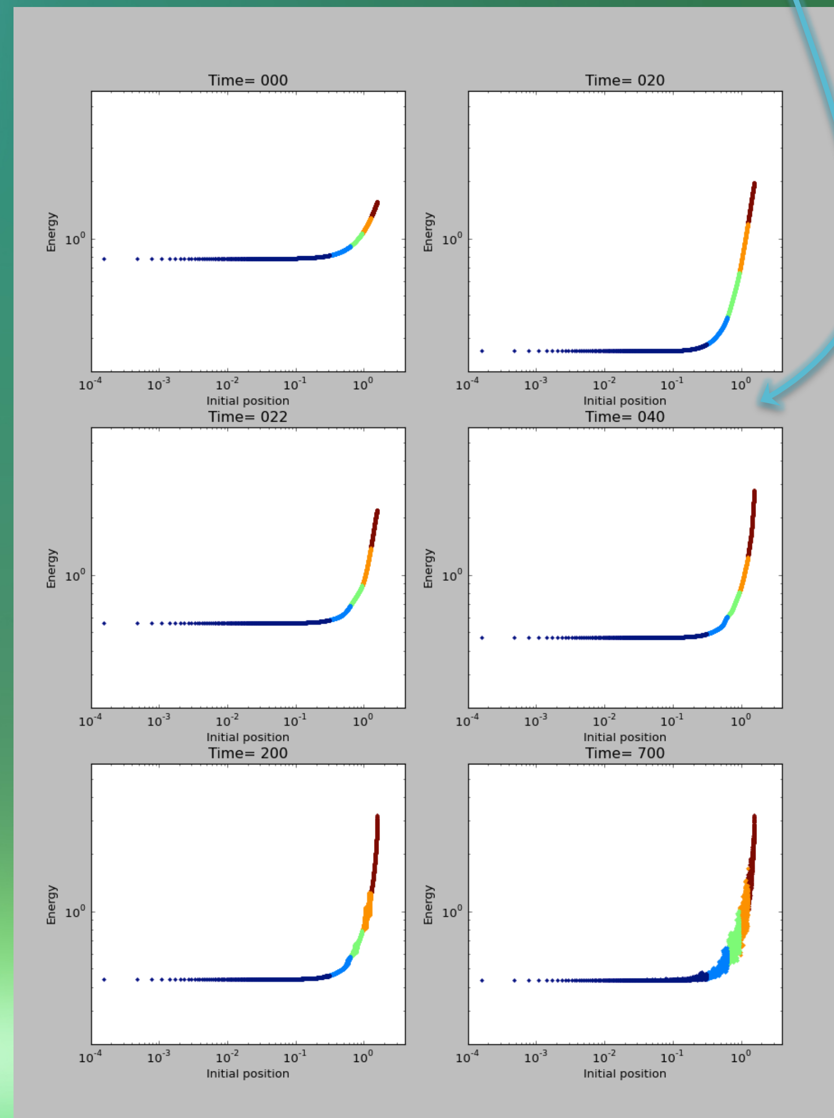
Energy Slides

Energy is a continuous variable along the winds

E vs. Final position

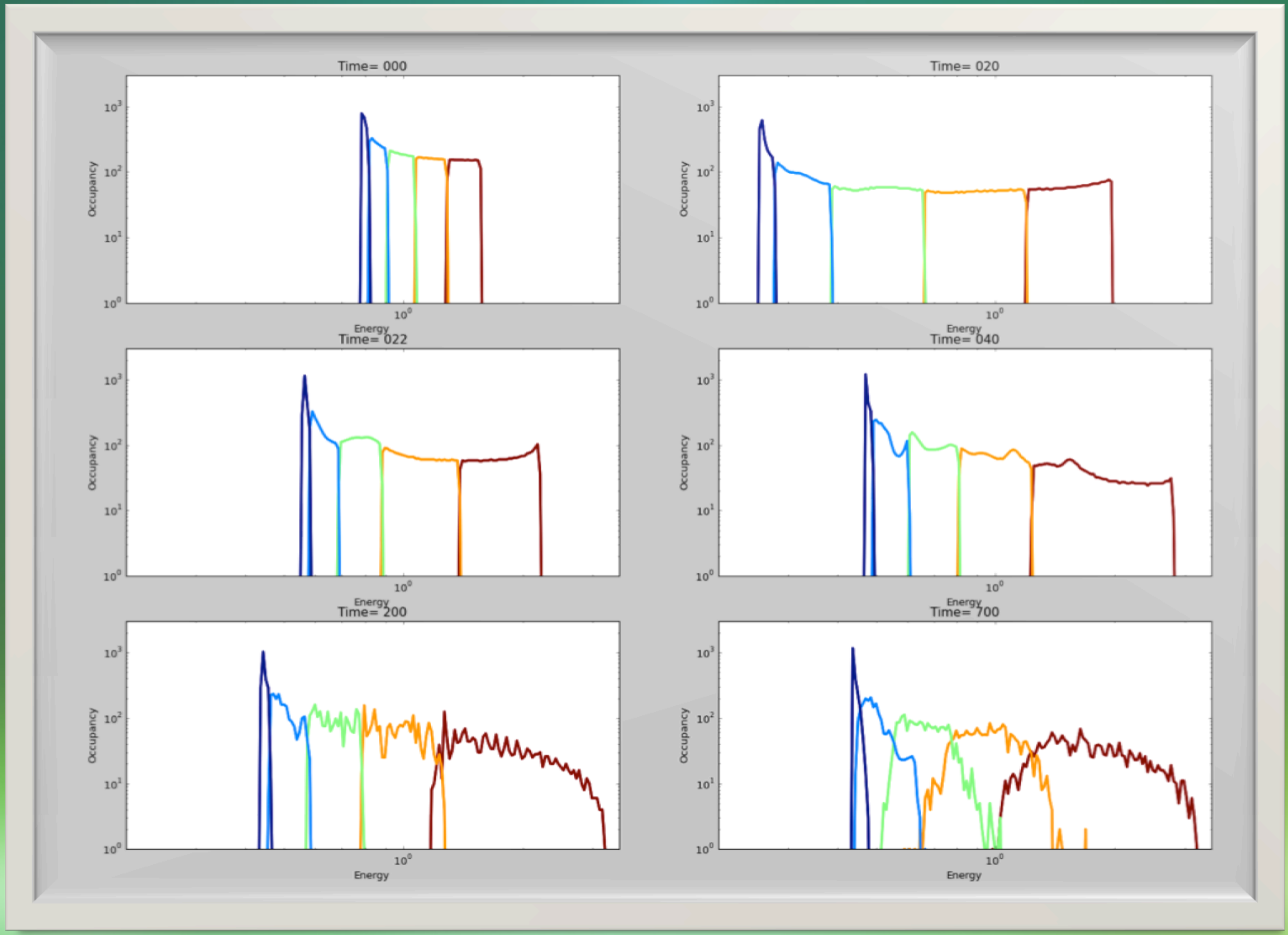


E vs. Initial position



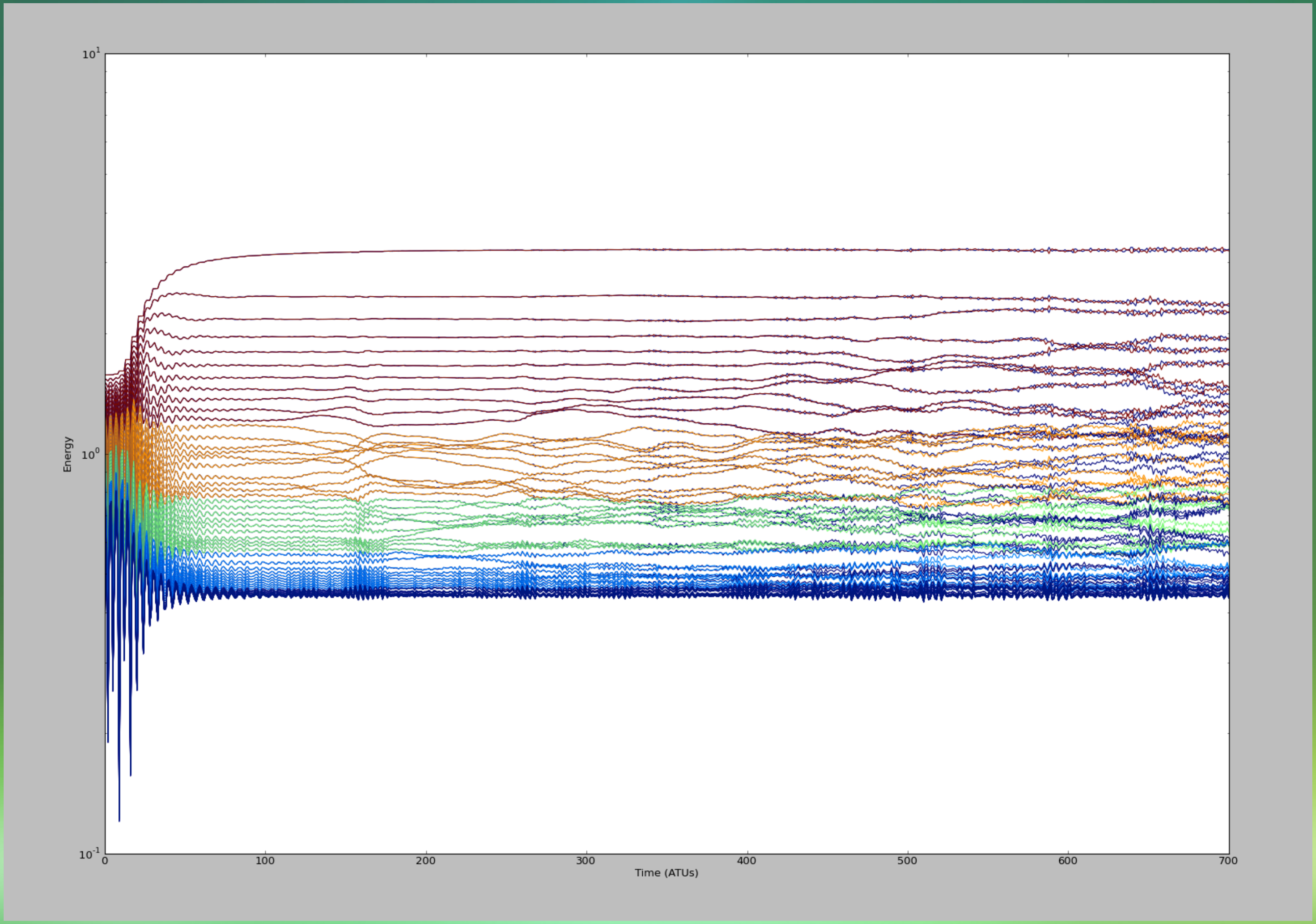
Energy slides

- Energies mix only at late times
- Scale as a power law

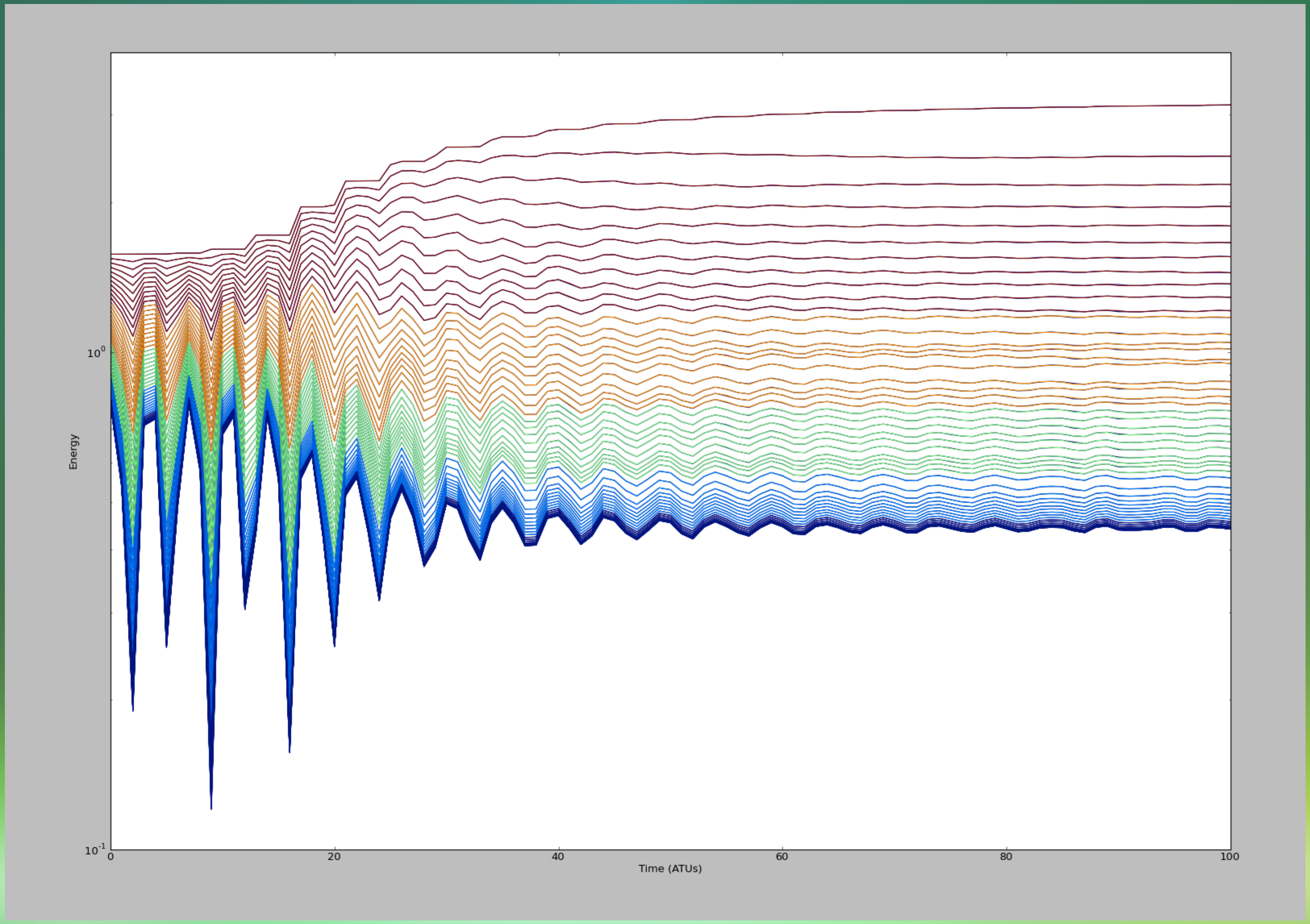


Energy Slides

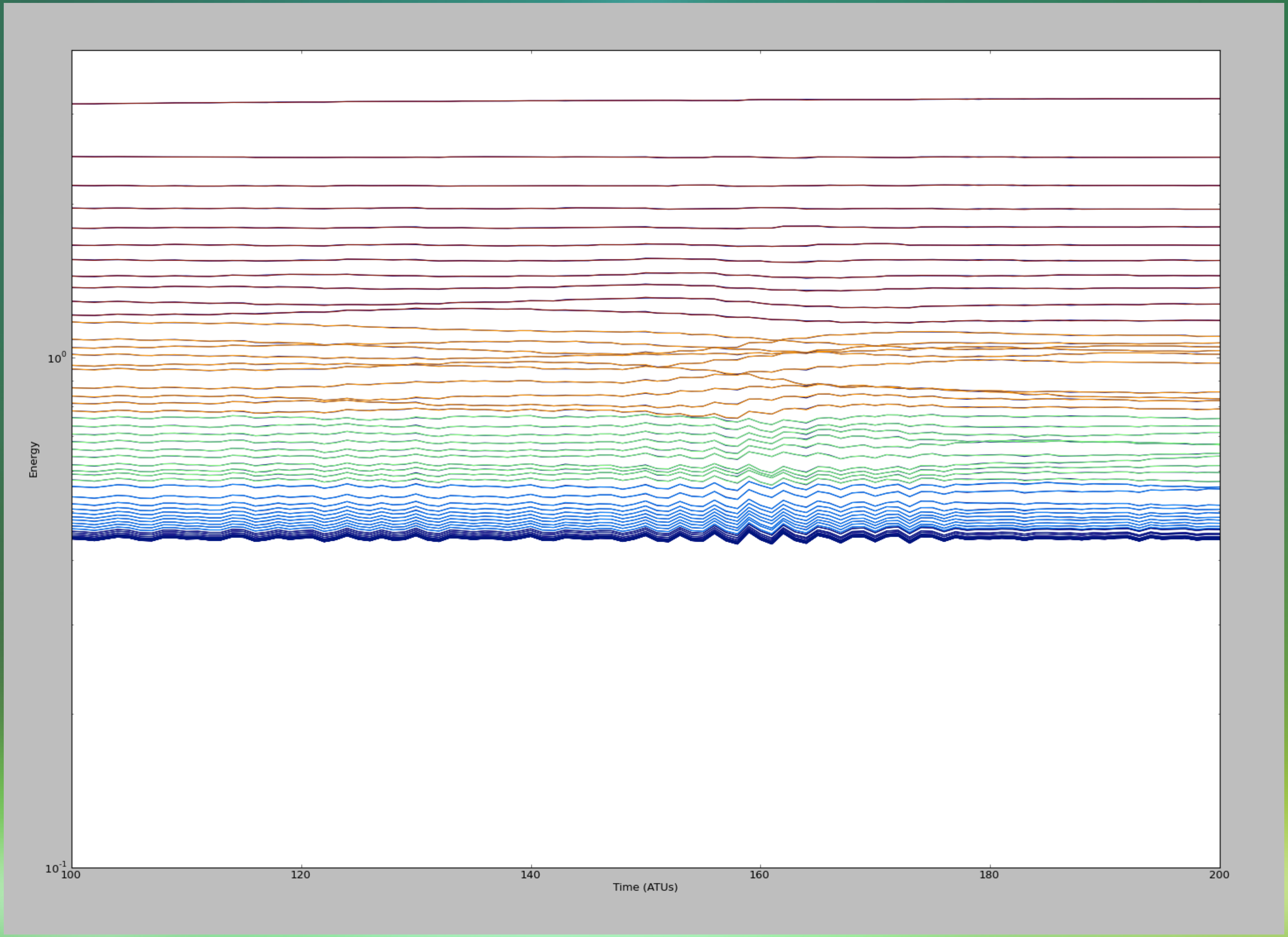
- The evolution of the energies with time



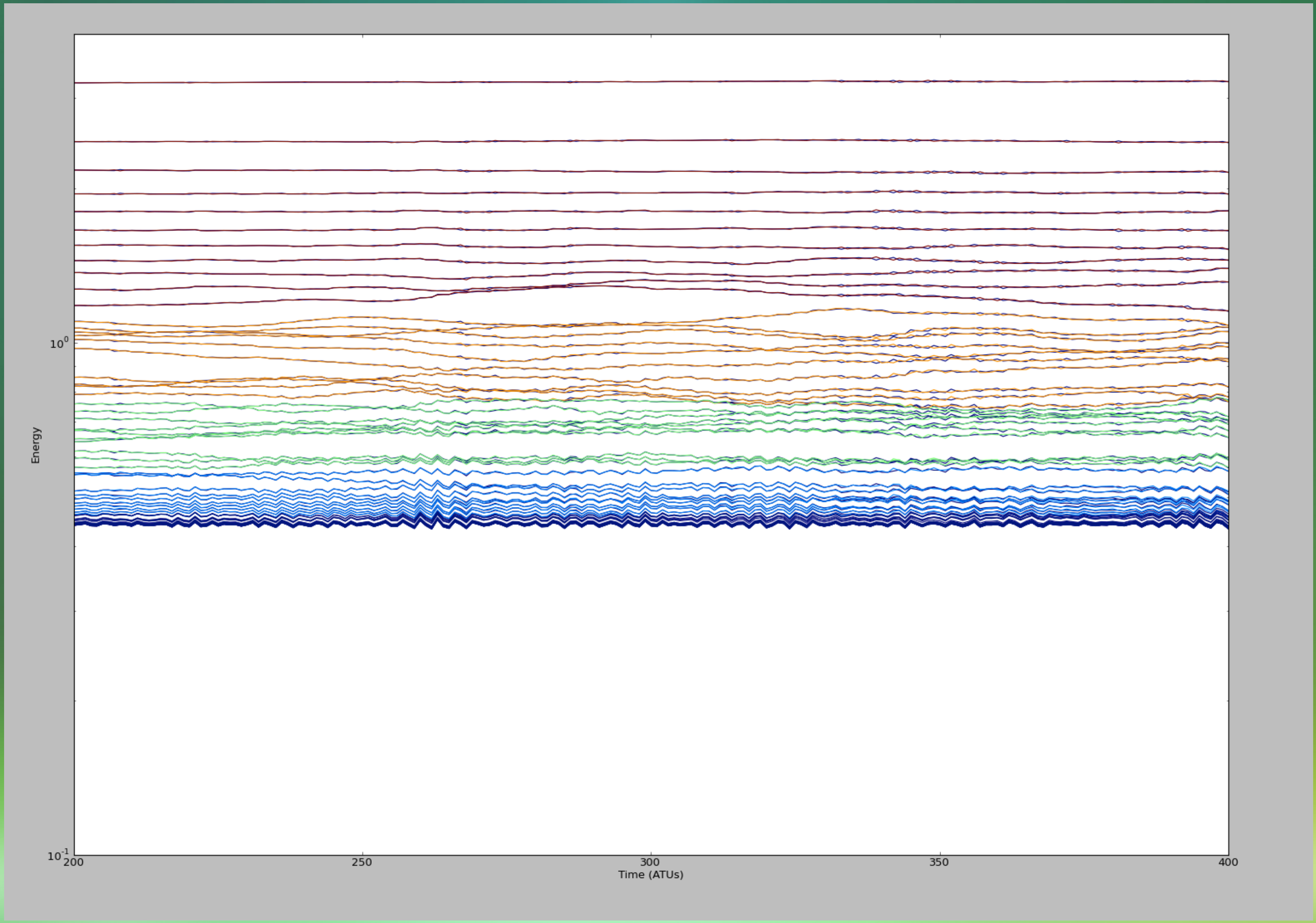
Energy slides



Energy Slides



Energy plots



Energy slides