Needs to Be Shorter: This question needs to be shorter. Questions should be as concise as possible. The question details field should be used to provide context and added detail.

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The question will have limited distribution. Please do not remove this tag unless the question has been suitably improved.

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Mathematics: Needs to Be Shorter Calculus Needs Attention

Mathematics: Why does John Gabriel state that standard calculus is flawed, while mainstream academia considers it to be true?...

2 Comments • Share • Report • Options

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Jack Huizenga
The question title and question description are not the place to put material that essentially attempts to answer the question. Your own personal answer is the place for this material. If that answer has been downvoted and collapsed, that is no excuse to cut and paste the same material into the question details.
Reply... Upvote • Share • Report • 10h ago

Tatiana Estévez
John Gabriel You keep vandalising this question, you cannot do this. However as this question is about you, I'm more than happy to delete this as per Quora's Policy on Protecting Individuals?

Tatiana, Quora Admin
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John Gabriel
A very good question indeed. Why is standard calculus flawed?

I could write several books on the matter, but English is not my first language, and I hate writing in any language. If my unpublished work is ever published some day (approx. 2000 pages), much more will be revealed.

In the meantime, I think that pictures are far more powerful than words and essays. So I have included a few links to applets that educate and entertain. There are no viruses or spy software in any of these applets. The applets are dynamic, that is, there are sliders and movable points. Use these to see how the standard calculus is indeed flawed.

If you have any questions or comments, I suggest you post them on my Facebook page: New Calculus | Facebook

Also, there are 65 articles at the following link addressing this question, and much, much more!

https://www.filesanywhere.com/fs...

Applets:

A comparison of the flawed calculus and the New Calculus derivative:
(This applet shows the initial flaw with Newton's definition)

https://www.filesanywhere.com/fs...

Cauchy's kludge:
(This applet shows how Cauchy in fact did nothing by adding the limit concept to Newton's definition. In fact, Cauchy made it worse!)

https://www.filesanywhere.com/fs...

Riemann's Kludge:

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(This applet shows that considering the integral to be the limit of an infinite number of rectangular areas is also counter-intuitive and a very bad idea)

https://www.filesanywhere.com/fs...

The New Calculus Derivative at a glance:

https://www.filesanywhere.com/fs...

The New Calculus Integral at a glance:

https://www.filesanywhere.com/fs...

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John Gabriel

So Anders Kaseorg (with Math Prof Jack Huizenga) and a few Anons, recently launched a full-fledged attacked on my New Calculus.

Kaseorg is reasonably intelligent, but he has been brainwashed to a good extent by mainstream maths. So much so, that it clouds his ability to think clearly.

As for Jack, he didn't know until recently that sin(x) only takes radians as an argument.

Both these gentlemen have had much to say about my New Calculus - all of it wrong.

Read my responses to see how they try to skirt the issues, produce handwaving arguments and outright assertions without any substantiation.

Also download my applets to learn why the standard calculus is flawed.

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19 people are following this question.

4 Answers

85 **Anders Kaseorg, MIT 2008, S.B. in Mathematics with Computer Science** Edit Bio

Votes by **Jack Huizenga** (Math Professor, University of Illinois at Chicago), **David Joyce** (Professor of Mathematics at Clark University), **Alon Amit** (PhD in Mathematics; Mathcircler.), **Anurag Bishnoi** (Ph.D. student in Mathematics at Ghent University.), **Jitendra Prakash** (PhD Candidate), and 79 more.

I was curious about this enough for some reason to take the time to have a patient(ish) discussion with John. What I found was that the root of his disagreement with mainstream mathematics is that he is a strict finitist : he does not accept the existence of any infinite set, such as the set $\mathbb{N}$ of natural numbers (postulated by the axiom of infinity in Zermelo–Fraenkel set theory). As such, what he means by “set” and other terms (let alone “tangent line”!) is different from what mainstream mathematicians mean by these terms—he is not speaking the same language.

When he asserts that a standard construction is “juvenile and invalid” and that “ignorant educators…never understood mathematics” and that “You are a pathetic joke. Yes, you really are that stupid”, what I think he really means is that the principles of strict finitism do not support the justification of these constructions, and that most mathematicians operate under a system that is incompatible with strict finitism. That is indeed correct. If one replaces ZFC with a weaker system such as GST that includes no infinite sets, one loses the ability to define the real numbers (using dedekind cuts , Cauchy sequences , etc.) and develop a theory of real analysis that formalizes calculus.

To me this is just a good reason not to replace ZFC, but let’s roll with this for a minute (one can certainly do interesting mathematics as a finitist) and consider his other claim. John claims to have invented “the first rigorous formulation of calculus in history”, so presumably what he means is that he has a way to formalize calculus within a strictly finitist axiom system, up through at least integrals (he says “any function can be systematically integrated”). I’m not sure exactly what axiom system John subscribes to—if I find out, I’ll let you know—but in any case, this is basically impossible. Reverse mathematics has shown that proving the integrability of every continuous function on the closed unit interval requires a much stronger system—at least as strong as WKL_0, which includes an axiom that I’m sure John wouldn’t touch with an eleven-foot pole: the weak König's lemma (every infinite binary tree has an infinite branch).

Given the impossibility of what John aims to achieve and the extent of its irrelevance to real mathematics, the many specific problems with his “formulation” of calculus look almost trivial and redundant by comparison. Perhaps that's why John feels the need to so viciously attack anyone who contradicts his work. Which is a shame, because some of his work (especially his interactive GeoGebra applets) would actually do a pretty good job of illustrating how standard calculus works, if he were willing to use the same common language of mathematics for long enough to see this.

Edit

3 Comments • Share • Delete • • 22 Mar

Comment

Sridhar Ramesh 3 votes (show)

In fairness, one might well develop a form of integral calculus without the property that absolutely every continuous function on the closed unit interval is integrable... Or with that property, but nontraditional in some other way, such that the weak Koenig's lemma failed to follow [RCA_0 not having been adopted in full].

Not that I think John Gabriel's approach to calculus is worth much consideration. I just think bringing up WKL_0 in objection to it is not particularly damning.

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Jack Huizenga 14 votes (show)

Your level of patience in that conversation is admirable. I'm afraid the issue isn't merely strict finitism, but also his desire to use "dictionary" definitions of terms instead of mathematical ones. These imprecise definitions and axioms (together with his stubbornness) make his work essentially unfalsifiable in his view.

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John Gabriel

Dictionary terms? :-) You mean like "tangent line" that has been around since 1582? I have news for you: that's not just a dictionary term, even the Britannica used it in exactly the same way in most of its publications.

Sorry, but you are not allowed to change definitions mid-stream. Besides, even if I let you claim that the cubic has a tangent line at x=0, you still have a problem of then reconciling what it means to be a derivative.

The problems are in mainstream maths, not in the New Calculus.

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Alex Kritchevsky 3 votes (show)

god you are insufferable

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John Gabriel

Do you enjoy making a nuisance of yourself? Please, do not respond again! No one cares or wants to know what you think of me or anyone else. Behave yourself if that's at all possible. Tsk, tsk

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Alex Kritchevsky

To answer your first question: yes.

I love the chance to annoy someone who is so sad as to post their drivel on every post they can find, desperately pleading for attention and validation.

Reply... Upvote • Share • Report • 24 Mar

Ari Zax 3 votes (show)

'When he asserts that a standard construction is "juvenile and invalid" and that "ignorant educators…never understood mathematics" and that "You are a pathetic joke. Yes, you really are that stupid", what I think he really means is that the principles of strict finitism do not support the justification of these constructions, and that most mathematicians operate under a system that is incompatible with strict finitism.'

Your calm, dispassionate tone makes it impossible for me to read this with a straight face.

Reply... Upvoted (Undo) • Share • Report • 25 Mar

John Gabriel

I don't like labels, but if you want to think about me as a finitist, you are already assuming that I agree with most of the finitist views, which is false.

"he does not accept the existence of any infinite set, such as the set
$\mathbb{N}$
of natural numbers (postulated by theaxiom of infinity in Zermelo–Fraenkel set theory). "

Of course I don't agree. Do you have any idea what goes into the construction of natural numbers and rational numbers? The spaghetti axioms (Peano) assume all these things.

"As such, what he means by “set” and other terms (let alone “tangent line”!) is different from what mainstream mathematicians mean by these terms—he is not speaking the same language."

That was exactly what Georg Cantor meant when he defined set - a finite set.

"what I think he really means is that the principles of strict finitism do not support the justification of these constructions, and that most mathematicians operate under a system that is incompatible with strict finitism."

Not true. Mainstream mathematics does a pretty good job of shooting itself in the head. No finitism is required.

"John claims to have invented “the first rigorous formulation of calculus in history”, so presumably what he means is that he has a way to formalize calculus within a strictly finitist axiom system, up through at least integrals (he says “any function can be systematically integrated”). "

The formulation of calculus has nothing to do with the foundations of mathematics. It assumes the basic properties and existence of rational numbers and incommensurable magnitudes. It would work just fine whether ZFC was invented or not.

"I’m not sure exactly what axiom system John subscribes to—if I find out, I’ll let you know—but in any case, this is basically impossible."

I don't use ZFC axioms. It's pure rot. Archimedes used calculus just fine long before the idiot Peano came to be. Same for Newton and all the others. Their work was just as rigorous as anything in existence today.

"Reverse mathematics has shown that proving the integrability of every continuous function on the closed unit interval requires a much stronger system—at least as strong as WKL_0, which includes an axiom that I’m sure John wouldn’t touch with an eleven-foot pole: the weak König's lemma (every infinite binary tree has an infinite branch)."

False. Every continuous and smooth function is integrable in the New Calculus. I'll share all the details if I am ever recognised. There is a small example of the normal distribution function. That's all I sharing at this time.

"Given the impossibility of what John aims to achieve and the extent of its irrelevance to real mathematics, the many specific problems with his “formulation” of calculus look almost trivial and redundant by comparison."

Just more assertions....

"Perhaps that’s why John feels the need to so viciously attack anyone who contradicts his work."

That's a joke!

"Which is a shame, because some of his work (especially his interactive GeoGebra applets) would actually do a pretty good job of illustrating how standard calculus works,..."

More like how it does *not work*. :-)

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Anders Kaseorg 1 vote by Ricky Kwok

If you claim that mainstream mathematics shoots itself in the head without requiring finitism, then you should be able to explain to me why the Dedekind cut construction of the real numbers fails *without denying the existence of infinite sets* like the set $\mathbb{Q}$ of rational numbers, and the sets A and B that I constructed in the other thread. I understand that *you* think these sets are invalid, but mainstream mathematics says that they're valid. Can you show what problems arise from this construction within mainstream mathematics?

Reply... Share • Report • 22 Mar

John Gabriel

The sets say nothing about the real number. In fact, the way you defined sqrt(2) is how you would have to define every real number, that is, you would have to write these as some function using the same set notation. You can't do this for all the real numbers - it's impossible. You don't even have a general method for doing it. Therefore you don't have a valid construction.

In essence, you would need an infinite vector array pointing to functions for each real number. It's absurd. You don't have all those functions, so they can't be defined. You can't even call it a set, because there can be more than one way to describe certain numbers.

As I said, mainstream maths does a pretty good job of shooting itself in the head.

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Anders Kaseorg 4 votes (show)

You tell me that I can't have this infinite vector array, but ZFC tells me that I can. You tell me that there's more than one way to describe certain numbers, but I can prove in ZFC that there is exactly one. This is not a self-contradiction in ZFC, it's just another restatement of your refusal to accept ZFC.

(Remember, ZFC does not require us to write down every individual real number before we can write down the set $\mathbb{R}$ of real numbers. It's true that we can't write down every individual real number, because there are only countably many things we could ever write down. Still, we can obtain the entire set $\mathbb{R}$ at once from the ZFC axioms, notably the axioms of power set and specification, which let us construct an uncountable set by specifying the properties of its elements.

We can write down a Dedekind cut for any specific real number one would ever actually care about, though, including plenty of irrational numbers.)

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John Gabriel

ZFC does not tell you anything. ZFC cannot even produce the number 0 or 1 without the Natural numbers in place.

You are writing a lot of nonsense.

You can't even produce a D cut for square 2.

As long as you can't tell me the limit of the lower set, you can't deduce anything about the sets. sqrt(2) is not the limit. You only assert that it is the limit.

Finally, there are many functions that can be used to represent the same real number, contrary to your claims. For square root of two, here are two functions:

$f(x) = \dfrac{2x+2}{x+2}$ and

$g(x) = \sqrt{x}$

$f(\sqrt{2}) = g(2) = \sqrt{2}$

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Anders Kaseorg 3 votes (show)

ZFC tells me exactly nine things (seven axioms and two axiom schema). One of the nine things is the axiom of infinity: $\exists \mathbb{N}[\varnothing \in \mathbb{N} \wedge \forall n[n \in \mathbb{N} \implies n \cap \{n\} \in \mathbb{N}]]$. You don't have to "agree" with those things, but you can't *deny that ZFC says them*.

Denying that there is a symbol called $\mathbb{N}$ in ZFC with this property is like denying that there's a piece called a knight in the game of chess with certain moves. Nobody can force you to play the game of chess, and nobody can make you believe that there's a correspondence between what chess says a knight is and what the British monarchy says a knight is, but none of that changes the fact that *there is a piece called a knight in the game of chess*—that's just part of the rules of chess. And I don't need to presuppose the existence of knights to say that.

Within the rules of ZFC, I can construct the (ZFC-)sets of natural numbers, integers, and rationals; I can construct a Dedekind cut for $\sqrt{2}$ without presupposing the existence of $\sqrt{2}$ or saying anything whatsoever about "limits"; I can prove that the two Dedekind cuts corresponding to your $f(\sqrt{2})$ and your $g(2)$ are identical. And I can construct the set of real numbers and do rigorous calculus with them.

You can assert that ZFC is "nonsense", that you have a "simpler" way to formalize these things, that everything I have said is a "joke"—but none of this changes the fact that I did what I did within the rules of ZFC, and this is a valid way to formalize calculus.

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John Gabriel

I have never denied there is a symbol N. You appear to be unable to grasp the fact that you can't just assume its existence. That's what Peano and mainstream maths has done. :-)

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Anders Kaseorg 2 votes (show)

These two statements are different:

(a) "There exists a set $\mathbb{N}$ with these properties."

(b) "*One of the axioms of ZFC asserts* that there exists a set $\mathbb{N}$ with these properties."

(Where by "these properties", I mean $\varnothing \in \mathbb{N} \wedge \forall n[n \in \mathbb{N} \implies n \cap \{n\} \in \mathbb{N}]$.)

If I were asserting (a), then it might be reasonable for you to reply that I "can't just assume its existence". But for the purposes of this discussion, I'm not asserting (a). I'm only asserting (b). By asserting (b), I'm not assuming the existence of anything. I'm just quoting one of the rules of the game called ZFC.

Again, my purpose in saying this is not to argue that ZFC is "true" and that you should assume without question everything that ZFC tells you. My purpose is only to justify that ZFC is one possible basis for a rigorous formalization of calculus. Not the only one, not even necessarily the best one, just one. There might be interesting discussions to be had about which of many possible formalizations is best. But you'd need a much more sophisticated argument to demonstrate that ZFC is *flawed*—you'd need to find a contradiction within the rules of ZFC, in the same way that Russell's paradox was a contradiction within the rules of naive set theory.

You haven't come anywhere close, because you refuse to consider the idea of even *hypothetically* working within the rules of ZFC. You won't even acknowledge that within the rules of ZFC, the ZFC axiom of infinity holds. How can you purport to demonstrate that within the rules of ZFC, there's a flaw, when you won't even acknowledge that within the rules of ZFC, we have the rules of ZFC?

Reply... Share • Report • 24 Mar

Jack Huizenga, Math Professor, University of Illinois at Chicago

47 Votes by **Ricky Kwok** (Ph.D. student in Applied Math at UC Davis), **David Joyce** (Professor of Mathematics at Clark University), **Anurag Bishnoi** (Ph.D. student in Mathematics at Ghent University.), Anders Kaseorg, and 42 more.

Mainstream academia considers calculus to be true because it is built on a history of proof and rigor over the past 400 years. Everything in it follows from extremely simple axioms of set theory, and all the major issues of set theory which existed 150 years ago have been solved since then.

Mathematicians are not concerned with the objections raised by John Gabriel, because every argument he makes that is contrary to established thought has some obvious flaw in it.

1. John objects to the fact that we can construct the real numbers. His understanding of the construction of reals by Dedekind cuts is here: What is an intuitive explanation of a Dedekind cut? by John Gabriel on Posts. A real number is not constructed as a Dedekind cut using only rational numbers of fixed or bounded denominators. The whole cut must be considered at once. He discredits this construction of the real numbers by choosing to portray it differently from how it actually is.
2. John objects to the fact that we have a rigorous definition of limit. It is completely true that most calculus classes give a terribly non-rigorous definition of limit, but one of the major points of a class in real analysis is to fix this gap rigorously via the $\epsilon - \delta$ definition of limit.
3. John discusses such things as the function $f(x) = \sin(x)/x$, and says it is "obviously defined at 0," using a power series to conclude that $f(0) = \sin(0)/0 = 1$. Expressions like this make a mathematician's eyes bleed. I can only conclude that there is a misunderstanding about the domain of definition of functions, and what it means to have a continuous extension of a function that is not everywhere defined. Academics never quite understood what sin (x)/x means! by John Gabriel on Posts
4. In order to discuss the aforementioned function $f(x) = \sin(x)/x$, he expands it in a power series and says "look you can plug in 0 and it works." But if, say, you wish to define the sine function via a power series, you had better know what the limit of a sequence of real numbers is. John is inconsistent with his principles of what is and is not allowed, and changes the rules at his own whim. (Admittedly, limits of sequences are slightly different from limits of functions, although really not in an essential way.)

Most of John's discussion on his objections to calculus essentially says "go read my life's work." There needs to be one, absolute, certifiable problem with calculus for anybody to care about "fixing" calculus or believe that calculus "needs" fixing. With set theory, there was Russel's paradox, which you could explain in 2 minutes and led to the necessity of revolutionizing set theory over the next 50 years. John has not provided any similar example with calculus.

Upvoted • 1 Comment • Share • Thank • Report • 21 Mar

Comment

Benjamin Schak

As a little self-promotion, I wrote a description of Dedekind cuts at How can the real numbers be constructed from the rational numbers? some time ago. I've always found Dedekind's definition of the reals to be more beautiful than Cauchy's, but less frequently taught in undergrad analysis. So for anyone who hasn't seen the construction before, I hope that my outline might be more useful than Mr. Gabriel's post, which is frankly incoherent.

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John Gabriel

Your construction is juvenile and invalid.

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Benjamin Schak 1 vote by Varun K Shetty

It would be fruitless to debate the validity of my (or, to give credit where credit is due, Mr. Dedekind's) construction of a complete ordered field in which the rationals are naturally embedded. You surely won't be convinced, and essentially everyone else is already convinced, so why bother.

But juvenile?

What makes a construction "juvenile"? Are all "juvenile" constructions also invalid, or are there valid "juvenile" constructions as well? Is there some kind of age limit? (If so, you should know that I didn't see Dedekind cuts until sophomore year of college, and I don't think that I personally had enough sophistication to understand them until late high-school, at the very earliest. This is no compass-and-straightedge construction that any intelligent 5th-grader can play with!) And is this more or less juvenile than "Weiner measure" or the "Tits Alternative"?

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John Gabriel

"Mainstream academia considers calculus to be true because it is built on a history of proof and rigor over the past 400 years."

Quite the opposite. There is no evidence of proof or rigour.

"Everything in it follows from extremely simple axioms of set theory, and all the major issues of set theory which existed 150 years ago have been solved since then."

Are you referring to the Peano Axioms? These are a joke!

"Mathematicians are not concerned with the objections raised by John Gabriel, because every argument he makes that is contrary to established thought has some obvious flaw in it."

That's just simple false and you have not been able to provide any evidence of that.

"A real number is not constructed as a Dedekind cut using only rational numbers of fixed or bounded denominators. The whole cut must be considered at once."

Except that the cut says nothing about the real number, except that it is bounded by sets of rational numbers on both sides. But every cut has this property! So, all the cuts are the same! :-)
In order to claim otherwise, you would have to be able to give every "real" number a NAME. You can't! Conclusion: D. Cuts do not represent a valid construction of anything.

"He discredits this construction of the real numbers by choosing to portray it differently from how it actually is."

False. I have never misrepresented the facts. Mainstream theory does a pretty good job of discrediting itself.

"John objects to the fact that we have a rigorous definition of limit. "

False. I object to the fact that it adds any rigour to calculus.

"It is completely true that most calculus classes give a terribly non-rigorous definition of limit, but one of the major points of a class in real analysis is to fix this gap rigorously via the
$\epsilon - \delta$
definition of limit."

The
$\epsilon - \delta$
definition is no different from the same definition in words. Stating a word definition using numbers and symbols does not add any rigour.

"John discusses such things as the function
$f(x) = \sin(x)/x$
, and says it is "obviously defined at 0," using a power series to conclude that
$f(0) = \sin(0)/0 = 1$
. Expressions like this make a mathematician's eyes bleed. "

If he is an ignoramus, yes.

"I can only conclude that there is a misunderstanding about the domain of definition of functions, and what it means to have a continuous extension of a function that is not everywhere defined."

What are you babbling about? Sin x / x is defined at every point of its domain, and it is everywhere conitinuous.

Academics never quite understood what sin (x)/x means! by John Gabriel on Posts

"In order to discuss the aforementioned function
$f(x) = \sin(x)/x$
, he expands it in a power series and says "look you can plug in 0 and it works." But if, say, you wish to define the sine function via a power series, you had better know what the limit of a sequence of real numbers is."

Nonsense. In most cases you have NO IDEA what is the limit! You only know that it exists as an incommensurable magnitude. In fact, through the ratio test, you know that a limit exists, BUT IN MOST CASES, YOU HAVE NO IDEA WHAT IS THIS LIMIT!!!!

Besides, your argument is null and void because to use power series, all we need to know is that a limit exists, not what it is!
Hope you tell your students what I just taught you!

"John is inconsistent with his principles of what is and is not allowed, and changes the rules at his own whim."

False. You ought to look in a mirror Jackie boy! Tsk, tsk.

"(Admittedly, limits of sequences are slightly different from limits of functions, although really not in an essential way.) "

So why mention this?

"Most of John's discussion on his objections to calculus essentially says "go read my life's work."

Nonsense. Nowhere have I claimed that. In fact I tell others not to believe a word of what I say, but to prove everything.

"There needs to be one, absolute, certifiable problem with calculus for anybody to care about "fixing" calculus or believe that calculus "needs" fixing."

There are many certifiable problems and I have mentioned all of them to my knowledge.

"John has not provided any similar example with calculus."

Oh, the examples are there in many forms. I suppose that if you shut your eyes, you won't see. :-)

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Anders Kaseorg 5 votes (show)

Mathematicians will happily discard an axiom system because you can formally demonstrate that it leads to serious problems, such as when Russell's paradox forced us to discard the system of naive set theory. We will not, however, discard an axiom system that has served us well for a hundred years just because someone asserts that it is "a joke". If you think the axioms lead to a contradiction, go ahead and exhibit one.

The axiomatic foundations of calculus are so well-understood that they have been formalized in a computerized theorem-proving language with a well-established metatheory. Even if you haven't carefully studied these foundations (which I have), this is pretty good evidence that they work just fine.

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John Gabriel

"Mathematicians will happily discard an axiom system because you can formally demonstrate that it leads to serious problems, "

I don't know of many mathematicians, only ignorant academics.

Never have I claimed that calculus theory needs to be discarded because it is a joke. That's just your false assertion. I have shown serious flaws in the theory of calculus. You can produce your typical strawman arguments and other platitudes in the form of ZFC axioms or anything else you please.

This only shows that you fail to understand that calculus is flawed.

Finally, that theorems work is no evidence the foundations are sound.

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Fred Feinberg Suggest Bio

4 Votes by Ross Rheingans-Yoo, Denis Oakley, and Anonymous.

From reading the site that he's linked to below, the critique appears to be a casual use of infinity. There is definitely something to this criticism; the founders of calculus were not aiming at what today would be viewed as a rigorous approach, but we aimed at an intuitive set-up with repeatable, understandable derivations THAT GAVE CORRECT ANSWERS. This last part cannot be overestimated: the integral calculus allowed calculations of, for example, volumes and arc lengths that were inaccessible before. Nothing anyone has done since has changed the answers to these questions.

Gabriel's remarks about rigor need to be taken with a grain of salt. Saying that all professional mathematicians have missed something is a tall claim, and needs to be backed up by clear, patient argument, not a web site that starts out asking that the author be nominated for the Abel Prize, or hurling invective at anyone who disagrees.

The claims about lack of rigor by prior investigators is "not even wrong". For example, there is a VERY rigorous theory of nonstandard analysis, pioneered by Abraham Robinson, and calculus is very intuitive in that framework. There is even a beginner's textbook using it (Elementary Calculus: An Infinitesimal Approach).

Anyway, physics is notorious for attracting people who are fond of extreme claims.

You see less in mathematics because one immediately asks for proof, and there is no recourse to experiment.

Upvote • Comment • Share • Thank • Report • 25 Mar

Comment

John Gabriel

Another comment by someone who did not bother to study my New Calculus. Mr. Feinberg, you don't understand the New Calculus by just browsing through the website. Believe it or not, you have to do some studying.

As for your claims about Robinson's non-standard analysis, that's just baloney. Many mathematicians don't accept non-standard analysis, with good reason - it's anti-mathematical rubbish.

I have only one claim, and it happens to be fact: The New Calculus if the first and only rigorous formulation of calculus in human history. Perhaps if you bothered to study it, you might learn more.

Reply... Upvote • Share • Report • 25 Mar

Anonymous

7 Votes by Denis Oakley, Stephen McInerney, Marco Böhm, Ross Rheingans-Yoo, and 2 more.

On a basic level, he dislikes infinity and its use in any context. This is a valid position, held by many philosophers and mathematicians.

Of course, he ignores that fact that "traditional calculus" does not use infinity in any essential way. For example, to differentiate f(x) = x^3, you can just simplify the expression (x^3 - y^3)/(x-y) and then set x=y (to obtain x^2 + xy + y^2, and then 3x^2). No infinity required, no irrational numbers required, and 100% rigorous. Yet this is precisely the limit definition. It's only because mathematicians want a broader, more powerful theory of differentiation that the full "Cantorian rot" (as Gabriel calls it) enters the definitions. But in the simple, everyday cases, there's really no infinity to speak of.

But mathematicians bear some blame for this. Just look at calculus textbooks: they define the "limit" and then set the students to work on extremely pathological cases (jump discontinuities and so forth). So it makes it look like it's essential. But you could teach the core of calculus, the important stuff (working with nice smooth functions only), with none of the pathology, in 2 weeks. Gabriel is right about that.

A second reason that I've gleaned from many remarks here is that he believes that words have "correct" definitions. A good case in point is "tangent line." Conventional mathematicians were perfectly willing to give it a new definition to suit their needs and encompass a broader range of cases (for example, by ensuring that smooth curves have tangent lines at *all* points, including inflection points or points with vertical tangent line). To Gabriel, though, there is one correct definition, and you are not allowed to change it. Again, this is a valid position. Even Grothendieck, known for his iconoclastic redefining of centuries-old mathematical words (and who Gabriel would surely denounce with unrestrained venom) would become angry with people when they made definitions that looked wrong (however rigorous and logically sound).

In defense of mathematicians: they are well aware that they can't just change definitions *in media res*. A new definition always means tearing down what's on top of the old one and starting fresh (or alternatively, re-expressing the old definition in terms of the new one and "translating" the superstructure). Always.

His worldview comes from very valid sources. Both sides could learn from each other, if the dialog were not so antagonistic.

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2 Answers Collapsed (Why?)

Downvoted:

Anonymous

Vote by Jake McNamara.

2 Because he's an amateur, while mainstream mathematicians are made up of some of the smartest most educated people on the planet.

Downvoted • 3+ Comments • Share • Thank • Report • 21 Mar

Comment

David Joyce 4 votes (show)

I wouldn't denigrate amateurs. Fermat was one of the best and most innovative mathematicians, but he was an amateur.

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Anonymous

True. But the proof of his pudding was in the theorems. Amateurs whose only output is definitions don't get the same treatment.

Reply... Upvote • Share • Report • 21 Mar

John Gabriel

So far all that nobodies like you can do, is throw feces from the sidelines.

@Joyce: Sorry, but you denigrate anyone with views different to yours. :-)

Reply... Downvoted (Undo) • Share • Report • 22 Mar

Stephen McInerney

You need to give specific comments.

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Richard Pietrasz

Many professionals dismiss amateurs based on who they are and not their arguments. I am not commenting on Gabriel per se, just on common human behavior.

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John Gabriel

I am not an amateur by any standard, but of course if one is not part of a clique, his arguments are rejected by the closemindedness of those in the clique.

I have given up approaching the clique many years ago. It does not even bother me that my work is published in journals because I experience thousands of hits at my website. The idea is to spread this knowledge with or without the "blessing" of academic ignoramuses.

What will happen is that they shall find themselves left behind in the future when the New Calculus and my new mathematics replaces the flawed calculus and Cantorian rot.

I shall laugh last.

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John Gabriel

"Because he's an amateur," - Anonymous

And you are a nobody.

Amusing how one gets called an amateur after 35 years of research. Too funny really.

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2 **John Gabriel, Read about the New Calculus here: http://johngabrie1.wix.com/newcalculus**

Vote by Anonymous.

Mathematics: Why does John Gabriel state that standard calculus is flawed, while mainstream academia considers it to be true?

Jack Huizenga and Anders Kaseorg, have repeatedly vandalised this comment: Please DO NOT edit this for any reason whatsoever. Thank you!

There is a very good reason this text is included here. Readers can get to the New Calculus site and see firsthand for themselves before they read all the wrong comments posted on this question.

See The New Calculus

A very good question indeed. Why is standard calculus flawed?

I could write several books on the matter, but English is not my first language, and I hate writing in any language. If my unpublished work is ever published some day (approx. 2000 pages), much more will be revealed.

In the meantime, I think that pictures are far more powerful than words and essays. So I have included a few links to applets that educate and entertain. There are no viruses or spy software in any of these applets.
The applets are dynamic, that is, there are sliders and movable points. Use these to see how the standard calculus is indeed flawed.

If you have any questions or comments, I suggest you post them on my Facebook page: New Calculus | Facebook

Also, there are 65 articles at the following link addressing this question, and much, much more!

https://www.filesanywhere.com/fs...

Applets:

A comparison of the flawed calculus and the New Calculus derivative:
(This applet shows the initial flaw with Newton's definition)

ttps://www.filesanywhere.com/fs/v.aspx?v=8b6c6a8b5a607375ad6c

Cauchy's kludge:
(This applet shows how Cauchy in fact did nothing by adding the limit concept to Newton's definition. In fact, Cauchy made it worse!)

https://www.filesanywhere.com/fs...

Riemann's Kludge:
(This applet shows that considering the integral to be the limit of an infinite number of rectangular areas is also counter-intuitive and a very bad idea)

https://www.filesanywhere.com/fs...

The New Calculus Derivative at a glance:

https://www.filesanywhere.com/fs...

The New Calculus Integral at a glance:

https://www.filesanywhere.com/fs...

A very good question indeed. Why is standard calculus flawed?

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The New Calculus Derivative at a glance:

https://www.filesanywhere.com/fs...

The New Calculus Integral at a glance:

https://www.filesanywhere.com/fs...

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Comment

Anders Kaseorg 2 votes (show)

I took a look at a few of your applets (Newton, Cauchy, Riemann, but then I reached the FilesAnywhere download limit). I think they actually do a pretty good job of illustrating how limits work, and I don't understand why you think there's a problem.

Let's consider the Riemann applet, for example (it sounds like the problem you perceive is substantially similar in all three cases). It nicely demonstrates that, for all $\epsilon > 0$, we can make the partition fine enough that the area is between $4 - \epsilon$ and $4 + \epsilon$. Formally, we can choose $\delta > 0$ as a function of ϵ such that for all $0 < \Delta x < \delta, |Area_{\Delta x} - 4| < \epsilon$. (In this case it looks like the appropriate function is $\delta = \sqrt{\epsilon}$; for example, if I want the answer to be within $\epsilon = 0.0001$, it suffices to choose Δx to be within $\delta = 0.01$ of 0, as long as it does not equal 0.) This is precisely the formal definition that lets us state that the limit equals 4. We never need to set $\Delta x = 0$ and attempt to evaluate an "infinite sum of 0s" in order to define the limit. The definition does not require any particular behavior at $\Delta x = 0$, so all the ill-defined infinity-related concepts to which you object disappear completely.

So where's the problem? If you consider this formalization of limits ill-defined, what exactly goes wrong here?

(For the record, I'm not downvoting this answer, and I don't think others should either. Although I think you're wrong about calculus, you're certainly qualified to say why you personally think it's flawed in response to this question.)

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John Gabriel

"So where's the problem? If you consider this formalization of limits ill-defined, what exactly goes wrong here?"

The main problem is in the use of ill-defined concepts to formulate theory.

The derivative does not require a limit in any shape or form. In fact, Newton's finite difference quotient is almost 100% rigorous if interpreted as a general derivative. If f(x)=x^2, then the general derivative is f'(x)=2x and a numeric derivative is f'(k)=2k where k is some number.

In Newton's definition, the general derivative is never equal to the numeric derivative, but this does not matter, because one can find any numeric derivative from the general derivative. What Cauchy attempted to do, was to fix this problem with the limit concept.

As for the Riemann integral, it is in fact a product of two averages and I have proved this in the article called RiemannFaux.pdf at:

https://www.filesanywhere.com/fs...

The fact is that the limit is not at all required in calculus. Not for the derivative or the integral. The New Calculus is sound evidence of this.

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Anders Kaseorg

The limit is not an ill-defined concept. I *just gave you* a rigorous definition. If you still think that there's a problem with the ϵ–δ definition that makes it unsuitable as a foundation for calculus, then please present it. (Hint: there isn't one, but you're still welcome to try.)

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John Gabriel

I don't have to try. I have proved it is not rigorous a long time ago.

The first proof:

The limit depends on the existence of irrational numbers. However, there is no valid construction of the irrational numbers.

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Anders Kaseorg 1 vote by Razvan Baba

There isn't? Because the well-understood dedekind cuts and Cauchy sequences constructions are "juvenile and invalid"? You'll have to do better than that.

Reply... Share • Report • 22 Mar

John Gabriel

No. You'll have to do better than that. I have proved that D. Cuts and Cauchy sequences are invalid.

http://www.spacetimeandtheuniver...

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Anders Kaseorg 2 votes (show)

That post demonstrates a misunderstanding of Dedekind cuts. You claim to have exhibited multiple Dedekind cuts that represent the same irrational number, but your claim is incorrect, in that $(S_n(m), L_n(m))$ is not a Dedekind cut. The definition of a Dedekind cut stipulates that every rational number must be included on one side or the other, and $(S_n(m), L_n(m))$ does not satisfy this because both sides are finite sets. For example, $\frac{1}{7}$ is missing from both $S_6(9)$ and $L_6(9)$.

The *unique* Dedekind cut that represents $\frac{\sqrt{2}}{2}$ is (A, B), where

$$A = \left\{x \in \mathbb{Q} \mid x < 0 \vee x^2 < \frac{1}{2}\right\},$$
$$B = \left\{x \in \mathbb{Q} \mid x > 0 \wedge x^2 > \frac{1}{2}\right\}.$$

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John Gabriel

You demonstrate a misunderstanding of the post. It does not exhibit the D. Cut for a rational number, but that for an "irrational number".

Back to the drawing board for you!

For example,
$\frac{1}{7}$
is missing from both
$S_6(9)$

and
$L_6(9)$
.

It's not missing. It's not part of that cut. That was just an example.

BTW: Nothing is missing from my example. I suggest that you study it carefully!

Reply... Upvote • Share • Report • 22 Mar

Anders Kaseorg 2 votes (show)

What do you claim are the multiple Dedekind cuts that represent the same irrational number, if not $(S_n(m), L_n(m))$ (which themselves are not Dedekind cuts)?

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John Gabriel

The comment states clearly that the D. Cut is given by the limit as n increases without bound.

It turns out that no unique cut can be found.

Moreover, you can't assign the magnitude of the hypotenuse in a right-angled isosceles triangle the name or symbol $\sqrt{2}$ and claim that it is then defined. In order to do so, you would have to give every "real" number a *name*. You can't. If you could, the "real" numbers would be well defined and thus countable.

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Anders Kaseorg 1 vote by Nicholas Halderman

But that's not what happens. The Dedekind cut is not "increasing", or changing in any way. It is fixed, once and for all time, at (A, B), where A and B are the sets that I mentioned two comments ago.

And we don't claim that $\sqrt{2}$ is a real number just because we can give it that name. We claim it is a real number because we can write down a Dedekind cut that defines it, and *rigorously prove* that it is a positive number whose square is 2.

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John Gabriel

That's exactly what happens. The definition I give in the comment is better than anything Dedekind could do.

The problem is that there are no such sets A and B as you claim. That's what has been proved.

Since there are no such sets that can be constructed logically, the D. Cut is junk. In order for you to say that the sets are constructible, you would need to show that a unique limit exists, but you can't, unless you already assume it's sqrt(2)!

Yes, you do claim that

$\sqrt{2}$

is a real number just because you can give it that name/symbol.

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Anders Kaseorg 1 vote by Nicholas Halderman

Here's the Dedekind cut construction for $\sqrt{2}$:

$$A = \{x \in \mathbb{Q} \mid x < 0 \lor x^2 < 2\},$$
$$B = \left\{x \in \mathbb{Q} \mid x > 0 \land x^2 > 2\right\},$$
$$\sqrt{2} = (A, B).$$

There's nothing circular about this. I did not assume the existence of $\sqrt{2}$ to construct A and B. All I used is the ability to take a rational number, square it, and decide whether the square is greater or less than 2. (We can prove that no square of a rational number is equal to 2.) These are all rational number operations, and I think you agree with me that rational number operations are perfectly fine. For example, $\left(\frac{7}{5}\right)^2 = \frac{49}{25} < 2$, so $\frac{7}{5} \in A$.

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John Gabriel

It's completely circular. There are no such sets A and B.

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Anders Kaseorg 1 vote by Nicholas Halderman

Is there a set $\mathbb{Q}$ of rational numbers?

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John Gabriel

There is no infinite set.

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Anders Kaseorg 2 votes (show)

Okay, then that appears to be the primary point of disagreement.

However, the axioms of ZFC set theory do allow us to construct objects that behave like infinite sets, under certain well-defined rules, and manipulate them rigorously. You might believe that infinite sets don't exist, but certainly there are symbols that represent "infinite sets" under ZFC, and those symbols themselves exist. (For the purposes of this discussion, we could call them ZFC-sets, to distinguish them from whatever you think actual sets are.) That's enough to allow us to construct a foundation for real numbers including both rational and irrational numbers, and do calculus with them in the standard way.

(Unless you believe that the axioms of ZFC result in a contradiction. You could certainly shake the foundations of mathematics by exhibiting a contradiction in ZFC, but I don't think you have.)

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John Gabriel

To say "the axioms of ZFC set theory do allow us to construct objects that behave like infinite sets" is already circular! In that statement you have not only assumed the existence of infinite sets, but you have also assumed that they have properties.

You think that is not circular?! :-)

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Anders Kaseorg 1 vote by Nicholas Halderman

No, not when you unpack what I meant by that:

The axiom of infinity states that there exists a ZFC-set $\mathbb{N}$ such that $\varnothing$ is an element of I, and for every $x \in \mathbb{N}$, $x \cup \{x\} \in \mathbb{N}$. We call $\mathbb{N}$ "the (ZFC-)set of natural numbers". We construct $\mathbb{Z}$ from $\mathbb{N}$ and $\mathbb{Q}$ from $\mathbb{Z}$ in the standard way.

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John Gabriel

The problem is that the ZFC axioms presume the set N exists as a well-defined set. So, yes, your argument is circular.

The ZFC axioms presume a lot of things. Take away N and the ZFC axioms are worthless. I can construct the rational numbers without any pre-assumptions in the Euclidean way.

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Anders Kaseorg 2 votes (show)

The existence of the set $\mathbb{N}$ is an axiom of ZFC . You are welcome to dispute whether this axiom is "true", or whether that question even makes sense.

But the truth of that axiom is not relevant to my argument. The existence of the **ZFC-set**—the literal symbol $\mathbb{N}$—is indisputable: I just wrote it down. As long as the axioms of ZFC do not lead to a contradiction with each other, it doesn't matter whether they're "true". I can still use them to manipulate symbols according to its rules.

That's all we need for a rigorous foundation of calculus.

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John Gabriel

That is false. The very first axiom assumes its existence.

There are a lot more things that are wrong, which I shall not discuss.

Calculus does not depend at all on ZFC. My New Calculus is not about the foundations, it's about the way calculus is done. In fact, ZFC is completely irrelevant.

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Anders Kaseorg 1 vote by Nicholas Halderman

This question is about why you think standard calculus is flawed, i.e., is not a rigorous formulation (given that you claim to have discovered the first and only such rigorous formulation). From your comments in this thread, it sounds like the ultimate reason you think it's flawed is that it's based on ZFC, and you don't accept the axioms of ZFC as true.

The axiom of infinity in ZFC does state that $\mathbb{N}$ exists (with the above property). That's what it says. If you accept the axiom of infinity as true, then $\mathbb{N}$ exists for you; if you deny the axiom of infinity, then it doesn't. There's no assumption here except that which you choose to assume.

(Mathematicians do talk about theories in which the axiom of infinity is denied. For example, see hereditarily finite set , if you're interested.)

The thing is, you don't need to accept the axioms of ZFC as true to see that they constitute a rigorous foundation for calculus.

Reply... Share • Report • 22 Mar

John Gabriel

Not in the least. The arguments are not about ZFC. I just used that to show you that your ground assumptions are not valid.

Let's go to the definition of the derivative in standard calculus. It is circular.

a. Well, if you divide by h, then you are in effect calculating the slope of a non-parallel secant line, not the tangent line.

b. Assuming we can get past that, the next thing you do is set h=0, so that you can find the general derivative. In spite of the fact that h *cannot be zero* ever, because the limit does not allow that!

c. a) and b) aside, you then claim that the definition via means of the limit is sound, but you can't know the limit unless you find the derivative using the flawed first principles method.

Do you want me to list more flaws or can you deal with these?

"Cauchy had stated in his Cours d'analyse that irrational numbers are to be regarded as the limits of sequences of rational numbers. Since a limit is defined as a number to which the terms of the sequence approach in such a way that ultimately the difference between this number and the terms of the sequence can
be made less thanany given number, the existence of the irrational number depends, in the definition of limit, upon the known existence, and hence the prior definition, of the very quantity whose definition is being attempted.

That is, one cannot define the number sqrt(2) as the limit of the sequence 1, 1.4, 1.41,
1.414, ... because to prove that this sequence has a limit one must assume, in view
of the definitions of limits and convergence, the existence of this number aspreviously demonstrated or defined. Cauchy appears not tohave noticed the circularityofthe reasoning in this connection, but tacitly assumed that every sequence converging within itself has a limit."

The History of Calculus and its Conceptual Development' (Page. 281) Carl B. Boyer

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Anders Kaseorg 2 votes (show)

No. That is not the definition of the derivative in standard calculus.

The definition of the derivative in standard calculus is:

$f'(x)$ is the number m (iff it exists) such that, for all $\epsilon > 0$, there exists a $\delta > 0$ such that, for all $h \neq 0$ with $|h| < \delta$, $\left|\frac{f(x+h)-f(h)}{h} - m\right| < \epsilon$.

We never ever ever divide by 0. That is absolutely not allowed, and the definition excludes that possibility, as it needs to.

That is also not the definition of a Cauchy sequence. A Cauchy sequence is not defined as a sequence that approaches a limit. That would indeed be circular.

Instead, a Cauchy sequence is defined as a sequence $x_1, x_2, x_3, \ldots$ such that for all $\epsilon > 0$, there exists a natural number N such that for all $m, n > N$, $|x_m - x_n| < \epsilon$. This definition makes no reference to limits, only to the finite-indexed terms of the sequence that we already know.

There is no problem here to be fixed.

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John Gabriel

That's the definition I am referring to. That's the one that is problematic. May I suggest a reread thinking of the definition you provided? :-)

Now address the following with YOUR definition.

Let's go to the definition of the derivative in standard calculus. It is circular.

a. Well, if you divide by h(*), then you are in effect calculating the slope of a non-parallel secant line, not the tangent line.

(*) You do have to divide by h at some or other time. :-)

b. Assuming we can get past that, the next thing you do is set h=0, so that you can find the general derivative. In spite of the fact that h *cannot be zero* ever, because the limit does not allow that!

c. a) and b) aside, you then claim that the definition via means of the limit is sound, but you can't know the limit unless you find the derivative using the flawed first principles method.

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Anders Kaseorg 2 votes (show)

Let's differentiate $f(x) = x^2$, using the formal definition of the derivative.

For any $\epsilon > 0$, pick $\delta = \epsilon$. For any $h \neq 0$ with $|h| < \delta$, we have $\left|\frac{(x+h)^2 - x^2}{h} - 2x\right| = |h| < \delta = \epsilon$. Therefore, $2x$ satisfies the definition of $f'(x)$ (and we can prove generally that any number satisfying this definition must be unique), so $f'(x) = 2x$.

h has never been set equal to zero.

(Of course, this is not how we would differentiate this function in practice. We would do it by proving theorems such as the sum rule, product rule, and chain rule that let us rigorously get the same answer without going through the ε–δ definition every time. But you can always do it this way if you want to.)

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John Gabriel

Nonsense! How did you get 2x? You would have to differentiate first. Not even close!

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Anders Kaseorg 1 vote by Nicholas Halderman

A rigorous formalization of calculus only needs to tell me how to formally justify everything I assert. It doesn't need to say anything about how my brain comes up with those assertions before they are proven.

But if you are curious how I come up with $2x$, here's how:

I want to show that $\frac{(x+h)^2-x^2}{h}$ is arbitrarily to some unknown value m, as h gets arbitrarily small (remaining nonzero). For all $h \neq 0$, we can simplify $\frac{(x+h)^2-x^2}{h}$ to $2x+h$. Since this is a continuous function of h, the value that $2x+h$ gets arbitrarily close to (but never reaches) is $m = 2x$. Having found m, I justify it as in the previous comment, by plugging $2x$ into the formal statement that defines $f'(x)$ and demonstrating that the formula holds.

In fact, things get even better in the case of differentiation: it turns out that there's a simple algorithm that mechanically produces the derivative of any formula (using the sum rule, product rule, chain rule, etc.). One can implement this algorithm in a computer, and one can rigorously prove using the ε–δ definition that it always produces the correct derivative.

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John Gabriel

You do need to have a reason for why you come up with *anything*.

As I have stated before, you cannot use the finite difference quotient as you have to arrive at 2x for the reasons I stated. Look, the example you provided, which helps you to find the general derivative, that is, $\frac{x^3-y^3}{x-y}$ is just as flawed, because you start off with $x \neq y$, and then you set $x = y$ and you can't tell which derivative it might be, i.e. f'(x) or f'(y).

You cannot (for the same reason) start off with $h \neq 0$ and then set $h = 0$. No amount of handwaving will get you past that.

Let's get back to the definition of the derivative in standard calculus. It is circular. You have not addressed any one of the following issues. You tried to skirt these by introducing the irrelevant ZFC axioms.

a. If you divide by h, then you are in effect calculating the slope of a non-parallel secant line, not the tangent line.

b. Assuming we can get past that, the next thing you do is set h=0, so that you can find the general derivative. In spite of the fact that h *cannot be zero* ever, because the limit does not allow that!

c. a) and b) aside, you then claim that the definition via means of the limit is sound, but you can't know the limit unless you find the derivative using the flawed first principles method.

Now, unless you can answer these satisfactorily, you don't have any argument.

The New Calculus does not suffer from any of these flaws. It is rigorous and sound.

The Cauchykludge.ggb applet shows you how Cauchy erroneously defined the derivative as follows:

$$f'(x) = lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

In words: The derivative of f at x exists in an interval (a,b) iff all the other derivatives exist in the interval (a,b), except perhaps at x.

But the derivative at x is the one we want!
Suppose we have f(x)=x^2 and we want the derivative at x=0. Further suppose that f is not defined at x=0. By Cauchy's flawed definition, the derivative exists, and it is equal to 0. That's what the Cauchy limit definition means!

No, don't even try to tell me that it must also be defined there - the Cauchy definition states it as a limit, and we know that a function need not be continuous at a point, in order to have a limit.

You can add in rules (as mainstream math has) to justify Cauchy's rot, but it does not resolve the issues.

The New Calculus is the first and only rigorous calculus in human history. You can't dispute this fact in any way, shape or form.

Unless your next comment contains a substantial argument, I don't know if I will continue this discussion.

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Anders Kaseorg **1** vote by Joseph Heavner

I do have a reason I come up with everything, but it is valid for that reason to be “intuition”, as long as I’m able to formally justify it later, which I am.

You’re picking up the x^3 thing from a different thread. I can go and do exactly the same thing that I just did with x^2 in this thread to x^3, and once again, I get a completely rigorous formalization of the derivative of x^3 that does not ever involve setting $h = 0$. Your step (b) simply is not a thing in this formalization.

For all $h \neq 0$, $\frac{(x+h)^3-x^3}{h} = 3x^2 + h(3x+h)$.
We observe that this appears to get arbitrarily close to $3x^2$ for arbitrarily small $h \neq 0$. To prove it, for any $\epsilon > 0$, pick $\delta = \min\left\{\frac{\epsilon}{|3x+1|}, 1\right\}$; then, for any $h \neq 0$ with $|h| < \delta$,
$\left|\frac{(x+h)^3-x^3}{h} - 3x^2\right| = |h(3x+h)| < \delta|3x+1| \leq \epsilon$. Therefore, the derivative of x^3 is $3x^2$.

Since you seem to be absolutely insistent in making me show how *your argument* along the lines of (a), (b) can be formalized—which is something that I’m under no obligation to do, having already presented the standard formal argument that works without any appeal to (b)—I’ll do it for you anyway.

However, I’d like to emphasize that this argument does not directly use the formal definition of the derivative, but relies on a handful of theorems that can be proven rigorously from the formal definition of $\lim$. It is more convenient for someone who knows these theorems, but a complete treatment of this argument would have to include/reference the proofs of the theorems, which you can find in a real analysis textbook (they're all each about as long as my proof above).

If $f(x) = x^3$, then

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h}$$

(this is equivalent to the definition I used above, where I've just expanded out the ε–δ definition of $\lim$)

$$= \lim_{h \to 0} (3x^2 + h(3x + h))$$

(because I can divide by h for all $h \neq 0$)

$$= \lim_{h \to 0} g(h),$$

where $g(h) = 3x^2 + h(3x + h)$. We can prove that g is a *continuous* function of h, using the four theorems that the identity function, any constant function, the sum of any two continuous functions, or the product of any two continuous function, is continuous. (These theorems can be proven using the ε–δ definition of $\lim$.) The definition of continuity says that for every a,

$$\lim_{h \to a} g(h) = g(a).$$

Substituting $a = 0$, we have

$$\lim_{h \to 0} g(h) = g(0) = 3x^2.$$

Therefore, $f'(x) = 3x^2$.

This argument crucially relies on proving that g is a continuous function, so obviously we need to be careful to do that.

To quickly respond to your other claim: if we have a function $f(x) = x^2$ for $x \neq 0$ and $f(0)$ is undefined, then we agree that $f'(0)$ really is undefined, because we would have

$$f'(0) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$$

where the $f(0)$ on the right side is undefined.

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John Gabriel

Bravo! Bravo! Do you think you are the only one who understands the e-d definitions? :-)

To remind you once again, you used the derivative in both of your "proofs". In the other thread as well as this thread, you used $2x$ and $3x^2$ respectively.

You cannot use *that which you wish to prove* in your "proofs". The problem arises immediately in your first sentence:

"I do have a reason I come up with everything, but it is valid for that reason to be "intuition", as long as I'm able to formally justify it later, which I am.

Do you know what "intuition" means? I doubt it. If you did, you would not use the word incorrectly as you have. I have noticed that all those with similar ideas to you have been using the word incorrectly also.

Intuition: the ability to understand something immediately, without the need for conscious reasoning.

Now reread that sentence after you understood the meaning of intuition.

What I am trying to tell you is that your so-called proofs are circular. See, Anders, if we already know by intuition that $2x$ and $3x^2$ are the derivatives, this does not mean we can use the same in sound proofs. We have to show that these are the derivatives without using the same.

You still have not addressed any of these issues:

a. If you divide by h, then you are in effect calculating the slope of a non-parallel secant line, not the tangent line.

b. Assuming we can get past that, the next thing you do is set h=0, so that you can find the general derivative. In spite of the fact that h *cannot be zero* ever, because the limit does not allow that!

c. a) and b) aside, you then claim that the definition via means of the limit is sound, but you can't know the limit unless you find the derivative using the flawed first principles method.

Now, unless you can answer these satisfactorily, you don't have any argument.

If you want to use e-d "proofs", that's fine, but you cannot assume the derivatives in the "proofs".

e-d "proofs" only confirm what you already know about the derivatives. Nothing wrong with that! But
that does not mean they are proofs. :-)

We can say the same things using English words, but it does not prove anything.

The New Calculus never assumes any knowledge of the derivatives in arriving at the same.

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Anders Kaseorg

Anyone who has passed a course in real analysis understands the ε–δ definitions. At MIT this course is quite literally mathematics 100 , and it's a required course for everyone who graduates with a theoretical mathematics degree.

I have addressed every one of the issues that you pasted four times:

a. That's right: if I divide by h, I am in effect calculating the slope of a non-parallel secant line. I do that because that's exactly what is required by the ε–δ definition of the derivative. The ε–δ definition gives me the slope of the tangent line from the slopes of the secant lines. This is not an issue.

b. Either I never set $h = 0$, as in some of the proofs I've given in the comments above, or I set $a = 0$ only with justification in the form of proving that a certain function is continuous, as in that last proof. In either case I've made a correct formal argument. If I were to set a nonzero variable to zero, then I absolutely would be making a mistake, which is why I have been careful not to do so.

c. Observe that there's a difference between

(1) Because I intuitively believe the derivative to be $3x^2$, I'll use the fact that the derivative is $3x^2$ to show that the derivative is $3x^2$.

(2) Because I intuitively believe the derivative to be $3x^2$, I will see whether I can formally justify from base principles

that $3x^2$ indeed satisfies the definition of the derivative—if I am correct, the proof will go through, and if I am incorrect, the proof will not go through.

(1) is a circular argument and would absolutely be invalid. (2) is a perfectly fine formal argument. There is no way to use (2) to prove something that's false. If you think that (2) cannot yield a valid proof, then you don't understand what a proof is. If I have an idea intuitively, the whole purpose of writing a proof is to validate a correct intuition, or to invalidate a mistaken intuition.

However, if you look at my argument carefully, you'll observe that it is neither (1) nor (2). I didn't need to make an intuitive guess that $3x^2$ is the right answer in order to come up with that argument. I just simplified (for any $h \neq 0$):

$$\frac{(x+h)^3 - x^3}{h} = \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h}$$
$$= \frac{3x^2h + 3xh^2 + h^3}{h}$$
$$= 3x^2 + 3xh + h^2$$
$$= 3x^2 + h(3x + h)$$

and the $3x^2$ popped out by itself, with no intuition required. (I then still need to formally justify that the ε–δ definition of the derivative is satisfied, which I did.)

Another way to come up with $3x^2$—the way we'd usually come up with it in practice—is to prove the product rule from the ε–δ definition, prove the power rule from the product rule by induction, and apply the power rule to x^3.

So this "issue" is neither a necessary feature of mainstream differentiation, nor would it be a problem if it was necessary.

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John Gabriel

It's not just at MIT that a student has to pass real analysis to complete his degree, it's like that everywhere else.

You have failed to address the issues that I asked you to address.

a. An e-d argument (it's not a definition by the way!) is used to show that a limit exists. It says nothing about the finite difference quotient. In fact, it does not have to be a finite difference quotient at all, it can be a term of a Cauchy sequence. The mechanics apply in the same way.

So, the issue remains: you divide by h assuming it is not zero, and then afterward treat h as zero.

"b. In either case I've made a correct formal argument. If I were to set a nonzero variable to zero, then I absolutely would be making a mistake, which is why I have been careful not to do so."

But that's exactly what you have done in order to arrive at 2x and 3x^2. The "intuitive" nonsense you keep referring to, will not fly with me. ;-)

"(1) Because I intuitively believe the derivative to be
$3x^2$
, I'll use the fact that the derivative is
$3x^2$
to show that the derivative is
$3x^2$

."

I am sorry, you can't do *anything* intuitively. Newton did not arrive at his difference quotient "intuitively". No real mathematicians does anything intuitively. :-)

(2) is NOT a perfectly fine formal argument. To arrive at the derivative, you have to get past issues (a) and (b). You have not shown any evidence that you can do this. On the contrary, you have proved that it is not possible. The issues stand.

"*However,* if you look at my argument carefully, you'll observe that it is neither (1) nor (2). I didn't need to make an intuitive guess that

$3x^2$

is the right answer in order to come up with that argument."

It fails on issues (a) and (b).

"

$$= 3x^2 + h(3x + h)$$

and the

$3x^2$

popped out by itself, with no intuition required. (I then still need to formally justify that the ε–δ definition of the derivative is satisfied, which I did.)"

To get to

$$= 3x^2 + h(3x + h)$$

, you performed a lot of invalid arithmetic.

"Another way to come up with

$3x^2$

—the way we'd usually come up with it in practice—is to prove the product rule from the ε–δ definition, prove the power rule from the product rule by induction, and apply the power rule to

x^3

."

Um, no. Not even close. You could not go down that road without resolving issues (a) through (c)/

"So this "issue" is neither a necessary feature of mainstream differentiation, nor would it be a problem if it was necessary."

It is indeed an issue. :-)

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Anders Kaseorg

I didn't mean to suggest that MIT was the only place to learn about real analysis. Point is, no, I'm not the only one, there are lots of people who understand the ε–δ definitions.

a. We're talking about the (mainstream) definition of the derivative. The derivative is defined in terms of the limit of a function. The limit of a function is defined by ε–δ. This is the definition that I have been correctly using the whole time. (The limit of a sequence is given by a related ε–n_0 definition.)

b. See the "however" part of my last comment.

(1) We *agree* that my circular (1) argument is invalid, like I said in my last comment! It was presented as an example of an invalid argument, to contrast it with the valid argument (2).

(2) Addressed above.

“Invalid arithmetic”: what invalid arithmetic? Are you saying there’s something wrong with writing that, for all $h \neq 0$,

$$\frac{(x+h)^3 - x^3}{h} = \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h}$$
$$= \frac{3x^2h + 3xh^2 + h^3}{h}$$
$$= 3x^2 + 3xh + h^2$$
$$= 3x^2 + h(3x + h)?$$

By the way, in general, this conversation would be a lot less frustrating for both of us if, when pointing out a supposed problem in my argument, you would say where the problem is. There is a *difference* between a valid argument that Q follows from P which you nonetheless disagree with because you don’t accept P, and an invalid argument where Q does not follow even if you do accept P.

It would be even better if, when asserting that you disagree with P, you would tell me if there’s a related statement P′ that you would agree with, so we don’t need to waste our time arguing to establish points that we already agree on.

And when pointing out two supposed problems X and Y, it doesn’t help anyone when you respond to my reply to X with “no, you didn’t address Y” and to my reply to Y with “no, you didn’t address X”.

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John Gabriel

I too understand $\epsilon - \delta$ arguments. These are not the issue. The issue is that in order to use these arguments, you need to know a limit L. This limit L is what you *assume* when you construct a statement like $|f(x) - L| < \epsilon$.

The first issue I pointed out is that you can't assume $f'(x) = 2x = L$ or $f'(x) = 3x^2 = L$ because in order to do so, you have already used the finite difference quotient (without the limit) and performed invalid arithmetic on two occasions: $h \neq 0$ before you simplify the quotient and $h = 0$ after you simplify the quotient.

What you are telling me in effect, is that it's just fine to use your flawed *first principles method*, because you can check it later using $\epsilon - \delta$ arguments.

While this is TRUE, is does not make your flawed method any less flawed. :-)

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Anders Kaseorg

Why can’t you understand the difference between **assuming** that $f'(x) = 3x^2$, as a “fact” upon which to build further proofs, and **hypothesizing** that $f'(x)$ might equal $3x^2$, as a guess to be treated with extreme suspicion and checked using the definition *before* I’m allowed to write $f'(x) = 3x^2$?

I absolutely cannot, and did not, **assume** $f'(x) = 3x^2$. I can **hypothesize** whatever I want, for any reason or no reason—precisely because I treat the hypothesis as nothing more than a hypothesis until it is rigorously proven.

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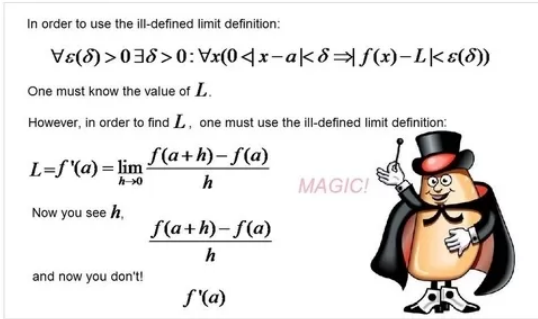
John Gabriel

You can hypothesize all you like. The problem is that your method of "first principles" is hopelessly flawed. No amount of e-d arguments can repair it or justify continued use of it.

It's like the superstitious blood-letting of days gone by, when the ill were thought to get better as a result of disposing bad blood.



Calculus was not rigorous until I discovered the new calculus. There can be no doubt about that any longer.

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Anders Kaseorg

I'll overlook that you wrote the ε–δ definition of limit incorrectly (ε is not a function of δ; δ is a function of ε).

It's still not true that one must know the value of L to apply the definition. One can very well attempt to apply the definition with the wrong value of L—and the conclusion will be that the wrong value of L does not satisfy the definition, as it should be.

If you could find a way to satisfy the formal definition of limit with the wrong value of L, *that* would be a real problem with the rigorousness of calculus. But you can't.

Alternatively, one can use theorems about $\lim$ to compute L rigorously without making any hypotheses at all, as in my second proof in this comment. This method avoids even the false appearance of the "circularity" you think you're complaining about.

Reply... Share • Report • 25 Mar

John Gabriel

"I'll overlook that you wrote the ε–δ definition of limit incorrectly (ε is not a function of δ; δ is a function of ε)."

Wrong. e is indeed a function of d. See link:

http://www.spacetimeandtheuniver...

Sorry Anders, you are beating a dead horse. I suggest you admit that you are wrong and move on.

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Anders Kaseorg

If you start with such a blatant misunderstanding of the ε–δ definition, it is not a surprise that you will find your misunderstood definition to be flawed. Your link only confirms this misunderstanding.

"For all ε > 0, there exists δ > 0" is exactly what it means for δ to be a function of ε.

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John Gabriel
If you study the link, you will see that *your* understanding is blatantly wrong!
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Anders Kaseorg
I am studying the link. Where did you get the idea that the right choice of δ for ε = 0.01 is δ = 0.8? As you later demonstrated, that's clearly not the right choice of δ.
Reply... Share • Report • 25 Mar

John Gabriel
Why don't you first read the whole thing. It will all become quite clear. :-)
Reply... Upvote • Share • Report • 25 Mar

Anders Kaseorg
The problem with your proof in that post isn't that the definition of limit is wrong, but merely that you chose the wrong relationship between δ and ε.

Here is a correct proof, using the correct definition of limit, where δ is a function of ε—namely, in this case, $\delta = \min\left\{\sqrt{\epsilon}, \frac{\pi}{2}\right\}$.

For example, when ε = 0.01, this proof uses δ = 0.1, so *x* = 0.8 is too far away to contradict the proof.
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John Gabriel
That is not true. I can choose any relationship I like, provided I am able to substantiate it, which I do.

You have not shown any point in my proof that is incorrect. The example shows that the definition as "understood" by most is not as rigorous as once thought.
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Anders Kaseorg
Huh? You showed that your own proof is incorrect. I thought that was your point. For $\epsilon = 0.01$, you chose $\delta = 0.81$, and found that at $x = 0.8$, we have $\left|\frac{\sin 0.8}{0.8} - 1\right| > \epsilon$, so the definition is not satisfied with that choice of δ. That just means that your choice of δ wasn't small enough to make the proof go through.
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John Gabriel
Nope. I showed that you can't just choose an ϵ and then find a δ for it. You have to start with δ, not ϵ. It should work for any ϵ, but obviously that's not true.
Reply... Upvote • Share • Report • 25 Mar

Anders Kaseorg
Nobody says you just choose an ε and find a δ for it. The definition says that **for all** ε you need to find a δ for it.
Reply... Share • Report • 25 Mar

John Gabriel
That's not what the definition says.

Here's the definition:

$$\forall \epsilon > 0 \;\; \exists \delta > 0$$

$\forall x(0 < |x-a| < \delta \Rightarrow 0 < |f(x) - L| < \epsilon)$

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Anders Kaseorg

That is the definition. Can you explain how you think that's different from what I said?

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John Gabriel

It's different because you claim that one needs to find an epsilon first, but that is obviously not the way it works. As you saw in my example, one can't just pick an epsilon and then be guaranteed to find a delta. Therefore, one starts with |x-a|<delta and then proceeds to find an epsilon. In fact, it very much depends on the choice of some x very close to a.

At any rate, we are not disagreeing that it can be done, only about how it is done.

None of this supports your argument that standard calculus is not flawed. I provided reasons, which you have not been able to refute, not even one yet!

The difference between us is that the method you use to find L for your secondary stage e-d argument, is flawed. The method I use in the New Calculus is 100% rigorous, does not require limits or additional arguments. ;-)

Now that's what I call the first and only rigorous formulation of calculus in history.

See, you can determine the derivative with absolute confidence that it is indeed the derivative. You are not required to produce any additional supporting evidence for Bishop Berkeley.

The New Calculus does not require any further justification. It is rigorous the first time, every time.

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Anders Kaseorg

I do not claim that one needs to "find an ε first"—one does not "find an ε" at all.

In order to prove that "for all ε > 0, there exists δ > 0, such that foo", you need a proof of the form "Given ε > 0, let δ = [chosen value based on ε]; then [proof of foo]". The prover does not get to choose ε. The proof has to work **for all** ε > 0, not just one. That's what $\forall$ means.

(And again with the "find L" thing that I've already addressed multiple times in two completely separate ways. In case you've forgotten again, the first is that it doesn't matter how I find the hypothesis for L as long as I formally justify it. And the second is that it doesn't matter that it doesn't matter, because I've shown you multiple ways to find L in a 100% rigorous way without making any such hypotheses, if they make you uncomfortable for some ridiculous reason—see this comment, and this comment starting at the sixth paragraph. You're still wrong that any of these methods involve setting a nonzero variable to zero.)

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John Gabriel

"The proof has to work **for all** ε > 0, not just one. That's what

$\forall$

means."

It does not work for all $\varepsilon > 0$. So it can't be a valid argument then, can it? :-)

"In case you've forgotten again, the first is that it doesn't matter how I find the hypothesis for L as long as I formally justify it."

You are wrong. It does matter because no hypothesis for L is required.

"And the second is that it doesn't matter that it doesn't matter, because I've shown you multiple ways to find L in a 100% rigorous way without making any such hypotheses,"

Nonsense. You've not shown that you can find L in any rigorous way.

"You're still wrong that any of these methods involve setting a nonzero variable to zero.)"

Rubbish. You are wrong!

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John Jazzer
Warning: I have detected malware in both the applets and the PDF files.

Reply... Upvote • Share • Report • 24 Mar

John Gabriel
You are a liar!

Reply... Upvote • Share • Report • 25 Mar

John Jazzer
There is a trojan in the applets; maybe not the PDFs, that may have been a false alarm (but I shall double check).

Reply... Upvote • Share • Report • 25 Mar

John Gabriel
That's untrue. There is no malicious software of any kind at my site. If you picked up a trojan, it's not from the Geogebra applets. More than likely from some other site you visited.

Reply... Upvote • Share • Report • 25 Mar

John Jazzer
I simply scanned your applets for viruses and found a trojan.

Reply... Upvote • Share • Report • 26 Mar

John Gabriel
You are lying. I have scanned them also. They are perfectly safe to run.

Reply... Upvote • Share • Report • 26 Mar

Locked questions do not accept new answers.

exercise. What should I do?

Murtaza Kainan Ibrahim, pokemon trainer. 1

Start with the exercise.

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programming languages?

Tikhon Jelvis, consistency is all I ask 21

The primary purpose of types is not in-memory representation. (Unless you're working in C, you poor soul.) Instead, the primary purpose is abstraction[1].

We have a lot of types because there are a ...

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8 year old kids?

Andrew Weimholt 1

apples and a knife.

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