THE EFFECT OF BUBBLE/BUBBLE INTERACTIONS ON LOCAL VOID DISTRIBUTION IN TWO-PHASE FLOWS

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Closure Laws for Multiple-Group Model of Different Size Bubbles: Coalescence (1/2)

- The coalescence terms, $m_{i,co}^{m}$, for the individual groups are:

  $m_{b1,co}^{m} = -m_{b1,b1\rightarrow b2}^{m} - m_{b1,b2\rightarrow b2}^{m} - m_{b1,b2\rightarrow b3}^{m} - m_{b1,b3\rightarrow b3}^{m} - \ldots$

  $m_{b2,co}^{m} = m_{b1,b1\rightarrow b2}^{m} + m_{b1,b2\rightarrow b2}^{m} - m_{b2,b2\rightarrow b3}^{m} - m_{b2,b3\rightarrow b3}^{m} - \ldots$

  $m_{b3,co}^{m} = m_{b1,b2\rightarrow b3}^{m} + m_{b1,b3\rightarrow b3}^{m} + m_{b2,b2\rightarrow b3}^{m} + m_{b2,b3\rightarrow b3}^{m} - m_{b3,b3\rightarrow b4}^{m} - \ldots$

- The rate of coalescence between bubbles of group-$i$ and group-$j$ to form a group-$k$ bubble is given by

  $m_{ij\rightarrow k}^{m} = c_{ij\rightarrow k}\rho_{v}f_{ij}P_{ij}\alpha_{d,i}\alpha_{d,j}$

- $f_{ij}$ represents the frequency of collisions between group-$l$ and group-$j$ bubbles

  $f_{ij\rightarrow k} = \left[\frac{|u_{i} - u_{j}|d_{i} + |u_{j} - u_{i}|d_{j}}{\pi(d_{i}^{2} + d_{j}^{2})} + \delta_{ij}\frac{|\nabla u_{i}|}{n_{k}(\alpha_{cr} - \alpha_{hi})^{1/3}}\right]$ with

  $\delta_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{otherwise} \end{cases}$

- $P_{ij}$ is the relative probability of coalescence; a typical estimated range is:

  - for small bubbles coalescence $0.4 < P_{sb, sb} < 0.8$
  - for small-to-large bubble coalescence $0.1 < P_{sb, lb} < 0.4$
The group coefficients, $c_{m_{ij} \rightarrow k}$, take into consideration the distribution of bubble sizes inside each group-$i$ around a mean diameter $d_i$.

Using a transport equation for the probability density function, and integrating the bubble birth rate due to coalescence over the range of $i$-bubble volume, $[v_{i-1/2}, v_{i+1/2}]$, we obtain the following definition:

$$c_{jk \rightarrow i} = \frac{\text{surface}(F_i \cap ([v_{j-1/2}, v_{j+1/2}] \times [v_{k-1/2}, v_{k+1/2}])}{\Delta v_j \Delta v_k}$$

The surface corresponding to the intersection $F_i \cap ([v_{j-1/2}, v_{j+1/2}] \times [v_{k-1/2}, v_{k+1/2}])$ is represented by the shaded area.

Another approach can also be used, based on directly using a discrete combinatorical method. Both approaches lead to same results.

The present coalescence model allows for bubble repartition with respect to three or more groups of bubble sizes.
Closure Laws for Multiple-Group Model of Different-Size Bubbles: Breakup (1/2)

- The breakup terms, $m_{i,br}^{\text{m}}$, for the individual groups are:

  \[
  m_{b1,br}^{\text{m}} = \sum_{k=b2}^{N} m_{k\rightarrow b1}^{\text{m}} \\
  m_{b2,br}^{\text{m}} = \sum_{k=b2}^{N} m_{k\rightarrow b2}^{\text{m}} - m_{b2\rightarrow b1}^{\text{m}} \\
  m_{b3,br}^{\text{m}} = \sum_{k=b3}^{N} m_{k\rightarrow b3}^{\text{m}} - m_{b3\rightarrow b1}^{\text{m}} - m_{b3\rightarrow b2}^{\text{m}} \\
  \ldots
  \]

- The rate of breakup of large bubbles to form smaller bubbles is

  \[
  m_{k\rightarrow i}^{\text{m}} = c_{k\rightarrow ij} \rho_v f_{br,k\rightarrow i} \alpha_{b,k}
  \]

- $f_{br,k\rightarrow i}$ represents the frequency of bubble breakup

  \[
  f_{br,k\rightarrow i} = \left( \frac{2\sigma}{\rho_l} \right)^{0.4} \frac{6\eta |u_{rk}|^{0.2}}{(1 - \frac{\rho_v}{\rho_l})d_k^{1.4} \left[ \left( \frac{d_k}{d_i} \right)^3 - 1 \right]}
  \]
Closure Laws for Multiple-Group Model of Different-Size Bubbles : Breakup (2/2)

- $\eta$ is a measure of the effect of departure of large bubble shape from spherical

- The group coefficients, $c_{k\to ij}$, account for the distribution of bubble sizes inside each group-$i$ around a mean diameter $d_j$ and can be obtained as follows:

$$c_{k\to ij} = \frac{\text{surface}(F_k \cap [v_{i-1/2}, v_{i+1/2}] \times [v_{j-1/2}, v_{j+1/2}])}{\text{surface}(F_k)}$$

- The present breakup model allows for bubble repartition with respect to three or more groups of bubble sizes.
Consider three group sizes:

- **Size-1**: small nearly-spherical bubbles (\(0 = d_0 < d_{bi} < d_1 = 0.005m\))
- **Size-2**: intermediate size distorted bubbles (\(d_1 < d_{bi} < d_2 = 0.02m\))
- **Size-3**: large Taylor-type bubbles (\(d_2 < d_{bi} < d_3 = \infty\))

Let note

\[
\alpha_k = \sum_{d_{(k-1)} < d_{bi} < d_k} \alpha_{bi}, \quad (k=1,2, \text{and} \ 3)
\]

\[
\alpha = \sum_{i=1}^{3} \alpha_i
\]

\[
j_{v1} = \sum_{d_{bi} < d_1} \alpha_{bi} u_{vi}
\]

\[
j = j_{v1} + \sum_{d_1 < d_{bi} < d_2} \alpha_{bi} u_{vi} + \sum_{d_{bi} > d_2} \alpha_{bi} u_{vi} + u_l (1 - \alpha)
\]
The drag coefficient for small (size-1) bubbles is given by a standard expression

$$C_{D1,bi} = \frac{24}{Re_{bi}}(1 + 0.1Re_{bi})^{0.75}, \quad \text{for} \quad d_{bi} < d_1$$

For the intermediate distorted bubbles (bubbly-to-churn flow transition), the following expression is used

$$C_{D2,bi} = \frac{4}{3}(1 - \frac{\rho_v}{\rho_l})gd_{bi}\frac{(1 - \alpha)^3}{\left[(C_2 - 1)j + V_2 - C_2(\alpha - \alpha_2)j + C_3\alpha_3j - (\alpha - \alpha_2)V_2 + \alpha_3V_3 + j_{v1}\right]^2}$$

for $d_1 < d_{bi} < d_2$

Similarly, the drag coefficient for large bubbles is based on the experimental evidence for Taylor bubbles in slug flows, and is given by

$$C_{D3,bi} = \frac{4}{3}(1 - \frac{\rho_v}{\rho_l})gd_{bi}\frac{(1 - \alpha)^3}{\left[(C_3 - 1)j + V_3 - C_3(\alpha - \alpha_3)j + C_2\alpha_2j - (\alpha - \alpha_3)V_3 + \alpha_2V_2 + j_{v1}\right]^2}$$

for $d_1 < d_{bi} < d_2$

with

$$C_2 = 1.1, \quad C_3 = 1.2, \quad V_2 = 1.53\left[(1 - \frac{\rho_v}{\rho_l})\frac{\sigma g}{\rho_l}\right]^{0.25}(1 - \alpha)^{1.5}, \quad V_3 = 0.35\left[(1 - \frac{\rho_v}{\rho_l})gD\right]^{0.5}$$
Model testing and validation

- **Code**: an advanced CMFD code called OVAP.

  - Two-phase flow in a vertical channel 3.061 m long and \( D = 50.8 \text{ mm} \)
  - Inlet boundary conditions:
    - \( j_v \) and \( j_l \) ranging from small (\( \sim 0.5 \text{ m/s} \)) to large (5 m/s)
    - Vapor volume fraction ranging from 0.22 to 0.36
  - Exit boundary condition:
    - \( p = 1 \text{ bar} \)
  - Interfacial area concentration was measured at \( z/D = 6, 30.3, \) and 53.5.

- **Test case 2: Boiling in a vertical heated channel**
  - Two-phase flow in a vertical channel 3 m long
  - Operating conditions: designed to give insight into the enhanced predictive capabilities of the multifield model compared to a two-fluid model, especially concerning the evolution of axial phase distribution; calculations were performed for two system pressures, 7 MPa and 1 MPa.
Model validation: Bubble column

Figure 1. Interfacial area concentration calculated using a 10-field model:
(a) low agitated medium (5-field model), (b) high agitated medium.

The agreement between the calculations based on the current model and the data is quite good, thus demonstrating the correctness and consistency of the proposed approach.
Model testing: Boiling in a vertical heated channel (1/2)

Figure 3. Partial volume fraction for individual bubble-size groups along a boiling water channel at a pressure of 7 MPa: (a) 2-group model, (b) 3-group model, (c) 4-group model, (d) 5-group model.

The volume fractions for the individual bubble-size groups reflect the effect of bubble/bubble interactions (in particular, coalescence) between the individual groups. Comparing the results for the 4-group and 5-group models clearly indicates that as the number of groups increases, the process of coalescence becomes more gradual, so that the sum of individual volume concentrations becomes less sensitive to the number of bubble groups.
Model testing: Boiling in a vertical heated channel (2/2)

Figure 4. Five-group model. Partial volume fraction for individual bubble-size groups along a boiling water channel at a pressure of 7 MPa. Comparison between Roe and SEDES numerical results.
Conclusion

- A multifield model is presented.
- The model allows for bubble repartition with respect to three or more groups of bubble sizes.
- The specific objective was to assess the effect of local bubble coalescence and breakup on the hydraulic performance of the channel. The model formulation used in the analysis was based on two-, three-, four-, five-, six-, and ten-field ensemble averaged multifluid models.
- The overall model was validated against experimental data for a bubble column, and a good agreement was observed between the results of calculations and the measurements.
- The results show that with the increasing number of groups of different bubble sizes the predicted total axial void fraction converges to a fixed distribution.
- Work is still needed, including thorough comparisons against experimental data, in order to significantly reduce the predictive uncertainties of void fraction distribution in boiling channels.