Introduction to *Absolute Generality*

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1. The Problem of Absolute Generality. Absolutely general inquiry is inquiry concerning absolutely everything there is. A cursory look at philosophical practice reveals numerous instances of claims that strive for absolute generality. When a philosopher asserts (1), for example, we generally take the domain of her inquiry to comprise absolutely everything there is:

(1) There are no abstract objects.

When presented with a purported counterexample, we do not regard it as open to the philosopher to reply that certain abstract objects are not relevant to her claim because, despite the fact they exist, they lie outside of her domain of inquiry.

Whether or not we achieve absolute generality in philosophical inquiry, most philosophers would agree that ordinary inquiry is rarely, if ever, absolutely general. Even if the quantifiers involved in an ordinary assertion are not explicitly restricted, we generally take the assertion's domain of discourse to be implicitly restricted by context.[1] Suppose someone asserts (2) while waiting for a plane to take off:

(2) Everyone is on board.

We would not wish to attribute to her the claim that absolutely everyone in the universe is on board, only the claim that everyone in a group of contextually relevant people is on board.

The topic of this volume is the question whether we are able to engage in absolutely general inquiry, and, more importantly, whether we do as a matter of fact engage in absolutely general

[1] The question of how this restriction takes place is a delicate and hotly contested issue. According to the standard approach, the phenomenon of quantifier domain restriction is a semantic phenomenon. But Bach (2000) has argued that it is best understood as a pragmatic phenomenon. In what follows, we shall assume the semantic approach for expository purposes. For a characterization of the standard view, and a discussion of the various forms it might take, see Stanley and Szabó (2000).

inquiry in philosophical and non-philosophical practice. This question breaks down into two related but distinct subquestions:

> THE METAPHYSICAL QUESTION
> Is there an all-inclusive domain of discourse?
>
> THE AVAILABILITY QUESTION
> Could an all-inclusive domain be available to us as a domain of inquiry?

In the special case of linguistic inquiry, it is natural to suppose that the availability question comes down to the question of whether our utterances could ever involve genuinely unrestricted quantifiers—quantifiers unburdened by any (non-trivial) restriction whatever, contextual or otherwise.

It may be of interest to note that the possibility of unrestricted quantification does not immediately presuppose the existence of an all-inclusive domain. One could deny that there is an all-inclusive domain and nevertheless grant that some of our quantifiers are sometimes *unrestricted*.[2] One could claim, for example, that although there is no all-inclusive domain, there are utterances of (1) in which no linguistic or contextual mechanisms impose any restrictions whatever on the quantifier. Such utterances would not be absolutely general since the quantifiers would not range over an all-inclusive domain, but they would nonetheless be unrestricted.[3] (It is an interesting question, however, what the truth-conditions of an unrestricted but non-absolutely general utterance would consist in.[4]) If, on the other hand, one believed both that there is an all-inclusive domain and that our quantifiers are sometimes genuinely unrestricted, then one should presumably believe that our discourse is sometimes absolutely general.

A word on our use of the term 'domain'. We shall be careful not to assume that the existence of an all-inclusive domain requires the existence of a set (or set-like object) of which all objects are

[2] As far as we know this point was first emphasized by Kit Fine. For further discussion, see Fine, Hellman and Parsons' contributions below.

[3] Someone who combines the view that there is no all-inclusive domain with the view that our quantifiers are sometimes absolutely unrestricted might give an affirmative answer to the availability question in the special case of linguistic inquiry. It might be claimed, in particular, that since there are no linguistic or contextual mechanisms restricting the relevant quantifiers, the all-incluisve domain would be available to us as a domain of inquiry, if only it existed. Were the world to cooperate, absolute generality would be achievable.

[4] For relevant discussion, see Lavine's contribution to the volume.

members. More generally, when we speak of a domain consisting of certain objects, we shall not assume that there must be a set (or set-like object) of which all and only the objects in question are members; the only requirement we take for granted is that there are such objects. We will return to this point in section 2.2.

1.1. A Disclaimer It would be disingenuous to suggest that we have taken a neutral stance in our characterization of the debate. Notice, for example, that our very statement of the topic of the volume—"whether there is (or could be) inquiry concerned with absolutely everything"—itself purports to be concerned with absolutely everything.

Notice, moreover, that in characterizing an 'absolutist' as a proponent of (3):

(3) There is (or could be) inquiry concerned with absolutely everything,

one tacitly presupposes that the debate has been settled in favor of the absolutist, since (3) is concerned with absolutely everything on its intended interpretation. Similarly, in characterizing a 'non-absolutist' as a proponent of the negation of (3):

(4) There isn't (or couldn't be) inquiry concerned with absolutely everything.

one tacitly presupposes that the debate has again been settled in favor of the absolutist, since (4), like (3), is concerned with absolutely everything on its intended interpretation.

Of course one might insist that domains of (3) and (4) should be regarded as somehow restricted. But then (3) and (4) would be beside the point. Each of the two claims prejudges the debate in favor of the absolutist when taken at face value and is irrelevant to the debate when not taken at face value. Absolutists might take this to be a point in their favor. They might suggest that whereas they are in a position to give an adequate statement of the debate from their point of view, it dubious whether it is possible to state the view under consideration from the point of view of a non-absolutist.[5]

[5]Non-absolutists might try to articulate their position with the help of a conditional:

(5) If D is a domain, then there is some individual not in D.

But here it is crucial that (5) not be read as a universally quantified sentence ranging over all domains. It is to be regarded as 'typically ambiguous' (or 'systematically ambiguous'). But ambiguous between what and what? The obvious response: 'ambiguous between *all* domains', will presumably not do. This has led some philosophers to doubt whether the required kind of ambiguity may be adequately elucidated. For discussion, see Parsons (1974), Parsons (1977), Glanzberg (2004) and Williamson (2003). See also Hellman, Lavine and Parsons' contributions below.

Even if one is convinced by the absolutist, one should remember that the mere fact that one is not able to characterize a certain state of affairs need not imply that the state of affairs in question fails to obtain. Moreover, non-absolutists might be in a position to gain philosophical ground even if they are not in a position to produce a statement of their view. Conspicuously, they can attempt to derive a *reductio* from the absolutist view, as characterized by the absolutist.[6]

In order to facilitate our exposition in the remainder of this introduction, we will continue to describe the debate from an absolutist perspective, while doing our best to ensure that it does not affect the justice with which non-absolutist arguments are presented.

2. Skeptical Arguments. In this section we will discuss some influential arguments against the possibility of absolutely general inquiry. The arguments support a negative answer to the metaphysical question or to the availability question or to both.

2.1. Indefinite Extensibility. An influential strategy for casting doubt on the prospects of absolute generality derives from the work of Michael Dummett.[7] It is based on the thought that certain concepts are *indefinitely extensible*. Indefinitely extensible concepts are usually taken to be ones lacking definite extensions. They are instead said to be subject to principles of extendibility which yield a hierarchy of ever more inclusive extensions. The concepts *set* and *ordinal* are often taken to be paradigm cases of indefinite extensibility. Accordingly, should one attempt to specify an extension for *set* or *ordinal*, proponents of indefinite extensibility would claim to be able to find a more inclusive extension by identifying a set that is not in the extension one had specified.[8]

Indefinite extensibility considerations are motivated by a certain view of the set-theoretic antinomies. To appreciate this, it may be helpful to begin by considering and contrasting two different

[6] As Williamson emphasizes in his contribution to this volume, the possibility of a *reductio* would appear to be incompatible with the idea that one could find a less-than-all-inclusive domain D such that each of the absolutist's purportedly absolutely general assertions would be true when restricted to D. For further discussion of this sort of idea, see section 2.3 below.

[7] For example, Dummett (1963) and Dummett (1991) pp. 316-319.

[8] Bertrand Russell had identified what appears to be the same pattern in his Russell (1906). See Shapiro and Wright's contribution for a discussion of Russell's thought and for an independent characterization of indefinite extensibility generally. For further discussion of indefinite extensibility, see Fine and Hellman's contributions below. It is worth noting that considerations of indefinite extensibility have played an positive role in the foundations of set theory; see chapter 2 of Hellman (1989).

attitudes one might take towards Russell's Paradox.[9] The Paradox arises from the observation that the schema of Naïve Comprehension:

(6) $\exists y \forall x(x \in y \leftrightarrow \phi(x))$, where $\phi(x)$ is any formula that doesn't contain 'y' free,

has as an instance:

(7) $\exists y \forall x(x \in y \leftrightarrow x \notin x)$.

But (7) entails each of the following in classical first-order logic:

(8) $\forall x(x \in r \leftrightarrow x \notin x)$,

(9) $r \in r \leftrightarrow r \notin r$.

And (9) is a contradiction.

A common line of response to the Paradox concedes that the argument from Naïve Comprehension to (9) is valid but insists that Naïve Comprehension—and, in particular, its instance (7)—should be rejected: there is no set of all non-self membered sets.

In contrast, friends of indefinite extensibility would contend that Naïve Comprehension is true, once properly interpreted, since the lesson of the paradox is that r should be taken to lie outside the range of 'x'. The inference from (8) to (9) is therefore invalid, and (6)–(8) can all be true even if (9) is false.[10]

This latter attitude towards the paradox plays an important role in a standard argument for the indefinite extensibility of the concept *set*. The argument is based on a challenge: to supply an extension for the concept *set*. Should one respond to the challenge by offering some candidate extension E, one will be asked to consider the set r of all and only sets in E that are not members of themselves:

(10) $\forall x(x \in r \leftrightarrow x \notin x)$, where '$x$' ranges over sets in E

The next step is to notice that if E contained all sets, r would have to lie within E and (10) would entail:

[9] Our discussion of the paradox closely follows Cartwright (1994).

[10] See, for example, the discussion of indefinite extensibility in Fine and Lavine's contributions.

(11) $r \in r \leftrightarrow r \notin r$,

which is a contradiction. Thus a contradiction has been reached from the assumption that E contains all sets. Most proponents of indefinite extensibility take the lesson to be that supplying a definite extension for the concept *set* is impossible, and that *set* is indefinitely extensible.[11]

Of course, one might protest that the lesson of Russell's Paradox is that there is no such set as r and that it was therefore illegitimate for proponents of indefinite extensibility to insist upon (10). But it is precisely at this point that the attitude that proponents of indefinite extensibility take towards the Paradox comes into play. They will insist instead that when we take the range of 'x' in Naive Comprehension to range over all and only members of E, a proper interpretation of Naive Comprehension will still deliver the existence of r as an immediate consequence. But r will lie outside the range of 'x' and will therefore fail to be a member of E. The moral of the Paradox is not, we are invited to suppose, that there is no such set as r, but rather that r is not a member of E.

A variant of this argument, based on the Burali-Forti Paradox, is sometimes used to motivate the conclusion that the concept *ordinal* is indefinitely extensible. And it might be suggested that it follows from either of these arguments that the concept *self-identical* is indefinitely extensible.[12] Admittedly, there is room for doubting whether the concept *self-identical* exemplifies the reproductive pattern illustrated by, e.g. *set*, since it is not obvious that there is an extensibility principle for *self-identical* paralleling the the extensibility principle for *set* (i.e. Naïve Comprehension). But perhaps one could argue for the indefinite extensibility of *self-identical* by making use of the observation that all sets are self-identical. From the assumption that a domain D contains all self-identical objects, one might reason that, since all sets are self-identical, D must have the domain E of all sets as a subdomain.[13] One might then use Naïve Comprehension to argue that there must be a set r outside the domain E, and hence a self-identical object outside D, contradicting one's original

[11] Not all, however. Shaughan Lavine suggests, in his contribution, a different route to the view that the concept *set* is indefinitely extensible.

[12] It may be of interest to note that Williamson (1998) supplies an account of indefinite extensibility whereby *set* is indefinitely extensible, but *self-identical* is not.

[13] While this reasoning may seem plausible, it is not beyond doubt. For one could presumably combine the assumption that D is the domain of all self-identical objects with the further thesis that no subdomain of D is, as a matter of fact, the domain of all self-identical objects that are sets.

assumption.

The claim that there are indefinitely extensible concepts is not quite the claim that absolute generality is unattainable. But proponents of indefinite extensibility are typically well-disposed to go from the one to the other. Considerations of indefinite extensibility have been used to question the prospects of absolute generality in at least two different ways. One route is to question the metaphysical presumption that there is an all-inclusive domain of all objects. If the concept *self-identical* lacks a definite extension, what reason could there be for thinking that there is an all-inclusive domain? Of course, absolutists will insist that it is incumbent on proponents of this position to provide some sort of account of what the world is like if there is to be no all-inclusive domain. One kind of answer is inspired by Ernst Zermelo's picture of the universe of set theory as an open-ended but well-ordered sequence of universes, where each universe is strictly more inclusive than its predecessor.[14]

A more modest approach would draw linguistic conclusions but refrain from setting forth any metaphysical theses. The claim would then be simply that considerations of indefinite extensibility show that our quantifiers are systematically restricted to less-than-all-inclusive domains. But, of course, this sort of position must be supplemented with some account of the mechanisms by which the relevant restrictions are supposed to take place.[15]

2.2. The All-in-One Principle. A related argument against absolute generality is based on a principle first identified—but never endorsed—by Richard Cartwright:

> **All-in-One Principle** The objects in a domain of discourse make up a set or some set-like object.

With the All-in-One Principle on board, one might argue as follows. Suppose for *reductio* that there is an absolutely general discourse. By the All-One-Princple, there is a set (or set-like object) with all objects as members. But the lesson of Russell's Paradox is that there is no set (or set-like object) with all objects as members. Contradiction.

[14]This picture of the universe of set theory is developed in Zermelo (1930). For a discussion of the open-endedness of mathematics generally, see Hellman's contribution below.

[15]For suggestions, see Parsons (1974), Parsons (1977), Glanzberg (2004), and Glanzberg and Parsons's contributions below.

There are two main lines of response to this argument. The first is to object that the argument presupposes that Russell's paradox entails that there is no universal set (or set-like object). But this is only the case in the presence of the Principle of Separation:

(12) $\forall z \exists y \forall x (x \in y \leftrightarrow \phi(x) \wedge x \in z)$, where $\phi(x)$ is any formula of the language that doesn't contain 'y' free

And there are set theories that countenance exceptions to the Principle of Separation in exchange for a universal set. (There are also theories of set-like objects that countenance exceptions to an analogue of Separation in exchange for an all-inclusive set-like object.)[16] A problem for this line of response is that the Principle of Separation seems to fall out of what are arguably the two best understood conceptions of set—the Iterative Conception and the Limitation of Size Conception. No set theory motivated by these conceptions allows for a universal set (or set-like object).

The second line of response is to object to the All-in-One Principle. Different motivations for the principle must be undercut by different lines of objection. One might be led to the All-in-One Principle by considerations of indefinite extensibility of the sort discussed in the last section. The obvious objection in that case would be to eschew the Principle of Naïve Comprehension and respond to Russell's Paradox by denying that there is a set of all and only nonselfmembered sets.

There is, however, an alternative motivation for the All-in-One Principle that does not immediately depend on considerations of indefinite extensibility. It begins with the observation that model theory—and, in particular, the model-theoretic characterization of logical consequence—requires quantification over all domains. But on the assumption that (singular) first-order quantification over objects is the only intelligible sort of quantification there is, model theory requires the truth of the All-in-One Principle.[17]

One response at this point, originally due to Richard Cartwright, accepts the claims that model theory requires quantification over domains and that first-order quantification over objects is the

[16] For an overview, see Forster (1995). Linnebo and Weir's contributions below both propose mixed theories of sets and properties that allow for properties applying to absolutely all there is. While Linnebo's contribution countenances exceptions to a principle of comprehension for properties, Weir's contribution countenances a revision of classical logic as the underlying logic.

[17] Timothy Williamson has recently articulated the motivation and confronted the argument against absolute generality that ensues in Williamson (2003). For additional discussions of the motivation and its role in arguments against absolute generality, see Linnebo and Parsons' contributions below.

only intelligible sort of quantification there is. Instead, it questions the claim that model theory requires quantification over *all* domains. Cartwright's strategy relies on Georg Kreisel's observation that the material adequacy of the standard model-theoretic characterization of logical truth and logical consequence requires only quantification over *set-sized* domains of discourse.[18] So one might argue that as far as the model-theoretic characterization of first-order logical truth and logical consequence is concerned, the needs of model theory will be met even if the All-in-One Principle fails. Unfortunately, this reply is relatively unstable. It will break down, for instance, if one enriches a first-order language with a quantifier 'Qx' such that '$Qx\ Fx$' is true just in case there are more Fs than there are sets.[19] More generally, Kreisel's argument depends on the existence of a sound and complete deductive system for the language in question, so Cartwright's reply is not guaranteed to be available when one is concerned with languages for which there is no sound and complete deductive system, such as higher-order languages.[20]

An alternative to Cartwright's response is to deny the assumption that first-order quantification is the only intelligible sort of quantification there is.[21] One might contend, in particular, that second-order quantification (and higher-order quantification in general) is just as intelligible as standard first-order quantification, and propose a second-order regimentation of domain-talk.[22]

If one thought of second-order quantification as quantification over first-level Fregean concepts, talk of domains might be regimented as talk of first-level concepts, which are not objects.[23] But, of course, a Fregean interpretation of higher-order quantification is not compulsory. One could instead

[18]Kreisel's argument is remarkably simple. Suppose that the first-order sentence ϕ is a logical consequence of the set of first-order sentences Γ. Then since every set-sized model corresponds to a legitimate interpretation of the language, ϕ must be true in every set-sized model of Γ. Conversely, suppose that ϕ is true in every set-sized model of Γ. Then, by completeness, ϕ must be a deductive consequence of Γ. But since the axioms of the first-order predicate calculus are valid and its rules are validity-preserving, this means that ϕ must be a logical consequence of Γ. The argument is given in Kreisel (1967).

[19]Vann McGee made this point in McGee (1992).

[20]Kreisel's result can be extended to the case of higher-order languages by assuming suitable reflection principles. But the principles in question are provably independent of the standard axioms of set-theory (if consistent with them). For further discussion, see Rayo and Uzquiano (1999).

[21]This type of response was first explicitly articulated in Williamson (2003).

[22]The intelligibility of higher-order quantification is a hotly contested issue. Proponents of intelligibility include Boolos (1984), Boolos (1985a), Boolos (1985b), Oliver and Smiley (2001), Rayo and Yablo (2001), Rayo (2002) and Rayo's contribution below; skeptical texts include Quine (1986) ch. 5, Resnik (1988), Parsons (1990) and Linnebo (2003). For a different sort of proposal, see Linnebo's contribution below.

[23]By a first-level Fregean concept, we mean a Fregean concept under which only objects fall. Correspondingly, a second-level Fregean concept is one under which only first-level Fregean concepts fall, and, in general, an $n+1$-th level Fregean concept is one under which only n-level Fregean concepts fall.

read higher-order quantification in terms of plural quantification, as in Boolos (1984). Apparently singular talk of domains could then be regimented as plural talk of the objects the domain consists of. For instance, the claim that the domain of Fs exists could be regimented as the claim that the Fs themselves exist. Plural quantifiers can be used to produce an adequate model-theory for first-order languages equipped with absolutely unrestricted quantifiers (and, when conjoined with plural predicates, an adequate model-theory for second-order languages).[24] A feature of this general line of response is that one finds oneself invoking in one's model theory logical resources that go well beyond the logical resources used in one's object language. So one faces a choice between doing without a model-theory for one's metalanguage or embracing an open-ended hierarchy of languages of ever-increasing strengths. Those who embrace a Fregean interpretation of higher-order quantification will presumably embrace such a hierarchy. But an open-ended hierarchy would seem to pose a special challenge for those who prefer a plural interpretation of second-order quantification, since it is doubtful that there is a English reading, e.g. for third-order quantifiers corresponding to the plural reading for second-order quantifiers.[25]

2.3. The Argument from Reconceptualization There is a tradition in philosophy according to which ontological questions are relative to a conceptual scheme, or to a language. It goes back to Rudolf Carnap and includes such recent philosophers as Hilary Putnam and Eli Hirsch.[26] Here is a suggestive example, due to Putnam.[27] We are asked to consider—as a Carnapian might put it—"a world with three individuals": x_1, x_2 and x_3. We are then asked to compare this description of the world with that of a "Polish logician" who, bound by the principles of classical mereology, believes that the world contains *seven* individuals: x_1, x_2, x_3, $x_1 + x_2$, $x_1 + x_3$, $x_2 + x_3$ and $x_1 + x_2 + x_3$.[28] Putnam suggests that it is nonsensical to ask which of the rival descriptions is correct. Each of them is correct relative to a particular conceptual scheme, but there is no conceptual-scheme-independent fact of the matter as to what individuals there are in the world. And if there is no

[24] The details are developed in Rayo and Uzquiano (1999). Rayo and Williamson (2003) employs a similar model theory for the purpose of proving a completeness theorem for first-order logic with absolutely unrestricted quantifiers.

[25] For an attempt to address the issue, see Rayo's contribution below.

[26] A canonical statement of the view is given in Carnap (1950). Hilary Putnam discusses the view in Putnam (1987). A more recent discussion of a similar view occurs in Hirsch (1993).

[27] The example is used in Putnam (1987), pp. 18-9.

[28] The examples presupposes that the Polish logician takes each of x_1, x_2 and x_3 to be mereological atoms.

conceptual-scheme-independent fact of the matter as to what there is, then it is nonsensical to speak of a domain that is objectively all-inclusive. The most one should hope for is a domain which is all-inclusive by the lights of some conceptual scheme or other.[29]

One response to this argument is to note that nothing in Putnam's argument rules out the possibility that the apparent lack of objectivity is merely a matter of linguistic equivocation. For all that has been said, the interlocutors in Putnam's example might have different concepts of existence and objecthood, and therefore mean different things by their quantifiers. On this view, Putnam's scenario poses no obstacle to the *metaphysical* thesis that there is an all-inclusive domain. At most, it would pose an obstacle to the *linguistic* thesis that our use of the quantifiers is univocal.[30] Issues of semantic indeterminacy will be the focus of the next section.

2.4. The Argument from Semantic Indeterminacy. Even if one takes for granted that there is such a thing as an all-inclusive domain, one might worry about access. One might worry, in particular, that any use of a quantifier that is compatible with the all-inclusive domain is also compatible with some less-than-all-inclsuive domain, and therefore that one could never *determinately* quantify over absolutely everything.

W.V. Quine and Hilary Putnam have famously set forth arguments that can be taken to support this sort of indeterminacy.[31] One of the most influential is based on the following technical result:

> Let L be a countable first-order language, and assume that each closed term in L has an intended referent and that each predicate in L has an intended extension. Call an interpretation I of L *apt* if it assigns to each term in L its intended referent and to each predicate in L the restriction of its intended extension to I's domain.[32]
>
> If the intended domain is uncountable, then it is provable that if every sentence of L in a set S is true according to some apt interpretation with an all-inclusive domain, then

[29]See Hellman and Parsons' contributions for discussions of this general strategy against absolute generality.

[30]Even the argument for equivocation might be resisted. See, for instance, Sider (forthcoming).

[31]The arguments are developed in Quine (1968) and Putnam (1980). Field (1998) contains a more recent discussion of the argument.

[32]Since there is no set of all objects, it will not do for present purposes to think of interpretations as set-sized models. For a more appropriate account of interpretation, see Rayo and Uzquiano (1999). The technical result that concerns us carries over without incident on the revised account.

> every sentence in S is also true according to an apt interpretation with a less-than-all-inclusive domain.[33]

The lesson of this result, it might be claimed, is that any use of a first-order quantifier compatible with an all-inclusive (uncountable) domain is also compatible with a less-than-all-inclusive domain, and therefore that first-order quantifiers never determinately range over an all-inclusive domain. But as with most philosophical morals extracted from technical results, the conclusion only follows in the presence of substantive philosophical assumptions. One such assumption might be the thesis that only domains of discourse that are incompatible with the truth of an utterance can be ruled out as unintended.

Even though the formal result is unassailable, there are various ways in which the auxiliary philosophical assumptions might be questioned. One might, in particular, claim that factors other than truth might help determine the domain of quantification of our utterances. One strategy, developed by David Lewis in a broader context, is based on the view that certain collections of individuals are objectively more 'natural' than others: to a larger extent, they 'carve nature at the joints'. The objective naturalness of collections of individuals might be used to argue that some candidate semantic values for an expression are objectively more natural than others. One could then argue that—other things being equal—an assignment of semantic value is to be preferred to the extent that it is objectively more natural than its rivals. This allows one to resist the indeterminacy argument on the grounds that it neglects to take into account the constraint that a semantic interpretation be objectively natural.[34] Ted Sider has explicitly deployed this sort of argument in the context of discussions of absolute generality by claiming that an all-inclusive interpretation of the quantifiers is especially natural.[35]

A different line of resistance has been proposed by Vann McGee. By taking absolutely general quantification for granted in the metalanguage, McGee argues that one's object-language quantifiers are guaranteed to range over absolutely everything on the assumption that they satisfy an *open-*

[33] As first noted by Putnam (1980), this result is provable on the basis of a strong version of the Löwenheim-Skolem Theorem.

[34] Lewis outlines the view in Lewis (1983) and Lewis (1984). The second paper is a response to a skeptical argument based on a formal result like the one quoted above.

[35] The suggestion occurs for example in pp. xx-xxiv of Sider (2003).

ended version of the standard introduction and elimination rules—that is, a version of the rules that is to remain in place even if the language is enriched with additional vocabulary. As McGee is at pains to insist, it is far from clear that simple standard introduction and elimination rules could exhaust the meaning of the English quantifiers. However, he suggests that open-endedness is a distinctive feature of our quantificational practice, and that this fact alone suffices to cast doubt on the indeterminacy argument, which fails to take considerations of open-endedness into account.[36]

A third strategy for resisting the indeterminacy argument is based on pragmatic considerations. Suppose a resourceful and fully cooperative speaker asserts 'I am speaking of absolutely everything there is' (and nothing else) in a conspicuous effort to clarify what her domain of discourse consists in. Since the sentence asserted can be true relative to any domain of discourse, the speaker was inarticulate. But the speaker is fully cooperative, so if she was inarticulate (and if there is nothing else to explain the inarticulacy), it must have been because she couldn't do any better. And since the speaker is resourceful, it is only plausible that she couldn't do any better if she intended her domain to be absolutely all-inclusive. (Otherwise she would be in a position to say at least that it was not incompatible with her intentions that the original domain was less-than-all-inclusive.) So there is some reason for thinking that the speaker intended her domain to be absolutely all-inclusive. By failing to take pragmatic considerations into account, the argument for indeterminacy is insensitive to this sort of phenomenon.[37]

It is worth emphasizing that the lines of objection we have considered in this section are not aimed at a skeptic of absolute generality. They are best understood as defensive maneuvers, intended to reassure absolutists of the coherence of their own position by explaining *why* it is that the indeterminacy argument fails, given that it does fail.

2.5. The Argument from Sortal Restriction. According to an influential account of quantification discussed by Michael Dummett "each domain for the individual variables will constitute the extension of some substantival general term (or at least the union of the extensions of a num-

[36] See McGee (2000) and McGee's contribution to this volume. For criticism of McGee, see the postscript to Field (1998) in Field (2001) and Lavine and Williamson's contributions to this volume. For additional discussion of open-endedness, see Parsons (1990), Feferman (1991) and Lavine (1994).

[37] This idea is developed in Rayo (2003).

ber of such substantival terms)".[38] By a substantival term, Dummett means a term that supplies a criterion of identity, and (following Frege) a criterion of identity is taken to be "a means for recognizing an object as the same again."[39]

This account of quantification is one step away from the thesis that absolutely general quantification is somehow illicit. All one needs to take the further step is the thesis that purportedly all-inclusive general terms—terms like 'thing', 'object' or 'individual'—do not supply criteria of identity, and are therefore not 'substantival'. This further thesis might perhaps be motivated by appeal to the claim that it is only sensible to ask a question of the form, for example, 'how many Fs does the room contain' when F is a substantival general term. It is nonsense, the suggestion would continue, to ask, without tacit restriction to one substantival term or another, 'how many *things* does the room contain?' (and similarly for other purportedly all-inclusive general terms).[40]

A proper assessment of the thesis that a domain of quantification must be restricted by a substantival general term—and that purportedly all-inclusive general terms are non-substantival—would require discussion of broad issues in the philosophy of language which are beyond the scope of this essay.[41]

3. Conflicts. We have considered a number of arguments designed to cast doubt on the prospects of absolutely general inquiry. Absolutists have developed responses, and optimistic absolutists might think that their view has emerged more or less unscathed. But even the optimists must acknowledge that engaging in absolutely general inquiry requires a great deal of caution. For absolutely general theories sometimes place non-trivial constraints on the size of the universe, and different theories might call for inconsistent constraints. Conflicts might also arise amongst absolutely general theories with different and in fact disjoint vocabularies, as in the case of certain standard formulations of applied set theory and a certain extension of classical extensional mereology. Certain plausible versions of these theories cannot both be true and absolutely general, for whereas the former requires that the universe be of a strongly inaccessible size, the latter requires

[38]See Dummett (1981) pp. 569 and 570, respectively.
[39]See Frege (1884) §64.
[40]See, for instance, Geach (1962), pp. 38, 153.
[41]Relevant texts in the more recent literature include Lowe (1989), Wiggins (2001) and Linnebo (2002).

that the universe be of a successor size.[42]

4. Summaries. We conclude with summaries of each of the contributions to the volume in the hope they may allow some readers to identify those most likely to be of interest to them.

Kit Fine. In "Relatively Unrestricted Quantification", Kit Fine reviews the classic argument from indefinite extensibility and suggests that it is unsatisfactory as its stands. While traditional indefinite extensibility considerations challenge the absolutist interpretation of the quantifiers, they have no force in the absence of an absolutist opponent. Fine's contribution develops a modal formulation of indefinite extensibility designed to overcome this defect. On the modal interpretation of indefinite extensibility, the concept of, e.g. *set* is said to be indefinitely extensible on the grounds that whatever interpretation of a quantifier ranging over sets one might come up with, it will be possible to find a more inclusive interpretation. This modal understanding of indefinite extensibility calls for the use of distinctive *postulational* modalities. What is special about postulational modalities is that they concern variation in interpretation rather than variation in circumstances. Since postulational necessities and possibilities are forms of interpretational necessities and possibilities, they are germane to the issue of unrestricted quantification.

Michael Glanzberg. In "Context and Unrestricted Quantification", Michael Glanzberg offers a development of the view that because of the semantic and set-theoretic paradoxes, seemingly unrestricted quantification is in fact restricted. Glanzberg's focus is not, however, on defending the argument from paradox. The main objective of the paper is to acquire a better understanding of what the quantificational restrictions required by the argument from paradox consist in. Glanzberg suggests that the argument from paradox results in a shift to a 'reflective context', which in turn leads to an enriched background domain of discourse. He goes on to argue that it is possible to specify a list of general principles governing the construction of the new domain. The process involves the setting of a context-dependent parameter in a sentence, and is said to be fundamentally related to the pragmatic processes governing non-paradoxical contexts. Glanzberg rounds off the

[42] The conflict has been discussed in Uzquiano (forthcoming) with a view to questioning the alleged absolute generality of mereology. For a general assessment of the difficulty, see Uzquiano's contribution below.

proposal by offering some formal models of what the enriched domain might look like.

Geoffrey Hellman. In "Against 'Absolutely Everything'", Geoffrey Hellman distinguishes two general kinds of arguments against absolute generality. One kind of argument is based on considerations of indefinite extensibility and the open-ended character of mathematical concepts and structures; the other is based on the relativity of ontology to a conceptual framework. While the first kind of argument springs from specifically mathematical concerns and relies on a platonist view of mathematical ontology, the second is much more general and threatens the prospects of absolute generality even for theorists with a nominalist view of mathematical ontology. This raises the question of what to make of seemingly absolutely unrestricted generalizations such as "No donkey talks". Hellman suggests a contextualist interpretation of such generalizations, according to which they are taken to contain a schematic element.

Shaughan Lavine. In "Something about Everything: Universal Quantification in the Universal Sense of Universal Quantification", Shaughan Lavine elucidates and defends the notion of a 'full scheme': a scheme whose instances are open-ended and automatically expand as the language in use expands. The notion is then used in support of two main theses. The first is that efforts to counter Putnam-style arguments for the indeterminacy of quantification and the non-existence of an all-inclusive domain through the use of open-endedness are ultimately unsuccessful. Lavine argues that the essential use of open-endedness in these arguments is best understood in terms of full schemes, and that full schemes would not give a proponent of these arguments what she needs. The second thesis is that opponents of absolutely unrestricted quantification do not suffer from crippling expressive limitations: full schemas give them all the expressive power they need. Lavine concludes that we have no positive reason for thinking that either the notion of absolutely unrestricted quantification or the notion of an all-inclusive domain of discourse is a coherent notion.

Øystein Linnebo. In "Sets, Properties and Unrestricted Quantification", Øystein Linnebo addresses the challenge of embracing absolutely general quantification while avoiding semantic pessimism: the view that there are legitimate languages for which an explicit semantics cannot be given.

Linnebo argues that efforts to address the challenge that proceed by postulating an open-ended hierarchy of logical types suffer from important expressive limitations, and lays the foundations for developing a type-free alternative. The alternative supplements standard ZFC set theory with a (first-order) notion of property with the following two features: on the one hand, there are enough properties to do interesting theoretical work; on the other, there are principled reasons for rejecting instances of a comprehension schema for properties that would lead to paradox. Linnebo's philosophical defense of the project is accompanied by a technical discussion of the resulting systems.

Vann McGee. In "There's a Rule for Everything", Vann McGee is engaged in a defensive project. He aims to explain—from the point of view of a generality absolutist—how it is that the meaning of an absolutely general quantifier might be fixed. The proposal is based on the idea that a rule of inference can be read open-endedly: it can be seen as continuing to hold even if the language expands. By working within a metalanguage in which absolutely general quantification is allowed, McGee uses a Tarski-style model-theoretic semantics to give a rigorous characterization of open-endedness, and argues that only an absolutely general quantifier could satisfy versions of the standard quantifier introduction and elimination rules that are open-ended in the regimented sense. McGee does not claim that English quantifiers should be seen as obeying the rules in question, but he takes his argument to show that considerations of open-endedness can help us understand how English speakers succeed in quantifying over absolutely everything.

Charles Parsons. In "The Problem of Absolute Universality", Charles Parsons raises the question of whether apparently absolutely unrestricted generalizations such as "Everything is self-identical" should be taken at face value and argues that they should not. He suggests that the acknowledgment of absolutely unrestricted quantification would commit one to metaphysical realism, understood as the thesis that there is some final answer to the question of what objects there are and how they are to be individuated. This is contrasted with the acknowledgment of merely unrestricted quantification, from which no grand metaphysical conclusion seems to follow. Parsons then presents what he takes to be some of the most important logical obstacles for the viability of absolutely unrestricted quantification. These obstacles emerge when we quantify over interpretations and look at what are now familiar Russellian paradoxes for interpretations. The paper concludes with a discussion of the suggestion that absolutely unrestricted generalizations such as "Everything is self-identical" should be taken to be systematically ambiguous.

Agustín Rayo. In "Beyond Plurals", Agustín Rayo has two main objectives. The first is to get a better understanding of what is at issue between friends and foes of higher-order quantification, and of what it would mean to extend a Boolos-style treatment of second-order quantification to third- and higher-order quantification. The second objective is to argue that in the presence of absolutely general quantification, proper semantic theorizing is essentially unstable: it is impossible to provide a suitably general semantics for a given language in a language of the same logical type. Rayo thinks that this leads to a trilemma: one must choose between giving up absolutely general quantification, settling for the view that adequate semantic theorizing about certain languages is essentially beyond our reach, and countenancing an open-ended hierarchy of languages of ever ascending logical type. Rayo concludes by suggesting that the hierarchy may be the least unattractive of the options on the table.

Stewart Shapiro and Crispin Wright. In "All Things Indefinitely Extensible", Stewart Shapiro and Crispin Wright are primarily concerned with indefinite extensibility. Their departure point is a conjecture they attribute to Bertrand Russell to the effect that a concept is indefinitely extensible only if there is an injection from the concept ordinal into it. After dispensing with apparent exceptions to Russell's conjecture, Shapiro and Wright argue for a more informative characterization of indefinite extendibility from which Russell's conjecture falls out as a consequence. Their characterization promises to cast new light upon the paradoxes of indefinite extensibility and have important ramifications for a variety of issues in the philosophy of mathematics. They suggest, for example, that indefinite extensibility is the key to a proper understanding of the Aristotelian notion of potential infinity. They also consider a restriction of Basic Law V based on indefinite extensibility and its potential for improvement on extant neo-logicist attempts to ground set theory on abstraction principles. Finally, they discuss the question of whether it is ever legitimate to quantify over all of the members of indefinitely extensible totalities, and suggest that no completely satisfactory answer seems to be available.

Gabriel Uzquiano. In “Unrestricted Unrestricted Quantification: The Cardinal Problem of Absolute Generality”, Gabriel Uzquiano raises an internal problem for absolutism. The problem arises when one notices the possibility of conflicts amongst absolutely general theories with different and, in fact, disjoint vocabularies. Uzquiano presents two different examples of such potential conflicts for consideration and casts doubts upon the prospects of a unified and systematic solution for them. Instead, Uzquiano suggests conflicts amongst absolutely general theories should generally be addressed on a case by case basis. Uzquiano argues that in at least some cases such conflicts are best solved by abandoning the claim to absolute generality for some of the theories involved. The paper concludes with a call for caution for the absolutist and a brief discussion of the prospects of an argument against absolute generality based on the possibility of conflicts amongst absolutely general theories with different vocabularies.

Alan Weir. In “Is It Too Much to Ask, To Ask for Everything?”, Alan Weir argues for a view based on the following three theses: (1) one can quantify over absolutely everything, (2) the domain of discourse of an interpreted language is an individual, and (3) one can characterize a notion of truth-in-an-interpretation for one’s object language in one’s object language. He proceeds by using a Kripke-style fixed-point construction to characterize a notion of property that satisfies naïve comprehension axioms, and a notion of truth-in-an-interpretation that takes domains to be properties. (Such notions would ordinarily lead to paradox, but Weir addresses the problem by proposing a revision of classical logic.) Weir concludes by arguing that, even though his proposal doesn’t deliver everything one might have hoped for, it is nonetheless to be preferred over theories that allow for absolutely general quantification but appeal to an ascending hierarchy of logical types to characterize the notion of truth-in-an-interpretation.

Timothy Williamson. In “Absolute Identity and Absolute Generality”, Timothy Williamson develops an analogy between identity and absolutely general quantification. He argues that just like an open-ended reading of the usual axioms of identity can be used to uniquely characterize the notion of identity, an open-ended reading of the usual introduction and elimination rules for the first-order quantifiers can be used to uniquely characterize the notion of absolutely general quantification. Williamson uses the analogy to to show that certain objections against absolutely general quantification can only succeed if corresponding objections succeed in the case of identity. He concludes by noting that foes of absolutely general quantification cannot coherently maintain both that everything the generality absolutist says can be reinterpreted as something the non-absolutist would accept and that the generality absolutist's position leads to paradox.

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