The University of St Andrews
Fife KY16 9AL, UK
slr@st-and.ac.uk

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Frege's correlation

AGUSTÍN RAYO

1. Let me remind you of an old puzzle about applied arithmetic:

> How can knowing anything about numbers be relevant to one's knowledge of the natural world? Why is it, for example, that knowing that the *number* of the apples is identical to the *number* of the children can be relevant to knowing that no apples will be left if each of the children eats an apple?

It is not unusual to think that the right answer to this puzzle is implicit in Frege's *Grundlagen* (see, for instance, Steiner 1998: 21–22). The Fregean answer is supposed to be this:

> There is a correlation between arithmetical truths, on the one hand, and certain second-order truths, on the other. It is embodied in the following principle:
>
> > *Hume's Principle*
> > The number of the Fs is identical to the number of the Gs if and only if the Fs are in one-one correspondence with the Gs.[1]

[1] Although plural quantifiers are used for expositional purposes, Hume's Principle is to be understood as involving second-order quantifiers.

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> In particular, the fact that the number of the apples is identical to the number of the children is correlated with the (second-order) fact that there is a one-one correspondence between the apples and the children, and Hume's Principle can be used to go from knowledge of the former to knowledge of the latter. This solves the puzzle because there is no special mystery about how knowledge of the fact that there is a one-one correspondence between the apples and the children could be relevant to knowledge of the fact that no apples will be left if each of the children eats an apple.

Even if Hume's Principle is assumed to be known and second-order quantification is assumed to be unproblematic, the Fregean answer leaves two important issues unresolved. Firstly, it would be desirable to explain more precisely what it means to say that an arithmetical truth is 'correlated' with a second-order truth. Secondly, there is a question about whether the Fregean answer is complete: we know that Hume's Principle can be used to explain how it is that knowledge of *numerical identities* can be relevant to knowledge about the natural world, but does it suffice to explain how other, more complex, arithmetical truths can be relevant to knowledge of the natural world? The aim of this note is to suggest a way of addressing these two unresolved issues.

2. Consider the following example:

(1) There is a number *n* such that *n* is the number of the planets.

Which second-order truth is correlated with the arithmetical truth expressed by (1)? It is not obvious what to say because, although Hume's Principle delivers a method for eliminating the number operator from contexts of the form 'the number of the Fs = the number of the Gs', it does not immediately deliver a method for eliminating the number operator from contexts of the form '*n* = the number of the Fs'.

Here is a suggestion for dealing with the problem. The first stage is to note that (1) admits of the following paraphrase:

(2) There are some things – the Fs – such that the number of the Fs is the number of the planets.

The second stage is to replace, in accordance with Hume's Principle, the numerical identity in (2) with an appropriate one-one correspondence:

(3) There are some things – the Fs – such that the Fs are in one-one correspondence with the planets.

And (3) gives us what we want, since the second-order truth expressed by (3) seems like a suitable Fregean correlate for the arithmetical truth expressed by (1).

This procedure can be easily generalized. Say that the *Frege-formula*, $\phi^{\mathcal{F}}$, of ϕ is the result of replacing occurrences of $\ulcorner \exists n_i \urcorner$ and $\ulcorner \forall n_i \urcorner$ in ϕ by $\ulcorner \exists Z_i \urcorner$ and $\ulcorner \forall Z_i \urcorner$, respectively, and replacing occurrences of $\ulcorner n_i = \#_x\, Gx \urcorner$, $\ulcorner \#_x\, Gx = n_i \urcorner$ and $\ulcorner n_i = n_j \urcorner$ in ϕ by $\ulcorner Z_i \approx G \urcorner$, $\ulcorner G \approx Z_i \urcorner$ and $\ulcorner Z_i \approx Z_j \urcorner$, respectively (where '#' is the number operator and '$\approx$' expresses one-one correspondence). It is then possible to prove the following two results for any formula in the language of second-order applied arithmetic:[2]

CORRELATION
Assume Hume's Principle. Then $\psi(n_i)$ is true of the number of the Fs just in case $\psi^{\mathcal{F}}(Z_i)$ is true of the Fs themselves (and similarly for formulae with more than one free variable).

COMPLETENESS
Hume's Principle $\vdash \phi \leftrightarrow \phi^{\mathcal{F}}$

3. We now have a proposal about what the Fregean correlation comes down to for any sentence ϕ in the language of second-order applied arithmetic: the arithmetical truth expressed by ϕ is correlated to the truth expressed by the corresponding Frege-formula $\phi^{\mathcal{F}}$. Moreover, as long as the number-operator is the only piece of mathematical vocabulary occurring in ϕ, it is easy to verify that $\phi^{\mathcal{F}}$ contains no mathematical vocabulary; so $\phi^{\mathcal{F}}$ can be said to express a second-order truth which is 'about the natural world'.

Let us return to the two unresolved issues from §1. In order to address the first we must explain more precisely what it means to say that an arithmetical truth is 'correlated' with a second-order truth. But, when Hume's Principle is in place, CORRELATION allows us to characterize the relationship between an arithmetical formula and the corresponding Frege-formula in very precise terms: an arithmetical formula $\psi(n_i)$ is true of the number of the Fs just in case the corresponding Frege-formula $\psi^{\mathcal{F}}(Z_i)$ is true of the Fs themselves.

The second unresolved issue concerned a doubt about the completeness of the Fregean response. But COMPLETENESS shows that the Fregean answer is indeed complete. For it delivers the result that, by making use of Hume's Principle, a suitable derivation can be used to go from knowledge of the arithmetical truth expressed by an arbitrary sentence ϕ in the lan-

[2] I assume that the language contains no mixed identities and no non-arithmetical predicates taking arithmetical variables as arguments. The transformation method is not entirely straightforward when it comes to sentences involving second-order variables with (first-order) arithmetical variables as arguments or sentences involving iterated applications of '#', but a precise characterization is supplied in Rayo 2002, together with proofs of results that entail CORRELATION and COMPLETENESS. For additional discussion of Frege-formulae see Fine 2002: II.5.

guage of second-order applied arithmetic to knowledge of the second-order truth expressed by $\phi^{\mathcal{I}}$.[3]

The University of St. Andrews
Fife KY16 9AL, UK

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[3] Many thanks to Gabriel Uzquiano.

Future contingents, non-contradiction, and the law of excluded middle muddle

CRAIG BOURNE

For whatever reason, we might think that contingent statements about the future have no determinate truth value. Aristotle, in *De interpretatione* IX, for instance, held that only those propositions about the future which are either necessarily true, or necessarily false, or 'predetermined' in some way have a determinate truth value. This led Łukasiewicz's in 1920 to construct a three-valued logic in an attempt to formalize Aristotle's position by giving the truth value 1/2 = indeterminate to future contingents and defining '~', '&' and '∨', where 1 = true and 0 = false, as:

~	
1	0
1/2	1/2
0	1

&	1	1/2	0
1	1	1/2	0
1/2	1/2	1/2	0
0	0	0	0

∨	1	1/2	0
1	1	1	1
1/2	1	1/2	1/2
0	1	1/2	0

We can see that the purely determinate entries match the tables of the classical two-valued system; thus, what needs justification are the other entries. Let us take negation to illustrate. We may treat indeterminateness as something to be resolved one way or the other: it will eventually be either