

# *On the Brink of Paradox:*

List of known errors, as of April, 2021<sup>1</sup>

## Chapter 1

- p. 10, table: “At most<sub>7</sub> as many members in A as in B”.
- p. 11: “select. [For example,] [i]t will be answered negatively”.
- p. 18: “Let the range of that function be the set  $S^{\mathbb{N}} = [\{s_0, s_1, s_2, \dots\}]$ ”.
- p. 22: Exercise 1 (and its answer on page 30) uses “ $B_0$ ” for the set that is introduced as “B” in the main text.

## Chapter 2

- p. 48: There’s an important omission here. I fail to mention that the claim that  $|A| + |B| = |A|$  when at least one of  $A$  and  $B$  is infinite and  $|B| \leq |A|$  assumes the axiom of choice. (The minimal correction here is to delete “The claim that  $|A| \otimes |B| = |A|$  whenever  $A$  is infinite,  $B$  is nonempty, and  $|B| \leq |A|$  assumes” and replace with “Nerdy Observation: Here I assume”.)
- p 48:  $|A| \otimes |B| = ~~|B| \oplus |A|~~ |B| \otimes |A|$

## Chapter 3

- pp. 70, 85, 295 “~~Bernadete~~-Benardete”.
- p. 75: “~~a~~-high table”

## Chapter 4

- p. 100: “correspond to points on the ~~dotted~~ [horizontal] line”.

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<sup>1</sup>With thanks to Richard Polt and Dustin Tucker.

## Chapter 5

- p. 135, second and fourth equations: “1BE” should be “2BE”.
- p. 134, last equation: “1BE” should be “2BE”.
- p. 135, first indented conditional: “she failed to do so” should be “she failed to take the trip”.

## Chapter 6

- There is a serious omission in section 6.1.1.

Even though this is not stated in the text, the Objective-Subjective connection is only plausible when one presupposes that a perfectly rational agent is always certain about the connection between events before  $t$  and the objective probabilities at  $t$ . Here is a proposed fix:

*(Nerdy observation:* Here I am tacitly presupposing that a perfectly rational agent is always certain about the connection between events before  $t$  and the objective probabilities at  $t$ . So, in particular, for each complete history of the world up to  $t$ ,  $H_t$ , there is a specification  $P_t$  of the objective probabilities at  $t$  such that the agent assigns credence one to the proposition [if  $H_t$  then  $P_t$ ]. This assumption is potentially controversial but adds simplicity to our discussion.)

With this fix in place one can give a formal proof—given assumptions—of the Principal Principle. Here is a proof for a particular instance:

Assume that  $x$ 's half life is  $7.04 \cdot 10^8$  years. Let  $D$  be the proposition that  $x$  will decay sometime within the next  $7.04 \cdot 10^8$  years. We show that you should believe  $D$  to degree 0.5.

It follows from the fact that  $x$ 's half life is  $7.04 \cdot 10^8$  years that the objective probability of  $D$  is 0.5. It then follows from the Objective-Subjective Connection that a perfectly rational agent with perfect information about the past (and none about the future) would assign credence 0.5 to  $D$ .

Now suppose you are perfectly rational and that—although you have not quite learned the full truth about the past—the information you have acquired,  $E$ , is entirely about the past. Suppose, moreover, that a rational agent would take  $E$  to be compatible with the proposition that  $p(D) = 0.5$ , were  $p$  is objective probability.

Because  $E$  is entirely about the past, it is equivalent to some disjunction  $H_t^1 \vee H_t^2 \vee \dots$  of possible histories-up-to- $t$ . (We must assume that the conjunction is either finite or countably infinite, to ensure Conglomerability later on.) Because perfectly rational agents are always certain about the connection between events before  $t$  and the objective probabilities up to  $t$ , each  $H_t^j$  is equivalent to  $H_t^j P_t^j$ , where  $P_t^j$  is a complete specification of the objective probabilities at  $t$ .

Because  $E$  (and therefore  $H_t^1 \vee H_t^2 \vee \dots$ ) is compatible with  $p(D) = 0.5$ , there are some  $H_t^{k_1}, H_t^{k_2}, \dots$  amongst the  $H_t^1, H_t^2$  such that each  $P_t^{k_i} H_t^{k_i}$  entails  $p(D) = 0.5$ . (Note that every  $H_t^j$  outside this list entails something incompatible with  $p(D) = 0.5$ .) So  $(p(D) = 0.5)E$  is equivalent to  $H_t^{k_1} \vee H_t^{k_2} \vee \dots$ :

$$c(D|(p(D) = 0.5)E) = c(D|H_t^{k_1} \vee H_t^{k_2} \vee \dots)$$

But, for each  $i$ , we know that  $c(D|H_t^{k_i}) = 0.5$ . So, by Conglomerability,

$$c(D|(p(D) = 0.5)E) = c(D|H_t^{k_1} \vee H_t^{k_2} \vee \dots) = c(D|H_t^{k_1}) = 0.5$$

And how do we know the Conglomerability holds? Here is a proof for the finite case. (The result also holds in the countably infinite case but requires Countable Additivity.)

$$\begin{aligned}
p(A|B_1) &= p(A|B_2) \\
\frac{p(AB_1)}{p(B_1)} &= \frac{p(AB_2)}{p(B_2)} \\
p(B_2) \cdot p(AB_1) &= p(B_1) \cdot p(AB_2) \\
p(B_2) \cdot p(AB_1) &= p(B_1)(p(AB_2) + p(AB_1) - p(AB_1)) \\
p(B_1) \cdot p(AB_1) + p(B_2) \cdot p(AB_1) &= p(B_1) \cdot p(AB_2) + p(B_1) \cdot p(AB_1) \\
p(AB_1)(p(B_1) + p(B_2)) &= p(B_1)(p(AB_2) + p(AB_1)) \\
\frac{p(AB_1)}{p(B_1)}(p(B_1) + p(B_2)) &= p(AB_2) + p(AB_1) \\
p(A|B_1)(p(B_1) + p(B_2)) &= p(AB_1) + p(AB_2) \\
p(A|B_1) &= \frac{p(AB_1) + p(AB_2)}{p(B_1) + p(B_2)} \\
p(A|B_1) &= \frac{p(AB_1 \vee AB_2)}{p(B_1 \vee B_2)} \\
p(A|B_1) &= \frac{p(A(B_1 \vee B_2))}{p(B_1 \vee B_2)} \\
p(A|B_1) &= p(A|B_1 \vee B_2)
\end{aligned}$$

We have now shown that  $c(D|(p(D) = 0.5)E) = 0.5$ . But the only restrictions on  $E$  are that it be entirely about the past and that it be compatible with  $p(D) = 0.5$ . So if you're fully rational, then as long as everything you've learned is entirely about the past and compatible that  $p(D) = 0.5$ , Update by Conditionalizing entails that you should believe  $D$  to degree 0.5.

- p. 169, “~~envelope~~ envelope”.
- p. 169, indented paragraph: “ $k$  dollars, say. [If  $k$  is odd, I should definitely switch. What about the case in which  $k$  is even? In that case] ~~This means that~~ the other envelope [...]”. (Also, replace two occurrences of “outcomes” in that paragraph with “scenarios”.)
- p. 173, answer to exercise 1 of section 6.1.1: “~~the coin will land Heads~~ the particle will decay”.

## Chapter 7

- p. 188, last line of first paragraph of 7.2.1: “~~Solovey~~ Solovay”
- p. 190, fourth line of second paragraph of 7.2.2.2: “ $1/2 - 1/6 = 2/3 - 1/3$ ”
- p. 190, third line of final paragraph: “~~Solovey~~ Solovay”

- p.191, final line before the second diagram: “counterclockwise—in this case a distance of  $\pm 1/2$ .”

## Chapter 8

- p. 209, “T” and “B” labels on diagram should be “U” and “D”, respectively.
- p. 218 “partitioning the surface of the ~~sphere~~ [ball]”.
- p. 219  $\left(\overleftarrow{R}\right)^e$  should be  $\left(\overleftarrow{R}_g\right)^e$ .
- p. 219  $\left(\overleftarrow{D}\right)^e$  should be  $\left(\overleftarrow{D}_g\right)^e$ .
- p. 219 “applies *simultaneously* to every graph [in]  $\mathcal{G}$ ”
- p. 220 “one can deal with the ~~vertices~~ [endpoints]”

## Chapter 9

- p. 201, “as we saw in ~~chapter 4~~ exercise 6 of section 6.3.2”
- p. 227, “Adam had named [a] number”
- p. 234, “~~family~~ [fairly] cumbersome”
- pp. 237, 252 “~~coding scheme~~ [coding scheme]”
- p. 244, “whether a purported ~~to-yes~~ [Yes] solution [to] the subset sum problem”

## Chapter 10

- p. 256, “and that every sentence in  $\mathcal{L}$  has a negation”

## Index

- p. 302, third line of final paragraph: “~~Solvey~~ [Solovay]”