In this paper I articulate and defend the claim that possibility is “open ended”: a given set of possibilities can always be used to characterize further possibilities.

1 Metaphysical Possibility

On the conception of metaphysical possibility that I prefer, a metaphysical possibility is simply a coherent way for the world to be.

That is a substantial view. There are conceptions of metaphysical possibility on which not every coherent way for the world to be counts as a metaphysical possibility. To illustrate this point, consider a conception of metaphysical possibility based on a simplified version of David Lewis’s Humean Supervenience Thesis (Lewis 1986). The picture proceeds by postulating a canonical spacetime structure and a set of “natural” properties, which are assumed to be intrinsic. It is then claimed that a metaphysical possibility is a distribution of pointwise instantiations of natural properties across the canonical spacetime structure.

On this conception of metaphysical possibility, there are seemingly coherent ways for the world to be that fail to count as metaphysical possibilities. Here is an example. For each set $X$ of distributions of natural properties across canonical spacetime, consider the result of uniting the distributions in $X$ into single superstructure, which consists of one copy of the canonical spacetime structure, along with its distribution of properties, for each distribution in $X$. There is nothing obviously incoherent about the world’s instantiating the superstructure generated by $X$. But Cantor’s Theorem entails there are more superstructures
than there are distributions of natural properties across canonical spacetime.\footnote{Here I rely on the assumption that whenever \( X \) and \( Y \) are different sets of distributions of natural properties across canonical space time, the superstructures generated by \( X \) and \( Y \) are distinct. But see Nolan 1996.}

So there are seemingly coherent ways for the world to be that fail to count as metaphysical possibilities.

I’d like to mention a second example of a conception of metaphysical possibility that fails to count seemingly coherent ways for the world to be as genuine metaphysical possibilities. Consider Williamson’s (2013) modal necessitism: the view that, as a matter of necessity, everything exists necessarily. Bob Stalnaker once pointed out to me that a problem with necessitism is that it delivers answers to questions that shouldn’t have answers. I take one such question to be the question of how many possible bald men in the doorway there are (Quine 1948), and another to be the question of how many possible angels there are (Hawthorne & Uzquiano 2011).

Why is the necessitist committed to thinking that these questions must have definite answers? Take the case of possible angels. According to necessitism, it is impossible for there to be an angel that is not identical to something that exists necessarily. So there is a definite totality of possible angels: it consists of all and only the individuals that might have been angels.\footnote{Here I really on the assumption that there is sense to be made of an absolutely general quantifier. I am not myself sympathetic towards this assumption, but Williamson is. See, for example, Williamson 2003.}

Notice, however, that once we have a definite totality of possible angels, we can use it to characterize a seemingly coherent way for the world to be that cannot be counted as metaphysically possible. For however large the relevant totality is, there is nothing obviously incoherent about a way for the world to be on which there are more angels than that.\footnote{I do not mean to suggest that there is no way of motivating a view according to which there is a fixed bound to the angels there could be, or even a principled such bound (See, for instance, Hawthorne & Uzquiano 2011. The point here is simply that such a view would result in a conception of metaphysical possibility on which seemingly coherent ways of the world to be fail to count as metaphysical possibilities.}

Another way to make the point is to focus on what I shall refer to as Fritz’s Puzzle (Fritz forthcoming). The puzzle is based on the observation that the following claims are mutually inconsistent, given standard modal principles:\footnote{As Jeremy Goodman pointed out to me, the necessitist can also respond to the puzzle by rejecting Modal Axiom B, which states that what is true is necessarily possible. I myself find it hard to wrap my mind around a conception of logical space that does not validate B. But

\[ \Box P \lor \Box \neg P \]

\[ \neg \Box P \lor \neg \Box \neg P \]

\[ \Box (P \lor \neg P) \]

\[ \neg \Box \neg (P \lor \neg P) \]
(P1) However many angels there might have been, there also might have been more.

(P2) It might have been that every possible angel is an angel.¹

Since (P1) and (P2) are mutually inconsistent, the necessitist must deny at least one. But whichever she chooses to deny, she will be left with a seemingly coherent way for the world to be that fails to be metaphysically possible. For however many angels there might have been, there is nothing obviously incoherent about a way for the world to be on which there are more angels than that. And assuming there is a definite totality of possible angels, as the necessitist believes, there is nothing obviously incoherent about a way for the world to be on which every possible angel in that totality is in fact an angel.

2 The Root of the Problem

We have considered two conceptions of metaphysical possibility: simplified Lewisianism and necessitism. Each is committed to non-trivial constraints on metaphysical possibility, and in each case the constraints can be used to characterize a way for the world to be that is seemingly coherent but cannot be counted as a genuine metaphysical possibility.

The root of the problem, I would like to suggest, is that in both cases the constraints impose fixed bounds on metaphysical possibility. Simplified Lewisianism imposes such bounds by demanding that every metaphysical possibility correspond to one of a fixed number of distributions of natural properties. Necessitism does so by demanding that every metaphysical possibility that concerns angels be about the individuals in a fixed totality of possible angels.

Why does imposing fixed bounds on metaphysical possibility lead to trouble? The answer I would like to defend is that logical space is open-ended: there is no such thing as a maximally specific space of coherent ways for the world to

¹As Fritz points out, one can state this claim precisely without appealing to mere possibilia if one is prepared to work within a language with suitably powerful modal operators. (For example, one might rely on the modal operators $\uparrow_i$ and $\downarrow_i$, which were first introduced in a postscript to (Fine & Prior 1977), and state the claim as “◊ $\uparrow_i \Box (\exists x (\text{Angel}(x)) \rightarrow \downarrow_i \exists y (y = x))$.” Here, however, I will continue to indulge in possible angel talk, in an effort to keep my exposition as straightforward as possible.

Goodman has an interesting research program that supplies independent reasons for rejecting it.
be. So a conception of metaphysical possibility that imposes fixed bounds on the metaphysical possibilities must thereby fail to count some coherent way for the world to be as a genuine metaphysical possibility.

This will need some unpacking. The first step is to say more about what I mean when I say that logical space is open-ended. Consider two coherent ways for the world to be:

$P_l$: There are lions.

$P_n$: There are no lions.

The set $\{P_l, P_n\}$ is an exhaustive and non-overlapping set of ways for the world to be. It is exhaustive because some member of the set will be realized however the world is; it is non-overlapping because no more than one member of the set will be realized however the world is. Whenever a set of coherent ways for the world to be is both exhaustive and non-overlapping, I shall say that it is a set of worlds.

Not every set of worlds is maximally specific. For a set of worlds to be maximally specific is for any member of the set to fully determine how the world is. The set $\{P_l, P_n\}$ is not maximally specific because settling the question of whether there are lions does not fully determine how the world is: for example, it does not settle the question of whether I am presently wearing an ascot.

With these definitions in place, I can say what I mean when I say that logical space is open-ended:

**Open-Endedness** There is no such thing as a maximally specific set of worlds.

In the next few sections I will offer a defense of Open-Endedness. I will then explain why I think it entails that a conception of metaphysical possibility that imposes fixed bounds on the metaphysical possibilities must thereby fail to count coherent ways for the world to be as genuine metaphysical possibilities.

### 3 Distinctions

My defense of Open-Endedness will rely on the notion of a *distinction*, so I would like to say a few words about how I will be thinking of distinctions.

Each side of a distinction corresponds to a coherent way for something to be. Consider, for example, the distinction between being red and not being red.
Being red is a coherent way for something to be; being not red is a coherent way for something to be.

A distinction can therefore be used to characterize an object. For instance, I can give a (partial) characterization of an apple by setting forth the distinction between being red and not being red, and saying which side of this distinction the apple falls on. (Equivalently: I can characterize the apple by saying that it is red, or by saying that it is not red.) If asked to characterize the apple further, I can set forth an additional distinction, and say which side of this second distinction the apple falls on.

A distinction without a difference is not a genuine distinction at all: it is a pseudo-distinction. Here is an example. There is no difference between being a woodchuck and being a groundhog: to be a woodchuck just is to be a groundhog. So any effort to distinguish between being a woodchuck and being a groundhog would misfire: it would fail to deliver in a genuine distinction. All one gets is the pseudo-distinction between being-a-woodchuck-but-not-a-groundhog and being-a-groundhog-but-not-a-woodchuck. Since neither side of this pseudo-distinction corresponds to a coherent way for something to be, it cannot be used to supply a genuine characterization of an object. One way to make this intuitive is to imagine that you’re told of an object that it is a woodchuck but not a groundhog. As long as you’re able to hold onto your (true) belief that there is no difference between being a woodchuck and being a groundhog, you will take yourself to have learned nothing at all about how the object is—though you may take yourself to have learned that your interlocutor is confused.

Not every respect in which things can differ is intrinsic. Suppose that $a$ and $b$, while numerically distinct, are perfect intrinsic duplicates of one another. Here is a respect in which they differ: one of them is identical to $a$, and the other is not. The distinction between being identical to $a$ and not is a non-intrinsic distinction.

It is natural to suppose that there is a close connection between distinctions and properties. In particular, it is natural to suppose that each distinction corresponds to two properties, one for each way of taking sides with respect to the distinction. For example, the distinction between being red and not corresponds to: (1) the property of being red, and (2) the property of not being red. I myself am attracted to this way of linking distinctions and properties, but I am also mindful of the fact that it carries an expository risk, since the notion of a property is associated with philosophical baggage that I would be reluctant to transfer onto the notion of a distinction. Here is one example of baggage: someone might have a sparse conception of properties, according to which there is
no such thing as the “disjunctive” property of being either my nose or the Eiffel Tower. But it would be a mistake to conclude from this that there is no such thing as the distinction between being-my-nose-or-the-Eiffel-Tower and not.

Here is another example of baggage: someone might have a conception of properties according to which one cannot talk about the property of redness without thereby making a non-trivial ontological demand on the way the world is. But it would be a mistake to conclude from this that talk of distinctions makes a non-trivial ontological demand on the way the world is. To introduce a distinction is to commit to thinking that the question of how an object is can be addressed (partially, perhaps) by taking sides on that distinction. For example, to introduce the distinction between being red and not is to commit to thinking that the question of how something is can be addressed by saying that it is red, and by saying that it is not red.

Because of these expository risks, I will be careful not to make property-talk an essential part of my arguments.

4 Possibilities

So far I have been focusing my attention on distinctions between coherent ways for objects to be. But one can also talk about distinctions between coherent ways for the world to be. We have seen that one can characterize an apple by setting forth the distinction between being red and not, and stating which side of the distinction the apple falls under. Similarly, one can characterize the world by setting forth the distinction between, e.g. there being lions and not, and stating which side of the distinction the world falls under.

Just like one might think of a property as a way for an object to be, so one might think of a possibility as a way for the world to be. Accordingly, each distinction between coherent ways for the world to be corresponds to exactly two possibilities: the possibility that the world fall on one side of the distinction and the possibility that the world fall on the other. Thus, the distinction between there being lions and not corresponds to: (1) the possibility that there be lions, and (2) the possibility that there not be lions. (For now I will remain neutral on the question of whether a every coherent way for the world to be—and therefore every possibility, in the present sense of “possibility”—should be counted as a metaphysical possibility.)

One can take a set of distinctions to “generate” a corresponding set of possibilities:
Generation If $D$ is a set of distinctions between ways for the world to be, the set of possibilities generated by $D$ is the set of coherent ways for the world to be that result from taking sides with respect to every distinction in $D$.

It is easy to verify that the set of possibilities generated by a set of distinctions between ways for the world to be will always be a set of worlds, in the sense of Section 2.\(^6\) (Here is an example. Suppose that $D$ consists of two distinctions: the distinction between there being lions and not, and the distinction between there being tigers and not. Then $D$ generates a set of four possible worlds: a world with lions but no tigers, a world with tigers but no lions, a world with both and a world with neither.)

We have seen that every set of distinctions between ways for the world to be generates a set of worlds. The converse is also true: every set of worlds can be used to characterize a set of distinctions between ways for the world to be that generates that set of worlds.\(^7\)

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\(^6\)Two important qualifications need to be in place. First, we must assume that each of the distinctions in our set is exhaustive; in other words: it is the distinction between something’s being the case, and not.

Second, we need to keep track of whether the distinctions in our set are independent of one another. For two distinctions to be independent of one another is for the following to be true: to take sides with respect to one of the distinctions is not to thereby take sides with respect to the other. Consider, for example, the distinction between there being lions and not, on the one hand, and the distinction between there being small lions and not, on the other. Although these are two different distinctions, they are not independent of one another, since one cannot fall on the there-being-small-lions side of the latter distinction without thereby falling on the there-being-lions side of the former. In using a set of distinctions to generate a set of worlds it is important to make sure not to include incoherent ways for the world to be that take sides with respect to dependent distinctions in conflicting ways.

With these qualifications in place, the proof is straightforward. We need to verify: (a) that at least one of the generated possibilities will be realized, however the world is; and (b) that at most one of the generated possibilities will be realized, however the world is. But (a) follows from the fact that each of our distinctions is exhaustive, and (b) follows from the fact that any two ways of taking sides with respect to every distinction in the set will differ on at least one distinction, and will therefore be incompatible with one another.

\(^7\)The technique described in Moss 2012 can be used to verify this claim. Assume, first, that there are $|2^\kappa|$ worlds in $W$, for some $\kappa$. Then there is a one-one mapping from our set of worlds onto the set of length-$\kappa$ sequences of zeroes and ones. So $D$ can be chosen to be the set of distinctions $\delta_\alpha$ for $\alpha < \kappa$, where:

\[
\delta_\alpha : \text{the distinction between the world's being such as to verify a world that is mapped onto a sequence with a one in the } \alpha \text{-th position, and not.}
\]
5 Building Distinctions

Give me a distinction, and I can use it to characterize another. Suppose, for example, that you give me

\[ \delta_l : \text{the distinction between there being lions, and not.} \]

Each side of this distinction can be used to characterize a further distinction; for example: the distinction between my desiring only that that side of the original distinction obtain, and not. In the case of \( \delta_l \), this yields:

\[ \delta_l^\uparrow : \text{the distinction between my desiring only that there be lions, and not.} \]
\[ \delta_l^\downarrow : \text{the distinction between my desiring only that there not be lions, and not.} \]

It is worth emphasizing there is no need to think of the introduction of \( \delta_l^\uparrow \) and \( \delta_l^\downarrow \) as imposing a non-trivial ontological demand on the world. As noted above, to introduce a distinction between ways for the world to be is to commit to thinking that the question of how the world is can be addressed by taking sides on that distinction. So, for instance, to introduce the distinction between my desiring only that there be lions, and not, is to commit to thinking that the question of how the world is can be addressed by saying that I desire only that there be lions, and that it can be addressed by saying that it’s not the case that I desire only that there be lions.

6 Open-Endedness

In Section 4 I pointed to a connection between distinctions and worlds: every set of distinctions generates a set of worlds, and every set of worlds can be used to characterize a set of distinctions that generates that set of worlds. In Section 5 I noted that there is a certain sense in which distinctions are open ended: if you give me a distinction, I can use it to characterize another. By bringing these two ideas together, we can show the following:

What if the cardinality of \( W \) is distinct from \( |2^\kappa| \) for every \( \kappa \)? Then for the smallest \( \kappa \) such that \( |W| < |2^\kappa| \) one can pick a world in \( W \) and use distinctions of the sort described in Section 5 to divide \( w \) into \( |2^\kappa| \) more specific worlds.
A set of worlds can always be used to characterize further possibilities: possibilities that are not entailed by any world in the set.\(^8\)

Here is one way of spelling out the argument:\(^9\)

Let \(W\) be a set of worlds. For each subset \(S\) of \(W\), we have the following distinction:

\[\delta_S : \text{the distinction between } S\text{'s being my favorite subset of } W, \text{ and not.}\]

Since \(\delta_S\) is a distinction between ways for the world to be, it can be used to characterize two coherent ways for the world to be, one for each way of taking sides with respect to the distinction. In particular, each \(\delta_S\) can be used to characterize the following way for the world to be:

\[\pi_S : S \text{ is my favorite subset of } W.\]

We know, however, that there are more subsets of \(W\) than worlds in \(W\). So there must be more ways for the world to be \(\pi_S\) than worlds in \(W\). But the \(\pi_S\) are incompatible with one another. So there must be some subset \(S^*\) of \(W\) such that \(\pi_{S^*}\) is not entailed by any world in \(W\).\(^{10}\)

We have seen that every set of possible worlds can be used to characterize further possibilities. An immediate consequence of this result is that Open-Endedness is true: there is no such thing as a \textit{maximally specific} set of worlds.

It is worth noting, moreover, that talk of possibilities is not essential to the present point. The basic phenomenon could have instead been characterized as

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\(^8\)Notation: a way for the world to be entails another if and only if for the world to be the former way is thereby for it to be the latter way.


\(^{10}\)As Gabriel Uzquiano pointed out to me, one can formulate a version of the argument that does not rely on cardinality considerations. Consider the set \(F\) of worlds \(w\) such that every set of worlds I prefer at \(w\) fails to contain \(w\). On pain of contradiction, \(F\) couldn’t be the only set of worlds I prefer.
A set of distinctions can always be used to characterize further distinctions: distinctions that are not reducible to the distinctions in the set. There is, of course, an important connection between Open-Endedness and the phenomenon of indefinite extensibility that has been discussed by Dummett and others. But there is also an important difference. Discussions of indefinite extensibility have been focused on possible worlds, while in this context we are interested in the distinction between different ways of taking sides with respect to the distinctions in the set. We can prove this claim by replicating our earlier proof in the new setting. Instead of possible worlds, we use “possible world distinctions”. More specifically, for each way $w$ of taking sides with respect to every distinction in $D$, we have the following possible world distinction:

$$\delta_w : \text{the distinction between the world's taking sides with respect to distinctions in } D \text{ in way } w, \text{ and not.}$$

Let us say that for the world to take sides with respect to $\delta_w$ in the “most specific way” is for it to take sides with respect to the distinctions in $D$ in way $w$, rather than not. (This definition is unambiguous whenever $D$ generates more than two possible worlds.) Instead of talking of sets of possible worlds, we use “possibility distinctions”. More specifically, for each set $S$ of possible world distinctions, we have the following possibility distinction:

$$\delta_S : \text{the distinction between the world's taking sides with respect to some } \delta_w \text{ in } S \text{ in the most specific way, and not.}$$

Each such $\delta_S$ can then be used to characterize a further distinction:

$$\delta^*_S : \text{the distinction between my hoping only that the world takes sides with respect to some } \delta_w \text{ in } S \text{ in the most specific way, and not.}$$

Let us now say that for the world to take sides with respect to $\delta^*_S$ “in the positive way” if for me to hope only that the world takes sides with respect to some $\delta_w$ in $S$ in the most specific way.

It is easy to verify that there is no way for the world to take sides with respect to more than one $\delta^*_S$ in the positive way. It is also easy to verify that a distinction $\delta$ is reducible to the distinctions in $D$ if and only if however one wishes to take sides with respect to $\delta$, one can do so by taking sides with respect to some possible world distinction. So we can prove our result by showing that there are more distinctions $\delta^*_S$ than there are possible world distinctions. But this follows immediately from the fact that there are more sets $S$ of possible world distinctions than there are possible world distinctions.

Notation: for distinction $\delta$ to be reducible to the distinctions in $D$ is for it to be the case that however one wishes to take sides with respect to $\delta$ one can do so by taking sides with respect to distinctions in $D$. Dummett’s discussion includes Dummett 1963, p. 195–6 and Dummett 1991, p. 316. More
extensibility are primarily concerned with ontological questions in the foundations of set-theory. Here we are instead concerned with our ability to give a full characterization of the way the world is.

7 Back to the Beginning

Tarski (1929) showed that first-order languages are open-ended, in the following sense. No interesting first-order language can contain its own truth predicate. So if you give me an interesting first-order language \( L \), I can always use it to build a more expressive one, by enriching \( L \) with a truth-predicate for \( L \). It follows from Open-Endedness that something similar is true of sets of worlds: if you give me a set of worlds \( W \), I can always use it to build a more specific one, by enriching \( W \) with possibilities that take sides on the question of which subsets of \( W \) I prefer.

Open-Endedness is a special case of a more general phenomenon: in a large range of cases, a definite totality of possibilities can be used to generate possibilities that lie outside that totality. I pointed to a couple of examples of the more general phenomenon in Section 1. We saw that Simplified Lewisianism postulates a definite totality of possibilities—those that correspond to distributions of intrinsic properties across canonical spacetime—and that this totality can be used to build further possibilities, by uniting different distributions in the totality into a single “superstructure”. We then saw that Necessitism postulates a definite totality of possibility angels—and therefore a definite totality of possibilities concerning the question of which angels exist—and that this totality can be used to build further possibilities, by considering a way for the world to be on which there are more angels than members of the totality.

The moral, it seems to me, is that one cannot hold onto both of the following ideas. On the one hand, the idea that any coherent way for the world to be should be counted as a metaphysical possibility. On the other, the idea that there is a maximally specific set of metaphysical possibilities—and, more generally, a definite totality of metaphysical possibilities.

My own view is that one hold onto the first idea by giving up on the second. One should think that a metaphysical possibility is just a coherent way for the world to be, and therefore that there there is no such thing as a definite totality of metaphysical possibilities. (The claim here is not that there is something

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“incomplete” or “open-ended” about the way the world is. On the contrary: there is a definite fact of the matter about how the world is. What we lack is the ability to identify a complete set of resources for characterizing the way the world is.)

One advantage of this view is that it allows for a satisfying resolution of Fritz’s Puzzle (Section 1). For once one gives up on the idea that there is a definite totality of metaphysical possibilities, it is natural to assume that there is no sense to be made of a definite totality of possible angels. Possible angels are open-ended: if you give me a definite totality of possible angels, I can use that totality to characterize even more, by considering, first, a possibility at which every possible angel in your totality is an angel, and, second, by considering a possibility at which there are more angels than that.

On this view, premise (P2) of the puzzle suffers from a false presupposition. For if there is no sense to be made of a definite totality consisting of every possible angel, there is no sense to be made of the claim that every member of that totality is an angel.

8 Models

I just suggested that one should take there to be no definite answer to the question of how many possible angels there are. In this section I would like to explain how one might give a model-theoretic treatment of a conception of metaphysical possibility based on this view.

We start with an analogy. Consider an unusually coarse-grained partition of logical space, on which there are only two worlds: \( w_e \) and \( \overline{w_e} \). At \( w_e \), there are elephants; at \( \overline{w_e} \), there are no elephants. \( \{w_e, \overline{w_e}\} \) counts as a set of possible worlds in the sense of Section 4: exactly one member of the set will be realized however the world is. But \( \{w_e, \overline{w_e}\} \) is not maximally specific. For example, it does not capture the distinction between there being exactly 17 elephants and not. How could one give a formal representation of a non-maximally-specific set of worlds like \( \{w_e, \overline{w_e}\} \)?

In the context of model theory, a possible world is sometimes represented as a pair \( \langle D, I \rangle \), where \( D \) is a domain of objects and \( I \) is a function that assigns an extension in \( D \) to each predicate of one’s object-language. Could such a pair be used to represent coarse-grained worlds like \( w_e \) and \( \overline{w_e} \)?

It can. Start with world \( w_e \), at which there are elephants. In order for a pair \( \langle D, I \rangle \) to represent \( w_e \), how many objects should \( I \) assign to the extension
of “Elephant”? Answer: at least one, but the precise number doesn’t matter. Here is why. When we use a pair $\langle D, I \rangle$ to represent a possible world, the representation should only be treated as accurate with respect to distinctions on which the possible world is intended to take sides.

Consider, for example, a pair $\langle D, I_{17} \rangle$ such that $I_{17}$ assigns 17 objects to the extension of “Elephant”. One might be tempted to think that $\langle D, I_{17} \rangle$ represents a world containing precisely 17 elephants. But this wouldn’t be right in a context in which $\langle D, I_{17} \rangle$ is used to represent $w_e$. For $w_e$ is only intended to take sides with respect to one distinction: the distinction between there being elephants and not. In this context, we should take $\langle D, I_{17} \rangle$ at face value when it comes to the question of whether there are elephants, but not when it comes to the question of whether there are exactly 17 elephants. Accordingly, we treat $\langle D, I_{17} \rangle$ as representing a world at which it is true that there are elephants, but is no more specific than that.

Moral: just because a world takes sides with respect to a limited set of questions, it doesn’t mean it can’t be represented using standard model-theoretic resources. We can use $\langle D, I_{17} \rangle$ to represent $w_e$, and a pair $\langle D, I_0 \rangle$ to represent $w_\bar{e}$, where $I_0$ assigns an empty extension to “Elephant”. The trick is to be careful not to take one’s model seriously when it comes to questions on which the relevant world is not intended to take sides.

When using model-theoretic resources in this way, it is important to make sure that one’s object-language is suitably restricted. More specifically: we need to make sure that any distinction that can be captured by sentences of the object-language is a distinction on which our worlds are intended to take sides. Otherwise, the resulting interpretation of the object-language will rely on features of the model-theory that are not supposed to be taken seriously.

Here is one way of satisfying this restriction in the context of $\{w_e, w_\bar{e}\}$. We let our object-language be a first-order language in which the non-logical vocabulary is limited to the predicate “Elephant”. In addition, we restrict the expressive power of the language in two ways. First, we insist that the language not contain an identity predicate; second we insist that every occurrence of a quantifier be restricted by the predicate “Elephant”. The resulting language contains sentences that can be used to capture the distinction between there being elephants, and there being none. But it contains no sentence that can be used to capture the distinction between there being precisely 17 elephants, and not.

With this as our background, let us turn to the question of how one might give a model-theoretic treatment of a conception of metaphysical possibility
according to which there is no definite answer to the question of how many possible angels there are.

In a sense, we have accomplished this goal already. We have seen that the set of pairs \( \{ \langle D, I_1 \rangle, \langle D, I_0 \rangle \} \) can be used to represent the set of worlds \( \{ w_e, w_\bar{e} \} \), which does not take a stand on the question of how many possible angels there are. But here we are hoping for something more ambitious: we want to model a conception of logical space that allows us to make interesting claims about possible angels without committing to a definite answer to the question of how many of them there are.

For concreteness, I will focus on a particular set of possible worlds, \( W^\kappa \), where \( \kappa \) is an infinite cardinal. I shall assume that \( W^\kappa \) satisfies the following two conditions:

1. The worlds in \( W^\kappa \) take a stand on whether each of \( \kappa \)-many possible angels exist.

2. The worlds in \( W^\kappa \) do not take a stand on any other questions. In particular, they do not take a stand on the question of how many possible angels there are.

Here is one way of representing \( W^\kappa \) model-theoretically. Let \( A^\kappa \) be an arbitrary set of size \( \kappa \), and model \( W^\kappa \) using the set of ordered pairs \( \langle D, I_D \rangle \) such that \( D \) is a subset of \( A^\kappa \) and the extension assigned to the predicate “Angel” by \( I_D \) is \( D \).

Think of each \( \langle D, I_D \rangle \) as representing a possible world that contains the possible angels represented by members of \( D \), but not the possible angels represented by members of \( A^\kappa - D \). Although \( D \) will never have more than \( \kappa \) elements, we do not think of \( \langle D, I_D \rangle \) as representing a world at which there are no more than \( \kappa \) angels. This is because a pair \( \langle D, I_D \rangle \) is only assumed to be accurate with respect to the distinctions on which the worlds we are representing take sides. And the worlds in \( W^\kappa \) are only intended to take sides with respect to the existence and non-existence of \( \kappa \)-many possible angels—the possible angels represented by members of \( A^\kappa \).

As before, our object language must be suitably restricted: we need to make sure that any distinction which can be captured by sentences of the object-language is a distinction on which our set of possible worlds takes sides. One way to secure this result is to use a first-order language with identity whose only non-logical vocabulary is the predicate “Angel”, and insist that every occurrence
of a quantifier be restricted by the predicate “Angel”. We could then enrich the language with a cardinality quantifier \( \exists_\alpha \) for each \( \alpha \leq \kappa \), where the sentence \( \exists_\alpha x \phi(x) \) is true if and only if there are at least \( \alpha \)-many objects satisfying \( \phi(x) \). For each cardinal \( \alpha \leq \kappa \), the resulting language is expressive enough to describe the possibility that there be at least \( \alpha \)-many angels. But for \( \gamma > \kappa \), the language is unable to express claims to the effect that there are at least \( \gamma \)-many angels. We can also enrich our language with a model operator “\( \Diamond \)”, which would put us in a position to describe further possibilities. For example, given any cardinalities \( \alpha \leq \beta \leq \kappa \), we can express the possibility that there be \( \beta \)-many angels and it be possible for \( \alpha \)-many of them to exist without \( \alpha \)-many of them.\(^{14}\)

9 Fritz’s Puzzle Revisited

In Section 7 I suggested that one can answer the Fritz’s Puzzle by claiming that there is no definite answer to the question of how many possible angels there are. It seems to me, however, that each of the Puzzle’s two premises contains an important grain of truth, and would like to end the paper by saying a few words about this.

Here is what I take to be true about (P2). If you give me a definite size, I can use that size to characterize a world according to which there are that many angels. The size can be a cardinality, in the set theoretic sense. But it needn’t be. For example, if you think there is a definite totality of set-theoretic cardinals, you can use it to characterize a world that contains as many angels as there are subtotalities of that totality.\(^{15}\)

Here is what I take to be true about (P1). If you give me a definite size, I can use it to characterize a bigger size. (For example, the number of subtotalities in a totality of the original size.) So however many angels exist at a world, I can use the size of that collection to characterize a bigger size, and use that size to characterize a world according to which there are that many angels.

These two ideas cannot coexist if there is a definite answer to the question of how many possible angels there are. But they can if there is no such answer.

\(^{14}\)Here is (a streamlined version of) a suitable sentence:

\[ \Diamond(\exists_\beta x (\text{Angel}(x)) \land \exists_\alpha x \exists_\alpha y (\text{Angel}(x) \land \text{Angel}(y) \land \Diamond(\exists z (\text{Angel}(z) \land z = x) \land \forall z (\text{Angel}(z) \rightarrow z \neq y))) \]

\(^{15}\)Here I assume that one does not accept the necessitation of the Urelement Set Axiom, according to which the non-sets form a set. See McGee 1997.
References


Quine, W. V. (1953), From a Logical Point of View, Harvard, Cambridge.


