Online appendix for Conditioning and Updating under Cumulative Prospect Theory

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Full proof of Theorem 1.

Sufficiency: We first prove that $f_* \in F_*$ is sufficient for the decomposition. First consider conditioning on $s \in A$. Let α be a function defined on $\{1, \ldots, n-k\}$ that orders the states in $A^{\complement}$ such that $f_*(\alpha(n-k)) \succsim \cdots \succsim f_*(\alpha(1))$. With the benchmark prospect $f_* \in F_*$, we have

$$\begin{aligned}
V(h_A f_*) &= V(h_{o(1)}, \ldots, h_{o(k)}, f_*(\alpha(1), \ldots, f_*(\alpha(n-k))) \\
&= \sum_{i=1}^{k} \left(w^-\big(\sum_{j=1}^{i} p_{o(j)}\big) - w^-\big(\sum_{j=1}^{i-1} p_{o(j)}\big) \right) v(h_{o(i)}) \\
&\quad + \sum_{i=1}^{n-k} \Big(w^+\big(\sum_{j=i}^{n-k} p_{\alpha(j)}\big) - w^+\big(\sum_{j=i+1}^{n-k} p_{\alpha(j)}\big) \Big) v(f_*(\alpha(i))).
\end{aligned}$$

and

$$\begin{aligned}
V(g_A f_*) &= V(g_{o(1)}, \ldots, g_{o(k)}, f_*(\alpha(1), \ldots, f_*(\alpha(n-k))) \\
&= \sum_{i=1}^{k} \left(w^-\big(\sum_{j=1}^{i} p_{o(j)}\big) - w^-\big(\sum_{j=1}^{i-1} p_{o(j)}\big) \right) v(g_{o(i)}) \\
&\quad + \sum_{i=1}^{n-k} \Big(w^+\big(\sum_{j=i}^{n-k} p_{\alpha(j)}\big) - w^+\big(\sum_{j=i+1}^{n-k} p_{\alpha(j)}\big) \Big) v(f_*(\alpha(i))).
\end{aligned}$$

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Thus,

$$
\begin{aligned}
& h_A f_* \succsim g_A f_* \\
\Leftrightarrow \quad & V(h_A f_*) \geq V(g_A f_*) \\
\Leftrightarrow \quad & \sum_{i=1}^{k} \left(w^-\big(\sum_{j=1}^{i} p_{o(j)}\big) - w^-\big(\sum_{j=1}^{i-1} p_{o(j)}\big) \right) v(h_{o(i)}) + \sum_{i=1}^{n-k} \Big(w^+\big(\sum_{j=i}^{n-k} p_{\alpha(j)}\big) - w^+\big(\sum_{j=i+1}^{n-k} p_{\alpha(j)}\big) \Big) v(f_*(\alpha(i))) \\
& \geq \sum_{i=1}^{k} \left(w^-\big(\sum_{j=1}^{i} p_{o(j)}\big) - w^-\big(\sum_{j=1}^{i-1} p_{o(j)}\big) \right) v(g_{o(i)}) + \sum_{i=1}^{n-k} \Big(w^+\big(\sum_{j=i}^{n-k} p_{\alpha(j)}\big) - w^+\big(\sum_{j=i+1}^{n-k} p_{\alpha(j)}\big) \Big) v(f_*(\alpha(i))) \\
\Leftrightarrow \quad & \sum_{i=1}^{k} \left(w^-\big(\sum_{j=1}^{i} p_{o(j)}\big) - w^-\big(\sum_{j=1}^{i-1} p_{o(j)}\big) \right) v(h_{o(i)}) \geq \sum_{i=1}^{k} \left(w^-\big(\sum_{j=1}^{i} p_{o(j)}\big) - w^-\big(\sum_{j=1}^{i-1} p_{o(j)}\big) \right) v(g_{o(i)}) \\
\Leftrightarrow \quad & \sum_{i=1}^{k} \frac{w^-\big(\sum_{j=1}^{i} p_{o(j)}\big) - w^-\big(\sum_{j=1}^{i-1} p_{o(j)}\big)}{w^-\big(\sum_{j=1}^{k} p_{o(j)}\big)} v(h_{o(i)}) \geq \sum_{i=1}^{k} \frac{w^-\big(\sum_{j=1}^{i} p_{o(j)}\big) - w^-\big(\sum_{j=1}^{i-1} p_{o(j)}\big)}{w^-\big(\sum_{j=1}^{k} p_{o(j)}\big)} v(g_{o(i)}),
\end{aligned}
$$

so preferences conditional on the information that $s \in A$ are represented by $E_L^e[h|s \in A]$.

Now consider conditioning on the complement $A^{\complement} = \{o(k+1), \ldots, o(n)\}$. We have

$$
\begin{aligned}
V(h_{A^{\complement}} f_*) &= V(x_*, \ldots, x_*, h_{o(k+1)}, \ldots, h_{o(n)}) \\
&= w^-\big(\sum_{j=1}^{k} p_{o(j)}\big) v(x_*) + \sum_{i=k+1}^{n} \left(w^-\big(\sum_{j=1}^{i} p_{o(j)}\big) - w^-\big(\sum_{j=1}^{i-1} p_{o(j)}\big) \right) v(h_{o(i)})
\end{aligned}
$$

and

$$
\begin{aligned}
V(g_{A^{\complement}} f_*) &= V(x_*, \ldots, x_*, g_{o(k+1)}, \ldots, g_{o(n)}) \\
&= w^-\big(\sum_{j=1}^{k} p_{o(j)}\big) v(x_*) + \sum_{i=k+1}^{n} \left(w^-\big(\sum_{j=1}^{i} p_{o(j)}\big) - w^-\big(\sum_{j=1}^{i-1} p_{o(j)}\big) \right) v(g_{o(i)}).
\end{aligned}
$$

Thus,

$$
\begin{aligned}
& h_{A^{\complement}} f_* \succsim g_{A^{\complement}} f_* \\
\Leftrightarrow \quad & V(h_{A^{\complement}} f_*) \geq V(g_{A^{\complement}} f_*) \\
\Leftrightarrow \quad & w^-\big(\sum_{j=1}^{k} p_{o(j)}\big) v(x_*) + \sum_{i=k+1}^{n} \left(w^-\big(\sum_{j=1}^{i} p_{o(j)}\big) - w^-\big(\sum_{j=1}^{i-1} p_{o(j)}\big) \right) v(h_{o(i)}) \\
& \geq w^-\big(\sum_{j=1}^{k} p_{o(j)}\big) v(x_*) + \sum_{i=k+1}^{n} \left(w^-\big(\sum_{j=1}^{i} p_{o(j)}\big) - w^-\big(\sum_{j=1}^{i-1} p_{o(j)}\big) \right) v(g_{o(i)}) \\
\Leftrightarrow \quad & \sum_{i=k+1}^{n} \left(w^-\big(\sum_{j=1}^{i} p_{o(j)}\big) - w^-\big(\sum_{j=1}^{i-1} p_{o(j)}\big) \right) v(h_{o(i)}) \geq \sum_{i=k+1}^{n} \left(w^-\big(\sum_{j=1}^{i} p_{o(j)}\big) - w^-\big(\sum_{j=1}^{i-1} p_{o(j)}\big) \right) v(g_{o(i)}) \\
\Leftrightarrow \quad & \sum_{i=k+1}^{n} \frac{w^-\big(\sum_{j=1}^{i} p_{o(j)}\big) - w^-\big(\sum_{j=1}^{i-1} p_{o(j)}\big)}{1 - w^-\big(\sum_{j=1}^{k} p_{o(j)}\big)} v(h_{o(i)}) \geq \sum_{i=k+1}^{n} \frac{w^-\big(\sum_{j=1}^{i} p_{o(j)}\big) - w^-\big(\sum_{j=1}^{i-1} p_{o(j)}\big)}{1 - w^-\big(\sum_{j=1}^{k} p_{o(j)}\big)} v(g_{o(i)}),
\end{aligned}
$$

so preferences conditional on the information that $s \in A^{\complement}$ are represented by $E_L^m[h|s \in A^{\complement}]$.

Since $p_A = \sum_{j=1}^k p_{o(j)}$, it now follows that in the loss domain

$$\begin{aligned}
& w^-(p_A)E_L^e[h|s \in A] + (1 - w^-(p_A))E_L^m[h|s \in A^{\complement}] \\
= \;& w^-(\sum_{j=1}^k p_{o(j)}) \sum_{i=1}^k \frac{w^-\big(\sum_{j=1}^i p_{o(j)}\big) - w^-\big(\sum_{j=1}^{i-1} p_{o(j)}\big)}{w^-\big(\sum_{j=1}^k p_{o(j)}\big)} v(h_{o(i)}) \\
& +(1 - w^-(\sum_{j=1}^k p_{o(j)})) \sum_{i=k+1}^n \frac{w^-\big(\sum_{j=1}^i p_{o(j)}\big) - w^-\big(\sum_{j=1}^{i-1} p_{o(j)}\big)}{1 - w^-\big(\sum_{j=1}^k p_{o(j)}\big)} v(h_{o(i)}) \\
= \;& \sum_{i=1}^n \left(w^-\big(\sum_{j=1}^i p_{o(j)}\big) - w^-\big(\sum_{j=1}^{i-1} p_{o(j)}\big) \right) v(h_{o(i)}),
\end{aligned}$$

which equals the unconditional CPT utility. Hence, we have the desired decomposition.

Necessity: We now turn to showing necessity. We proceed by showing that if $f \notin F_*$, then we can choose $w^-(\cdot)$ and $v(\cdot)$ such that conditional preferences are not represented as stated in part (a) of the theorem. That is, we can find functions $w^-(\cdot)$ and $v(\cdot)$ satisfying the stated conditions such that $g_{A^{\complement}}f \succ h_{A^{\complement}}f$ but $E_L^m[h|s \in A^{\complement}] \geq E_L^m[g|s \in A^{\complement}]$, or $g_A f \succ h_A f$ but $E_L^e[h|s \in A] \geq E_L^e[g|s \in A]$, for some acts $h, g \in \mathscr{H}_L$. A prospect $f \notin F_*$ if $f_s \succ x_*$ for some $s \in A$ or $0 \succ f_s$ for some $s \in A^{\complement}$.

We first show that if $f_s \succ x_*$ for some $s \in A$, then we can have $g_{A^{\complement}}f \succ h_{A^{\complement}}f$ but $E_L^m[h|s \in A^{\complement}] \geq E_L^m[g|s \in A^{\complement}]$. So suppose that there indeed exists $s \in A$ such that $f_s \succ x_*$. Since there are at least four outcomes in the loss domain and at least two states in $A^{\complement}$, we can choose prospects $h, g \in \mathscr{H}_L$ such that

$$0 \succsim h_{o(k+2)} \succ g_{o(k+2)} \succ g_{o(k+1)} \succ h_{o(k+1)} \succsim x_*$$

and $h_{o(i)} = g_{o(i)} = h_{o(k+2)}$ for all $i > k + 2$. Depending on f, there are different cases we need to consider.

The different cases arise because the details of the proof depend on how f relates to the outcomes in the loss domain (recall that all we require is that there are at least four outcomes in X_L). When there are more than four outcomes in X_L, we may simultaneously be able to use the approach of two different cases. The procedure is the same in each case: We calculate the utility of $h_{A^{\complement}}f$ and of $g_{A^{\complement}}f$ and derive conditions for $g_{A^{\complement}}f \succ h_{A^{\complement}}f$ and for $E_L^m[h|s \in A^{\complement}] \geq E_L^m[g|s \in A^{\complement}]$. We then show that there exist $w^-(\cdot)$ and $v(\cdot)$ such that both of these conditions are satisfied. Since such $w^-(\cdot)$ and $v(\cdot)$ exist, preferences are not represented as in statement (a) of Theorem 1.

To this end, we introduce the following notation: Let $\hat{A} = \{s \in A | f_s \succ x_*\}$ and let $\hat{A}_L = \{s \in \hat{A} | 0 \succ f_s\}$. Note that $\hat{A}$ is non-empty by the supposition above. Let $|\hat{A}|$ and $|\hat{A}_L|$ denote the number of states in $\hat{A}$ and $\hat{A}_L$, respectively, and let τ be a function defined on $\{1, \ldots, |\hat{A}|\}$ that orders the states in $\hat{A}$ such that $f_{\tau(|\hat{A}|)} \succsim \cdots \succsim f_{\tau(1)}$.

Suppose first that $f_{\tau(1)} \succsim 0$. Then

$$\begin{aligned} V[h_{A^\complement} f] &= w^-\Big(\sum_{\substack{j=1 \\ o(j)\notin \hat{A}}}^{k} p_{o(j)}\Big)v(x_*) + \sum_{i=k+1}^{n} \Big(w^-\Big(\sum_{\substack{j=1 \\ o(j)\notin \hat{A}}}^{i} p_{o(j)}\Big) - w^-\Big(\sum_{\substack{j=1 \\ o(j)\notin \hat{A}}}^{i-1} p_{o(j)}\Big)\Big) v(h_{o(i)}) \\ &\quad + \sum_{i=1}^{|\hat{A}|} \Big(w^+\Big(\sum_{j=i}^{|\hat{A}|} p_{\tau(j)}\Big) - w^+\Big(\sum_{j=i+1}^{|\hat{A}|} p_{\tau(j)}\Big)\Big) v(f_{\tau(i)}) \end{aligned}$$

and

$$\begin{aligned} V[g_{A^\complement} f] &= w^-\Big(\sum_{\substack{j=1 \\ o(j)\notin \hat{A}}}^{k} p_{o(j)}\Big)v(x_*) + \sum_{i=k+1}^{n} \Big(w^-\Big(\sum_{\substack{j=1 \\ o(j)\notin \hat{A}}}^{i} p_{o(j)}\Big) - w^-\Big(\sum_{\substack{j=1 \\ o(j)\notin \hat{A}}}^{i-1} p_{o(j)}\Big)\Big) v(g_{o(i)}) \\ &\quad + \sum_{i=1}^{|\hat{A}|} \Big(w^+\Big(\sum_{j=i}^{|\hat{A}|} p_{\tau(j)}\Big) - w^+\Big(\sum_{j=i+1}^{|\hat{A}|} p_{\tau(j)}\Big)\Big) v(f_{\tau(i)}). \end{aligned}$$

Since $h_{o(i)} = g_{o(i)} = h_{o(k+2)}$ for all $i > k+2$, we have

$$\begin{aligned} g_{A^\complement} f \succ h_{A^\complement} f &\Leftrightarrow V[g_{A^\complement} f] > V[h_{A^\complement} f] \\ &\Leftrightarrow \Big(w^-\Big(\sum_{\substack{j=1 \\ o(j)\notin \hat{A}}}^{k+1} p_{o(j)}\Big) - w^-\Big(\sum_{\substack{j=1 \\ o(j)\notin \hat{A}}}^{k} p_{o(j)}\Big)\Big)\big(v(h_{o(k+1)}) - v(g_{o(k+1)})\big) \\ &\qquad + \Big(w^-\Big(\sum_{\substack{j=1 \\ o(j)\notin \hat{A}}}^{k+2} p_{o(j)}\Big) - w^-\Big(\sum_{\substack{j=1 \\ o(j)\notin \hat{A}}}^{k+1} p_{o(j)}\Big)\Big)\big(v(h_{o(k+2)}) - v(g_{o(k+2)})\big) < 0. \qquad (8) \end{aligned}$$

Conditional preferences are not represented by $E_L^m[\cdot|s \in A^\complement]$ if we also have that $E_L^m[h|s \in A^\complement] \geq E_L^m[g|s \in A^\complement]$, which with $h_{o(i)} = g_{o(i)} = h_{o(k+2)}$ for all $i > k+2$ reduces to

$$\begin{aligned} &\Big(w^-\Big(\sum_{j=1}^{k+1} p_{o(j)}\Big) - w^-\Big(\sum_{j=1}^{k} p_{o(j)}\Big)\Big)\big(v(h_{o(k+1)}) - v(g_{o(k+1)})\big) \\ &+ \Big(w^-\Big(\sum_{j=1}^{k+2} p_{o(j)}\Big) - w^-\Big(\sum_{j=1}^{k+1} p_{o(j)}\Big)\Big)\big(v(h_{o(k+2)}) - v(g_{o(k+2)})\big) \geq 0. \qquad (9) \end{aligned}$$

Rearranging (8) and (9), conditional preferences are not represented by $E_L^m[\cdot|s \in A^\complement]$ if

$$\begin{aligned} \frac{w^-\big(\sum_{\substack{j=1 \\ o(j)\notin \hat{A}}}^{k+2} p_{o(j)}\big) - w^-\big(\sum_{\substack{j=1 \\ o(j)\notin \hat{A}}}^{k+1} p_{o(j)}\big)}{w^-\big(\sum_{\substack{j=1 \\ o(j)\notin \hat{A}}}^{k+1} p_{o(j)}\big) - w^-\big(\sum_{\substack{j=1 \\ o(j)\notin \hat{A}}}^{k} p_{o(j)}\big)} &< \frac{v(g_{o(k+1)}) - v(h_{o(k+1)})}{v(h_{o(k+2)}) - v(g_{o(k+2)})} \\ &\leq \frac{w^-\big(\sum_{j=1}^{k+2} p_{o(j)}\big) - w^-\big(\sum_{j=1}^{k+1} p_{o(j)}\big)}{w^-\big(\sum_{j=1}^{k+1} p_{o(j)}\big) - w^-\big(\sum_{j=1}^{k} p_{o(j)}\big)}. \qquad (10) \end{aligned}$$

The functions $w^-(\cdot)$ and $v(\cdot)$ can be chosen such that both inequalities in (10) are satisfied.

Therefore, preferences are not represented as in statement (a) of Theorem 1.

Suppose second that $0 \succ f_{\tau(1)}$. Define

$$\mu \equiv \begin{cases} \min\left\{j \in \{1,\ldots,|\hat{A}_L|\} \ : \ f_{\tau(j)} \succ f_{\tau(1)}\right\} & \text{if there exists } s \in \hat{A}_L \text{ such that } f_s \succ f_{\tau(1)} \\ |\hat{A}_L| + 1 & \text{if } f_s \sim f_{\tau(1)} \ \text{ for all } s \in \hat{A}_L. \end{cases}$$

Since there are at least four outcomes in the loss domain, there exists $\hat{x}$ such that $f_{\tau(1)} \succ \hat{x} \succ x_*$ and/or there exists x' such that $0 \succ x' \succ f_{\tau(1)}$. In the former case, since $0 \succ f_{\tau(1)}$, we can let $f_{\tau(\mu)} \succsim h_{o(k+2)} \succ g_{o(k+2)} = f_{\tau(1)} \succ g_{o(k+1)} \succ h_{o(k+1)} = x_*$. In the latter case, if it is also the case that $f_{\tau(\mu)} \succ \tilde{x} \succ f_{\tau(1)}$ for some $\tilde{x} \in X_L$, we can let $f_{\tau(\mu)} \succsim h_{o(k+2)} \succ g_{o(k+2)} \succ f_{\tau(1)} = g_{o(k+1)} \succ h_{o(k+1)} = x_*$. In either of these two cases,

$$\begin{aligned} V[h_{A^{\complement}} f] \quad = \quad & w^-\Big(\sum_{\substack{j=1 \\ o(j)\notin\hat{A}}}^{k} p_{o(j)}\Big) v(x_*) + \Big(w^-\Big(\sum_{\substack{j=1 \\ o(j)\notin\hat{A}}}^{k+1} p_{o(j)}\Big) - w^-\Big(\sum_{\substack{j=1 \\ o(j)\notin\hat{A}}}^{k} p_{o(j)}\Big)\Big) v(h_{o(k+1)}) \\ & + \sum_{i=1}^{\mu-1} \Big(w^-\Big(\sum_{\substack{j=1 \\ o(j)\notin\hat{A}}}^{k+1} p_{o(j)} + \sum_{j=1}^{i} p_{\tau(j)}\Big) - w^-\Big(\sum_{\substack{j=1 \\ o(j)\notin\hat{A}}}^{k+1} p_{o(j)} + \sum_{j=1}^{i-1} p_{\tau(j)}\Big)\Big) v(f_{\tau(i)}) \\ & + \sum_{i=k+2}^{n} \Big(w^-\Big(\sum_{\substack{j=1 \\ o(j)\notin\hat{A}}}^{i} p_{o(j)} + \sum_{j=1}^{\mu-1} p_{\tau(j)}\Big) - w^-\Big(\sum_{\substack{j=1 \\ o(j)\notin\hat{A}}}^{i-1} p_{o(j)} + \sum_{j=1}^{\mu-1} p_{\tau(j)}\Big)\Big) v(h_{o(i)}) \\ & + \sum_{i=\mu}^{|\hat{A}_L|} \Big(w^-\Big(\sum_{\substack{j=1 \\ o(j)\notin\hat{A}}}^{n} p_{o(j)} + \sum_{j=1}^{i} p_{\tau(j)}\Big) - w^-\Big(\sum_{\substack{j=1 \\ o(j)\notin\hat{A}}}^{n} p_{o(j)} + \sum_{j=1}^{i-1} p_{\tau(j)}\Big)\Big) v(f_{\tau(i)}) \\ & + \sum_{i=|\hat{A}_L|+1}^{|\hat{A}|} \Big(w^+\Big(\sum_{j=i}^{|\hat{A}|} p_{\tau(j)}\Big) - w^+\Big(\sum_{j=i+1}^{|\hat{A}|} p_{\tau(j)}\Big)\Big) v(f_{\tau(i)}) \end{aligned} \tag{11}$$

and

$$
\begin{aligned}
V[g_{A^{\complement}}f] \;=\;& w^{-}\Big(\sum_{\substack{j=1\\ o(j)\notin\hat{A}}}^{k} p_{o(j)}\Big)v(x_*) + \Big(w^{-}\Big(\sum_{\substack{j=1\\ o(j)\notin\hat{A}}}^{k+1} p_{o(j)}\Big) - w^{-}\Big(\sum_{\substack{j=1\\ o(j)\notin\hat{A}}}^{k} p_{o(j)}\Big)\Big)v(g_{o(k+1)})\\
&+\sum_{i=1}^{\mu-1}\Big(w^{-}\Big(\sum_{\substack{j=1\\ o(j)\notin\hat{A}}}^{k+1} p_{o(j)} + \sum_{j=1}^{i} p_{\tau(j)}\Big) - w^{-}\Big(\sum_{\substack{j=1\\ o(j)\notin\hat{A}}}^{k+1} p_{o(j)} + \sum_{j=1}^{i-1} p_{\tau(j)}\Big)\Big)v(f_{\tau(i)})\\
&+\sum_{i=k+2}^{n}\Big(w^{-}\Big(\sum_{\substack{j=1\\ o(j)\notin\hat{A}}}^{i} p_{o(j)} + \sum_{j=1}^{\mu-1} p_{\tau(j)}\Big) - w^{-}\Big(\sum_{\substack{j=1\\ o(j)\notin\hat{A}}}^{i-1} p_{o(j)} + \sum_{j=1}^{\mu-1} p_{\tau(j)}\Big)\Big)v(g_{o(i)})\\
&+\sum_{i=\mu}^{|\hat{A}_L|}\Big(w^{-}\Big(\sum_{\substack{j=1\\ o(j)\notin\hat{A}}}^{n} p_{o(j)} + \sum_{j=1}^{i} p_{\tau(j)}\Big) - w^{-}\Big(\sum_{\substack{j=1\\ o(j)\notin\hat{A}}}^{n} p_{o(j)} + \sum_{j=1}^{i-1} p_{\tau(j)}\Big)\Big)v(f_{\tau(i)})\\
&+\sum_{i=|\hat{A}_L|+1}^{|\hat{A}|}\Big(w^{+}\Big(\sum_{j=i}^{|\hat{A}|} p_{\tau(j)}\Big) - w^{+}\Big(\sum_{j=i+1}^{|\hat{A}|} p_{\tau(j)}\Big)\Big)v(f_{\tau(i)}). \qquad (12)
\end{aligned}
$$

Since $h_{o(i)} = g_{o(i)} = h_{o(k+2)}$ for all $i > k+2$, we have

$$
\begin{aligned}
& g_{A^{\complement}}f \succ h_{A^{\complement}}f\\
\Leftrightarrow\;& V[g_{A^{\complement}}f] > V[h_{A^{\complement}}f]\\
\Leftrightarrow\;& \Big(w^{-}\Big(\sum_{\substack{j=1\\ o(j)\notin\hat{A}}}^{k+2} p_{o(j)} + \sum_{j=1}^{\mu-1} p_{\tau(j)}\Big) - w^{-}\Big(\sum_{\substack{j=1\\ o(j)\notin\hat{A}}}^{k+1} p_{o(j)} + \sum_{j=1}^{\mu-1} p_{\tau(j)}\Big)\Big)\big(v(h_{o(k+2)}) - v(g_{o(k+2)})\big)\\
&+\Big(w^{-}\Big(\sum_{\substack{j=1\\ o(j)\notin\hat{A}}}^{k+1} p_{o(j)}\Big) - w^{-}\Big(\sum_{\substack{j=1\\ o(j)\notin\hat{A}}}^{k} p_{o(j)}\Big)\Big)\big(v(h_{o(k+1)}) - v(g_{o(k+1)})\big) < 0. \qquad (13)
\end{aligned}
$$

When $g_{A^{\complement}}f \succ h_{A^{\complement}}f$, conditional preferences are not represented by $E_L^m[\cdot|s \in A^{\complement}]$ if (9) also holds. Hence, rearranging (9) and (13), conditional preferences are not represented by $E_L^m[\cdot|s \in A^{\complement}]$ if

$$
\begin{aligned}
&\frac{w^{-}\big(\sum_{\substack{j=1\\ o(j)\notin\hat{A}}}^{k+2} p_{o(j)} + \sum_{j=1}^{\mu-1} p_{\tau(j)}\big) - w^{-}\big(\sum_{\substack{j=1\\ o(j)\notin\hat{A}}}^{k+1} p_{o(j)} + \sum_{j=1}^{\mu-1} p_{\tau(j)}\big)}{w^{-}\big(\sum_{\substack{j=1\\ o(j)\notin\hat{A}}}^{k+1} p_{o(j)}\big) - w^{-}\big(\sum_{\substack{j=1\\ o(j)\notin\hat{A}}}^{k} p_{o(j)}\big)}\\
&< \frac{v(g_{o(k+1)}) - v(h_{o(k+1)})}{v(h_{o(k+2)}) - v(g_{o(k+2)})} \leq \frac{w^{-}\big(\sum_{j=1}^{k+2} p_{o(j)}\big) - w^{-}\big(\sum_{j=1}^{k+1} p_{o(j)}\big)}{w^{-}\big(\sum_{j=1}^{k+1} p_{o(j)}\big) - w^{-}\big(\sum_{j=1}^{k} p_{o(j)}\big)}. \qquad (14)
\end{aligned}
$$

Again, the functions $w^{-}(\cdot)$ and $v(\cdot)$ can be chosen such that both inequalities in (14) are satisfied. Therefore, preferences are not represented as in statement (a) of Theorem 1.

If there does not exist $\hat{x} \in X_L$ such that $f_{\tau(1)} \succ \hat{x} \succ x_*$ and there does not exist $\tilde{x} \in X_L$ such that $f_{\tau(\mu)} \succ \tilde{x} \succ f_{\tau(1)}$, we can let $h_{o(k+2)} \succ g_{o(k+2)} = f_{\tau(\mu)} \succ f_{\tau(1)} = g_{o(k+1)} \succ h_{o(k+1)} = x_*$, which is possible since there are at least four outcomes in the loss domain. Define

$$\xi \equiv \begin{cases} \min\left\{j \in \{1,\ldots,|\hat{A}_L|\} \ : \ f_{\tau(j)} \succ f_{\tau(\mu)}\right\} & \text{if there exists } s \in \hat{A}_L \text{ such that } f_s \succ f_{\tau(\mu)} \\ |\hat{A}_L| + 1 & \text{if } f_{\tau(\mu)} \succsim f_s \ \text{ for all } s \in \hat{A}_L. \end{cases}$$

Then,

$$\begin{aligned}
V[h_{A^\complement} f] \;=\;& w^-\Big(\sum_{\substack{j=1\\ o(j)\notin\hat{A}}}^{k} p_{o(j)}\Big)v(x_*) + \Big(w^-\Big(\sum_{\substack{j=1\\ o(j)\notin\hat{A}}}^{k+1} p_{o(j)}\Big) - w^-\Big(\sum_{\substack{j=1\\ o(j)\notin\hat{A}}}^{k} p_{o(j)}\Big)\Big)v(h_{o(k+1)}) \\
&+ \sum_{i=1}^{\xi-1}\Big(w^-\Big(\sum_{\substack{j=1\\ o(j)\notin\hat{A}}}^{k+1} p_{o(j)} + \sum_{j=1}^{i} p_{\tau(j)}\Big) - w^-\Big(\sum_{\substack{j=1\\ o(j)\notin\hat{A}}}^{k+1} p_{o(j)} + \sum_{j=1}^{i-1} p_{\tau(j)}\Big)\Big)v(f_{\tau(i)}) \\
&+ \sum_{i=k+2}^{n}\Big(w^-\Big(\sum_{\substack{j=1\\ o(j)\notin\hat{A}}}^{i} p_{o(j)} + \sum_{j=1}^{\xi-1} p_{\tau(j)}\Big) - w^-\Big(\sum_{\substack{j=1\\ o(j)\notin\hat{A}}}^{i-1} p_{o(j)} + \sum_{j=1}^{\xi-1} p_{\tau(j)}\Big)\Big)v(h_{o(i)}) \\
&+ \sum_{i=\xi}^{|\hat{A}_L|}\Big(w^-\Big(\sum_{\substack{j=1\\ o(j)\notin\hat{A}}}^{n} p_{o(j)} + \sum_{j=1}^{i} p_{\tau(j)}\Big) - w^-\Big(\sum_{\substack{j=1\\ o(j)\notin\hat{A}}}^{n} p_{o(j)} + \sum_{j=1}^{i-1} p_{\tau(j)}\Big)\Big)v(f_{\tau(i)}) \\
&+ \sum_{i=|\hat{A}_L|+1}^{|\hat{A}|}\Big(w^+\Big(\sum_{j=i}^{|\hat{A}|} p_{\tau(j)}\Big) - w^+\Big(\sum_{j=i+1}^{|\hat{A}|} p_{\tau(j)}\Big)\Big)v(f_{\tau(i)})
\end{aligned}$$

and

$$\begin{aligned}
V[g_{A^\complement} f] \;=\;& w^-\Big(\sum_{\substack{j=1\\ o(j)\notin\hat{A}}}^{k} p_{o(j)}\Big)v(x_*) + \Big(w^-\Big(\sum_{\substack{j=1\\ o(j)\notin\hat{A}}}^{k+1} p_{o(j)}\Big) - w^-\Big(\sum_{\substack{j=1\\ o(j)\notin\hat{A}}}^{k} p_{o(j)}\Big)\Big)v(g_{o(k+1)}) \\
&+ \sum_{i=1}^{\xi-1}\Big(w^-\Big(\sum_{\substack{j=1\\ o(j)\notin\hat{A}}}^{k+1} p_{o(j)} + \sum_{j=1}^{i} p_{\tau(j)}\Big) - w^-\Big(\sum_{\substack{j=1\\ o(j)\notin\hat{A}}}^{k+1} p_{o(j)} + \sum_{j=1}^{i-1} p_{\tau(j)}\Big)\Big)v(f_{\tau(i)}) \\
&+ \sum_{i=k+2}^{n}\Big(w^-\Big(\sum_{\substack{j=1\\ o(j)\notin\hat{A}}}^{i} p_{o(j)} + \sum_{j=1}^{\xi-1} p_{\tau(j)}\Big) - w^-\Big(\sum_{\substack{j=1\\ o(j)\notin\hat{A}}}^{i-1} p_{o(j)} + \sum_{j=1}^{\xi-1} p_{\tau(j)}\Big)\Big)v(g_{o(i)}) \\
&+ \sum_{i=\xi}^{|\hat{A}_L|}\Big(w^-\Big(\sum_{\substack{j=1\\ o(j)\notin\hat{A}}}^{n} p_{o(j)} + \sum_{j=1}^{i} p_{\tau(j)}\Big) - w^-\Big(\sum_{\substack{j=1\\ o(j)\notin\hat{A}}}^{n} p_{o(j)} + \sum_{j=1}^{i-1} p_{\tau(j)}\Big)\Big)v(f_{\tau(i)}) \\
&+ \sum_{i=|\hat{A}_L|+1}^{|\hat{A}|}\Big(w^+\Big(\sum_{j=i}^{|\hat{A}|} p_{\tau(j)}\Big) - w^+\Big(\sum_{j=i+1}^{|\hat{A}|} p_{\tau(j)}\Big)\Big)v(f_{\tau(i)}).
\end{aligned}$$

Since $h_{o(i)} = g_{o(i)} = h_{o(k+2)}$ for all $i > k+2$, we have

$$
\begin{aligned}
& V[g_{A^{\complement}} f] > V[h_{A^{\complement}} f] \\
\Leftrightarrow\quad & \Big(w^-\big(\sum_{\substack{j=1\\ o(j)\notin\hat{A}}}^{k+2} p_{o(j)} + \sum_{j=1}^{\xi-1} p_{\tau(j)}\big) - w^-\big(\sum_{\substack{j=1\\ o(j)\notin\hat{A}}}^{k+1} p_{o(j)} + \sum_{j=1}^{\xi-1} p_{\tau(j)}\big)\Big)\big(v(h_{o(k+2)}) - v(g_{o(k+2)})\big) \\
& + \Big(w^-\big(\sum_{\substack{j=1\\ o(j)\notin\hat{A}}}^{k+1} p_{o(j)}\big) - w^-\big(\sum_{\substack{j=1\\ o(j)\notin\hat{A}}}^{k} p_{o(j)}\big)\Big)\big(v(h_{o(k+1)}) - v(g_{o(k+1)})\big) < 0. \qquad (15)
\end{aligned}
$$

Combining (15) with (9), conditional preferences are not represented by $E_L^m[\cdot|s \in A^{\complement}]$ if

$$
\begin{aligned}
& \frac{w^-\big(\sum_{\substack{j=1\\ o(j)\notin\hat{A}}}^{k+2} p_{o(j)} + \sum_{j=1}^{\xi-1} p_{\tau(j)}\big) - w^-\big(\sum_{\substack{j=1\\ o(j)\notin\hat{A}}}^{k+1} p_{o(j)} + \sum_{j=1}^{\xi-1} p_{\tau(j)}\big)}{w^-\big(\sum_{\substack{j=1\\ o(j)\notin\hat{A}}}^{k+1} p_{o(j)}\big) - w^-\big(\sum_{\substack{j=1\\ o(j)\notin\hat{A}}}^{k} p_{o(j)}\big)} \\
& < \frac{v(g_{o(k+1)}) - v(h_{o(k+1)})}{v(h_{o(k+2)}) - v(g_{o(k+2)})} \le \frac{w^-\big(\sum_{j=1}^{k+2} p_{o(j)}\big) - w^-\big(\sum_{j=1}^{k+1} p_{o(j)}\big)}{w^-\big(\sum_{j=1}^{k+1} p_{o(j)}\big) - w^-\big(\sum_{j=1}^{k} p_{o(j)}\big)}. \qquad (16)
\end{aligned}
$$

The functions $w^-(\cdot)$ and $v(\cdot)$ can be chosen such that both inequalities in (16) are satisfied. Therefore, preferences are not represented as in statement (a) of Theorem 1. The cases considered exhaust all possibilities for $f_s \succ x_*$ for some $s \in A$.

The second part of the necessity proof is to show that if $0 \succ f_s$ for some $s \in A^{\complement}$, then we can have $g_A f \succ h_A f$ but $E_L^e[h|s \in A] \ge E_L^e[g|s \in A]$. Since there are at least four outcomes in the loss domain, we can choose $h, g \in \mathscr{H}_L$ such that

$$0 \succsim h_{o(k)} \succ g_{o(k)} \succ g_{o(k-1)} \succ h_{o(k-1)} \succsim x_*$$

and $h_{o(i)} = g_{o(i)} = h_{o(k-1)}$ for all $i < k-1$. Again, depending on how f relates to the outcomes in the loss domain, there are different cases we need to consider.

The procedure is essentially the same as above: In each case, we calculate the utility of $h_A f$ and $g_A f$ and derive the conditions for $g_A f \succ h_A f$ and $E_L^e[h|s \in A] \ge E_L^e[g|s \in A]$. We then show existence of a $w^-(\cdot)$ and $v(\cdot)$ such that both conditions are satisfied.

Let $\tilde{A} = \{s \in A^{\complement} | 0 \succ f_s\}$ and $|\tilde{A}|$ denote the number of states in $\tilde{A}$. Let β be a function defined on $\{1, \dots, |\tilde{A}|\}$ that orders the states in $\tilde{A}$ such that $f_{\beta(|\tilde{A}|)} \succsim \cdots \succsim f_{\beta(1)}$. Define

$$\eta \equiv \begin{cases} \min\{j \in \{1,\dots,|\tilde{A}|\} \ : \ f_{\beta(j)} \succ f_{\beta(1)}\} & \text{if there exists } s \in \tilde{A} \text{ such that } f_s \succ f_{\beta(1)} \\ |\tilde{A}| + 1 & \text{if } f_s \sim f_{\beta(1)} \ \text{ for all } s \in \tilde{A}, \end{cases}$$

$$\rho \equiv \begin{cases} \min\{j \in \{1,\dots,|\tilde{A}|\} \ : \ f_{\beta(j)} \succ f_{\beta(\eta)}\} & \text{if there exists } s \in \tilde{A} \text{ such that } f_s \succ f_{\beta(\eta)} \\ |\tilde{A}| + 1 & \text{if } f_{\beta(\eta)} \succsim f_s \ \text{ for all } s \in \tilde{A}, \end{cases}$$

and

$$\sigma \equiv \begin{cases} \min\{j \in \{1,\dots,|\tilde{A}|\} \ : \ f_{\beta(j)} \succ f_{\beta(\rho)}\} & \text{if there exists } s \in \tilde{A} \text{ such that } f_s \succ f_{\beta(\rho)} \\ |\tilde{A}| + 1 & \text{if } f_{\beta(\rho)} \succsim f_s \ \text{ for all } s \in \tilde{A}. \end{cases}$$

Suppose first that $f_{\beta(1)} = x_*$. If there exist $\bar{z}, \check{z} \in X_L$ such that $f_{\beta(\eta)} \succ \check{z} \succ \bar{z} \succ x_*$, we can let $f_{\beta(\eta)} \succsim h_{o(k)} \succ g_{o(k)} \succ g_{o(k-1)} \succ h_{o(k-1)} = x_*$. Then,

$$\begin{aligned} V[h_A f] \;&=\; w^-\big(\sum_{j=1}^{\eta-1} p_{\beta(j)}\big)v(f_{\beta(1)}) + \sum_{i=1}^{k}\Big(w^-\big(\sum_{j=1}^{i} p_{o(j)} + \sum_{j=1}^{\eta-1} p_{\beta(j)}\big) - w^-\big(\sum_{j=1}^{i-1} p_{o(j)} + \sum_{j=1}^{\eta-1} p_{\beta(j)}\big)\Big)v(h_{o(i)}) \\ &\quad + \sum_{i=\eta}^{|\tilde{A}|}\Big(w^-\big(\sum_{j=1}^{k} p_{o(j)} + \sum_{j=1}^{i} p_{\beta(j)}\big) - w^-\big(\sum_{j=1}^{k} p_{o(j)} + \sum_{j=1}^{i-1} p_{\beta(j)}\big)\Big)v(f_{\beta(i)}) \\ &\quad + \sum_{\substack{i=k+1 \\ o(i)\notin\tilde{A}}}^{n}\Big(w^+\big(\sum_{\substack{j=i \\ o(j)\notin\tilde{A}}}^{n} p_{\beta(j)}\big) - w^+\big(\sum_{\substack{j=i+1 \\ o(j)\notin\tilde{A}}}^{k} p_{\beta(j)}\big)\Big)v(f_{\beta(i)}) \end{aligned}$$

and

$$\begin{aligned} V[g_A f] \;&=\; w^-\big(\sum_{j=1}^{\eta-1} p_{\beta(j)}\big)v(f_{\beta(1)}) + \sum_{i=1}^{k}\Big(w^-\big(\sum_{j=1}^{i} p_{o(j)} + \sum_{j=1}^{\eta-1} p_{\beta(j)}\big) - w^-\big(\sum_{j=1}^{i-1} p_{o(j)} + \sum_{j=1}^{\eta-1} p_{\beta(j)}\big)\Big)v(g_{o(i)}) \\ &\quad + \sum_{i=\eta}^{|\tilde{A}|}\Big(w^-\big(\sum_{j=1}^{k} p_{o(j)} + \sum_{j=1}^{i} p_{\beta(j)}\big) - w^-\big(\sum_{j=1}^{k} p_{o(j)} + \sum_{j=1}^{i-1} p_{\beta(j)}\big)\Big)v(f_{\beta(i)}) \\ &\quad + \sum_{\substack{i=k+1 \\ o(i)\notin\tilde{A}}}^{n}\Big(w^+\big(\sum_{\substack{j=i \\ o(j)\notin\tilde{A}}}^{n} p_{\beta(j)}\big) - w^+\big(\sum_{\substack{j=i+1 \\ o(j)\notin\tilde{A}}}^{k} p_{\beta(j)}\big)\Big)v(f_{\beta(i)}). \end{aligned}$$

Thus, since $h_{o(i)} = g_{o(i)} = h_{o(k-1)}$ for all $i < k-1$, we have

$$\begin{aligned} & V[g_A f] > V[h_A f] \\ \Leftrightarrow\;\; & \Big(w^-\big(\sum_{j=1}^{k-1} p_{o(j)} + \sum_{j=1}^{\eta-1} p_{\beta(j)}\big) - w^-\big(\sum_{j=1}^{k-2} p_{o(j)} + \sum_{j=1}^{\eta-1} p_{\beta(j)}\big)\Big)\big(v(h_{o(k-1)}) - v(g_{o(k-1)})\big) \\ & +\Big(w^-\big(\sum_{j=1}^{k} p_{o(j)} + \sum_{j=1}^{\eta-1} p_{\beta(j)}\big) - w^-\big(\sum_{j=1}^{k-1} p_{o(j)} + \sum_{j=1}^{\eta-1} p_{\beta(j)}\big)\Big)\big(v(h_{o(k)}) - v(g_{o(k)})\big) < 0. \end{aligned}$$

These preferences are not represented by $E^e_L[\cdot|s \in A]$ if $E^e_L[h|s \in A] \geq E^e_L[g|s \in A]$, which with $h_{o(i)} = g_{o(i)} = h_{o(k-1)}$ for all $i < k-1$ reduces to

$$\begin{aligned} & \Big(w^-\big(\sum_{j=1}^{k-1} p_{o(j)}\big) - w^-\big(\sum_{j=1}^{k-2} p_{o(j)}\big)\Big)\big(v(h_{o(k-1)}) - v(g_{o(k-1)})\big) \\ & +\Big(w^-\big(\sum_{j=1}^{k} p_{o(j)}\big) - w^-\big(\sum_{j=1}^{k-1} p_{o(j)}\big)\Big)\big(v(h_{o(k)}) - v(g_{o(k)})\big) \geq 0. \end{aligned} \tag{17}$$

We therefore have that conditional preferences are not represented by $E_L^e[\cdot|s \in A]$ if

$$\frac{w^-\big(\sum_{j=1}^{k} p_{o(j)} + \sum_{j=1}^{\eta-1} p_{\beta(j)}\big) - w^-\big(\sum_{j=1}^{k-1} p_{o(j)} + \sum_{j=1}^{\eta-1} p_{\beta(j)}\big)}{w^-\big(\sum_{j=1}^{k-1} p_{o(j)} + \sum_{j=1}^{\eta-1} p_{\beta(j)}\big) - w^-\big(\sum_{j=1}^{k-2} p_{o(j)} + \sum_{j=1}^{\eta-1} p_{\beta(j)}\big)}$$
$$< \frac{v(g_{o(k-1)}) - v(h_{o(k-1)})}{v(h_{o(k)}) - v(g_{o(k)})} \leq \frac{w^-\big(\sum_{j=1}^{k} p_{o(j)}\big) - w^-\big(\sum_{j=1}^{k-1} p_{o(j)}\big)}{w^-\big(\sum_{j=1}^{k-1} p_{o(j)}\big) - w^-\big(\sum_{j=1}^{k-2} p_{o(j)}\big)}. \tag{18}$$

As above, the functions $w^-(\cdot)$ and $v(\cdot)$ can be chosen such that both inequalities in (18) are satisfied. Therefore, preferences are not represented as in statement (a) of the theorem.

If there exist $\bar{z}, \hat{z} \in X_L$ such that $\hat{z} \succ f_{\beta(\eta)} \succ \bar{z} \succ x_*$, we can let $h_{o(k)} = \hat{z}$, $g_{o(k)} = f_{\beta(\eta)}$, $g_{o(k-1)} = \bar{z}$, and $h_{o(k-1)} = x_*$. If there exist $\hat{z}, \grave{z} \in X_L$ such that $f_{\beta(\rho)} \succsim \hat{z} \succ \grave{z} \succ f_{\beta(\eta)}$, we can let $h_{o(k)} = \hat{z}$, $g_{o(k)} = \grave{z}$, $g_{o(k-1)} = f_{\beta(\eta)}$, and $h_{o(k-1)} = x_*$. In either of these cases,

$$\begin{aligned}
V[h_A f] \;=\;& w^-\big(\sum_{j=1}^{\eta-1} p_{\beta(j)}\big)v(f_{\beta(1)}) + \sum_{i=1}^{k-1}\Big(w^-\big(\sum_{j=1}^{i} p_{o(j)} + \sum_{j=1}^{\eta-1} p_{\beta(j)}\big) - w^-\big(\sum_{j=1}^{i-1} p_{o(j)} + \sum_{j=1}^{\eta-1} p_{\beta(j)}\big)\Big)v(h_{o(i)}) \\
&+\Big(w^-\big(\sum_{j=1}^{k-1} p_{o(j)} + \sum_{j=1}^{\rho-1} p_{\beta(j)}\big) - w^-\big(\sum_{j=1}^{k-1} p_{o(j)} + \sum_{j=1}^{\eta-1} p_{\beta(j)}\big)\Big)v(f_{\beta(\eta)}) \\
&+\Big(w^-\big(\sum_{j=1}^{k} p_{o(j)} + \sum_{j=1}^{\rho-1} p_{\beta(j)}\big) - w^-\big(\sum_{j=1}^{k-1} p_{o(j)} + \sum_{j=1}^{\rho-1} p_{\beta(j)}\big)\Big)v(h_{o(k)}) \\
&+\sum_{i=\rho}^{|\tilde{A}|}\Big(w^-\big(\sum_{j=1}^{k} p_{o(j)} + \sum_{j=1}^{i} p_{\beta(j)}\big) - w^-\big(\sum_{j=1}^{k} p_{o(j)} + \sum_{j=1}^{i-1} p_{\beta(j)}\big)\Big)v(f_{\beta(i)}) \\
&+\sum_{\substack{i=k+1\\ o(i)\notin\tilde{A}}}^{n}\Big(w^+\big(\sum_{\substack{j=i\\ o(j)\notin\tilde{A}}}^{n} p_{\beta(j)}\big) - w^+\big(\sum_{\substack{j=i+1\\ o(j)\notin\tilde{A}}}^{k} p_{\beta(j)}\big)\Big)v(f_{\beta(i)})
\end{aligned} \tag{19}$$

and

$$\begin{aligned}
V[g_A f] \;=\;& w^-\big(\sum_{j=1}^{\eta-1} p_{\beta(j)}\big)v(f_{\beta(1)}) + \sum_{i=1}^{k-1}\Big(w^-\big(\sum_{j=1}^{i} p_{o(j)} + \sum_{j=1}^{\eta-1} p_{\beta(j)}\big) - w^-\big(\sum_{j=1}^{i-1} p_{o(j)} + \sum_{j=1}^{\eta-1} p_{\beta(j)}\big)\Big)v(g_{o(i)}) \\
&+\Big(w^-\big(\sum_{j=1}^{k-1} p_{o(j)} + \sum_{j=1}^{\rho-1} p_{\beta(j)}\big) - w^-\big(\sum_{j=1}^{k-1} p_{o(j)} + \sum_{j=1}^{\eta-1} p_{\beta(j)}\big)\Big)v(f_{\beta(\eta)}) \\
&+\Big(w^-\big(\sum_{j=1}^{k} p_{o(j)} + \sum_{j=1}^{\rho-1} p_{\beta(j)}\big) - w^-\big(\sum_{j=1}^{k-1} p_{o(j)} + \sum_{j=1}^{\rho-1} p_{\beta(j)}\big)\Big)v(g_{o(k)}) \\
&+\sum_{i=\eta}^{|\tilde{A}|}\Big(w^-\big(\sum_{j=1}^{k} p_{o(j)} + \sum_{j=1}^{i} p_{\beta(j)}\big) - w^-\big(\sum_{j=1}^{k} p_{o(j)} + \sum_{j=1}^{i-1} p_{\beta(j)}\big)\Big)v(f_{\beta(i)}) \\
&+\sum_{\substack{i=k+1\\ o(i)\notin\tilde{A}}}^{n}\Big(w^+\big(\sum_{\substack{j=i\\ o(j)\notin\tilde{A}}}^{n} p_{\beta(j)}\big) - w^+\big(\sum_{\substack{j=i+1\\ o(j)\notin\tilde{A}}}^{k} p_{\beta(j)}\big)\Big)v(f_{\beta(i)}).
\end{aligned} \tag{20}$$

Therefore, since $h_{o(i)} = g_{o(i)} = h_{o(k-1)}$ for all $i < k-1$, we have

$$
\begin{aligned}
& V[g_A f] > V[h_A f] \\
\Leftrightarrow \quad & \Big(w^-\big(\sum_{j=1}^{k-1} p_{o(j)} + \sum_{j=1}^{\eta-1} p_{\beta(j)}\big) - w^-\big(\sum_{j=1}^{k-2} p_{o(j)} + \sum_{j=1}^{\eta-1} p_{\beta(j)}\big)\Big)\big(v(h_{o(k-1)}) - v(g_{o(k-1)})\big) \\
& + \Big(w^-\big(\sum_{j=1}^{k} p_{o(j)} + \sum_{j=1}^{\rho-1} p_{\beta(j)}\big) - w^-\big(\sum_{j=1}^{k-1} p_{o(j)} + \sum_{j=1}^{\rho-1} p_{\beta(j)}\big)\Big)\big(v(h_{o(k)}) - v(g_{o(k)})\big) < 0.
\end{aligned}
$$

Together with (17) we have that conditional preferences are not represented by $E_L^e[\cdot|s \in A]$ if

$$
\begin{aligned}
& \frac{w^-\big(\sum_{j=1}^{k} p_{o(j)} + \sum_{j=1}^{\rho-1} p_{\beta(j)}\big) - w^-\big(\sum_{j=1}^{k-1} p_{o(j)} + \sum_{j=1}^{\rho-1} p_{\beta(j)}\big)}{w^-\big(\sum_{j=1}^{k-1} p_{o(j)} + \sum_{j=1}^{\eta-1} p_{\beta(j)}\big) - w^-\big(\sum_{j=1}^{k-2} p_{o(j)} + \sum_{j=1}^{\eta-1} p_{\beta(j)}\big)} \\
& < \frac{v(g_{o(k-1)}) - v(h_{o(k-1)})}{v(h_{o(k)}) - v(g_{o(k)})} \leq \frac{w^-\big(\sum_{j=1}^{k} p_{o(j)}\big) - w^-\big(\sum_{j=1}^{k-1} p_{o(j)}\big)}{w^-\big(\sum_{j=1}^{k-1} p_{o(j)}\big) - w^-\big(\sum_{j=1}^{k-2} p_{o(j)}\big)}. \qquad (21)
\end{aligned}
$$

Again, the functions $w^-(\cdot)$ and $v(\cdot)$ can be chosen such that both inequalities in (21) are satisfied. Therefore, preferences are not represented as in statement (a) of the theorem.

Finally, if there does not exist $\bar{z} \in X_L$ such that $f_{\beta(\eta)} \succ \bar{z} \succ x_*$ and there does not exist $\grave{z} \in X_L$ such that $f_{\beta(\rho)} \succ \grave{z} \succ f_{\beta(\eta)}$, then, since there are at least four outcomes in the loss domain, there exists $\tilde{z} \in X_L$ such that $\tilde{z} \succ f_{\beta(\rho)}$. In this case, we can let $g_{o(k-1)} = f_{\beta(\eta)}$ and $g_{o(k)} = f_{\beta(\rho)}$. Then,

$$
\begin{aligned}
V[h_A f] \;=\; & w^-\big(\sum_{j=1}^{\eta-1} p_{\beta(j)}\big) v(f_{\beta(1)}) + \sum_{i=1}^{k-1}\Big(w^-\big(\sum_{j=1}^{i} p_{o(j)} + \sum_{j=1}^{\eta-1} p_{\beta(j)}\big) - w^-\big(\sum_{j=1}^{i-1} p_{o(j)} + \sum_{j=1}^{\eta-1} p_{\beta(j)}\big)\Big) v(h_{o(i)}) \\
& + \Big(w^-\big(\sum_{j=1}^{k-1} p_{o(j)} + \sum_{j=1}^{\rho-1} p_{\beta(j)}\big) - w^-\big(\sum_{j=1}^{k-1} p_{o(j)} + \sum_{j=1}^{\eta-1} p_{\beta(j)}\big)\Big) v(f_{\beta(\eta)}) \\
& + \Big(w^-\big(\sum_{j=1}^{k-1} p_{o(j)} + \sum_{j=1}^{\sigma-1} p_{\beta(j)}\big) - w^-\big(\sum_{j=1}^{k-1} p_{o(j)} + \sum_{j=1}^{\rho-1} p_{\beta(j)}\big)\Big) v(f_{\beta(\rho)}) \\
& + \Big(w^-\big(\sum_{j=1}^{k} p_{o(j)} + \sum_{j=1}^{\sigma-1} p_{\beta(j)}\big) - w^-\big(\sum_{j=1}^{k-1} p_{o(j)} + \sum_{j=1}^{\sigma-1} p_{\beta(j)}\big)\Big) v(h_{o(k)}) \\
& + \sum_{i=\sigma}^{|\tilde{A}|}\Big(w^-\big(\sum_{j=1}^{k} p_{o(j)} + \sum_{j=1}^{i} p_{\beta(j)}\big) - w^-\big(\sum_{j=1}^{k} p_{o(j)} + \sum_{j=1}^{i-1} p_{\beta(j)}\big)\Big) v(f_{\beta(i)}) \\
& + \sum_{\substack{i=k+1 \\ o(i)\notin\tilde{A}}}^{n}\Big(w^+\big(\sum_{\substack{j=i \\ o(j)\notin\tilde{A}}}^{n} p_{\beta(j)}\big) - w^+\big(\sum_{\substack{j=i+1 \\ o(j)\notin\tilde{A}}}^{k} p_{\beta(j)}\big)\Big) v(f_{\beta(i)})
\end{aligned}
$$

and

$$
\begin{aligned}
V[g_A f] \;\; = \;\; & w^-\big(\sum_{j=1}^{\eta-1} p_{\beta(j)}\big)v(f_{\beta(1)}) + \sum_{i=1}^{k-1}\Big(w^-\big(\sum_{j=1}^{i} p_{o(j)} + \sum_{j=1}^{\eta-1} p_{\beta(j)}\big) - w^-\big(\sum_{j=1}^{i-1} p_{o(j)} + \sum_{j=1}^{\eta-1} p_{\beta(j)}\big)\Big)v(g_{o(i)}) \\
& + \Big(w^-\big(\sum_{j=1}^{k-1} p_{o(j)} + \sum_{j=1}^{\rho-1} p_{\beta(j)}\big) - w^-\big(\sum_{j=1}^{k-1} p_{o(j)} + \sum_{j=1}^{\eta-1} p_{\beta(j)}\big)\Big)v(f_{\beta(\eta)}) \\
& + \Big(w^-\big(\sum_{j=1}^{k-1} p_{o(j)} + \sum_{j=1}^{\sigma-1} p_{\beta(j)}\big) - w^-\big(\sum_{j=1}^{k-1} p_{o(j)} + \sum_{j=1}^{\rho-1} p_{\beta(j)}\big)\Big)v(f_{\beta(\rho)}) \\
& + \Big(w^-\big(\sum_{j=1}^{k} p_{o(j)} + \sum_{j=1}^{\sigma-1} p_{\beta(j)}\big) - w^-\big(\sum_{j=1}^{k-1} p_{o(j)} + \sum_{j=1}^{\sigma-1} p_{\beta(j)}\big)\Big)v(g_{o(k)}) \\
& + \sum_{i=\sigma}^{|\tilde{A}|}\Big(w^-\big(\sum_{j=1}^{k} p_{o(j)} + \sum_{j=1}^{i} p_{\beta(j)}\big) - w^-\big(\sum_{j=1}^{k} p_{o(j)} + \sum_{j=1}^{i-1} p_{\beta(j)}\big)\Big)v(f_{\beta(i)}) \\
& + \sum_{\substack{i=k+1 \\ o(i)\notin\tilde{A}}}^{n}\Big(w^+\big(\sum_{\substack{j=i \\ o(j)\notin\tilde{A}}}^{n} p_{\beta(j)}\big) - w^+\big(\sum_{\substack{j=i+1 \\ o(j)\notin\tilde{A}}}^{k} p_{\beta(j)}\big)\Big)v(f_{\beta(i)}).
\end{aligned}
$$

Thus, since $h_{o(i)} = g_{o(i)} = h_{o(k-1)}$ for all $i < k-1$, we have

$$
\begin{aligned}
& g_A f \succ h_A f \\
\Leftrightarrow \;\; & V[g_A f] > V[h_A f] \\
\Leftrightarrow \;\; & \Big(w^-\big(\sum_{j=1}^{k-1} p_{o(j)} + \sum_{j=1}^{\eta-1} p_{\beta(j)}\big) - w^-\big(\sum_{j=1}^{k-2} p_{o(j)} + \sum_{j=1}^{\eta-1} p_{\beta(j)}\big)\Big)\big(v(h_{o(k-1)}) - v(g_{o(k-1)})\big) \\
& + \Big(w^-\big(\sum_{j=1}^{k} p_{o(j)} + \sum_{j=1}^{\sigma-1} p_{\beta(j)}\big) - w^-\big(\sum_{j=1}^{k-1} p_{o(j)} + \sum_{j=1}^{\sigma-1} p_{\beta(j)}\big)\Big)\big(v(h_{o(k)}) - v(g_{o(k)})\big) < 0.
\end{aligned}
$$

This and (17) imply that conditional preferences are not represented by $E^e_L[\cdot|s \in A]$ if

$$
\begin{aligned}
& \frac{w^-\big(\sum_{j=1}^{k} p_{o(j)} + \sum_{j=1}^{\sigma-1} p_{\beta(j)}\big) - w^-\big(\sum_{j=1}^{k-1} p_{o(j)} + \sum_{j=1}^{\sigma-1} p_{\beta(j)}\big)}{w^-\big(\sum_{j=1}^{k-1} p_{o(j)} + \sum_{j=1}^{\eta-1} p_{\beta(j)}\big) - w^-\big(\sum_{j=1}^{k-2} p_{o(j)} + \sum_{j=1}^{\eta-1} p_{\beta(j)}\big)} \\
& < \frac{v(g_{o(k-1)}) - v(h_{o(k-1)})}{v(h_{o(k)}) - v(g_{o(k)})} \leq \frac{w^-\big(\sum_{j=1}^{k} p_{o(j)}\big) - w^-\big(\sum_{j=1}^{k-1} p_{o(j)}\big)}{w^-\big(\sum_{j=1}^{k-1} p_{o(j)}\big) - w^-\big(\sum_{j=1}^{k-2} p_{o(j)}\big)}. \qquad (22)
\end{aligned}
$$

The functions $w^-(\cdot)$ and $v(\cdot)$ can, once again, be chosen such that both inequalities in (22) are satisfied. Therefore, preferences are not represented as in statement (a) of the theorem.

Suppose second that there exist $\hat{x}$ such that $f_{\beta(1)} \succ \hat{x} \succ x_*$. Then we can let $f_{\beta(\eta)} \succsim h_{o(k)} \succ$

$g_{o(k)} = f_{\beta(1)} \succ g_{o(k-1)} \succ h_{o(k-1)}$. We thus have that

$$\begin{aligned}
V[h_A f] \;=\;& w^-\big(\sum_{j=1}^{k-2} p_{o(j)}\big) v(h_{o(k-1)}) + \Big(w^-\big(\sum_{j=1}^{k-1} p_{o(j)}\big) - w^-\big(\sum_{j=1}^{k-2} p_{o(j)}\big)\Big) v(h_{o(k-1)}) \\
&+\Big(w^-\big(\sum_{j=1}^{k-1} p_{o(j)} + \sum_{j=1}^{\eta-1} p_{\beta(j)}\big) - w^-\big(\sum_{j=1}^{k-1} p_{o(j)}\big)\Big) v(f_{\beta(1)}) \\
&+\Big(w^-\big(\sum_{j=1}^{k} p_{o(j)} + \sum_{j=1}^{\eta-1} p_{\beta(j)}\big) - w^-\big(\sum_{j=1}^{k-1} p_{o(j)} + \sum_{j=1}^{\eta-1} p_{\beta(j)}\big)\Big) v(h_{o(k)}) \\
&+\sum_{i=\eta}^{|\tilde{A}|} \Big(w^-\big(\sum_{j=1}^{k} p_{o(j)} + \sum_{j=1}^{i} p_{\beta(j)}\big) - w^-\big(\sum_{j=1}^{k} p_{o(j)} + \sum_{j=1}^{i-1} p_{\beta(j)}\big)\Big) v(f_{\beta(i)}) \\
&+\sum_{\substack{i=k+1 \\ o(i)\notin\tilde{A}}}^{n} \Big(w^+\big(\sum_{\substack{j=i \\ o(j)\notin\tilde{A}}}^{n} p_{\beta(j)}\big) - w^+\big(\sum_{\substack{j=i+1 \\ o(j)\notin\tilde{A}}}^{k} p_{\beta(j)}\big)\Big) v(f_{\beta(i)}) \qquad (23)
\end{aligned}$$

and

$$\begin{aligned}
V[g_A f] \;=\;& w^-\big(\sum_{j=1}^{k-2} p_{o(j)}\big) v(h_{o(k-1)}) + \Big(w^-\big(\sum_{j=1}^{k-1} p_{o(j)}\big) - w^-\big(\sum_{j=1}^{k-2} p_{o(j)}\big)\Big) v(g_{o(k+1)}) \\
&+\Big(w^-\big(\sum_{j=1}^{k-1} p_{o(j)} + \sum_{j=1}^{\eta-1} p_{\beta(j)}\big) - w^-\big(\sum_{j=1}^{k-1} p_{o(j)}\big)\Big) v(f_{\beta(1)}) \\
&+\Big(w^-\big(\sum_{j=1}^{k} p_{o(j)} + \sum_{j=1}^{\eta-1} p_{\beta(j)}\big) - w^-\big(\sum_{j=1}^{k-1} p_{o(j)} + \sum_{j=1}^{\eta-1} p_{\beta(j)}\big)\Big) v(g_{o(k)}) \\
&+\sum_{i=\eta}^{|\tilde{A}|} \Big(w^-\big(\sum_{j=1}^{k} p_{o(j)} + \sum_{j=1}^{i} p_{\beta(j)}\big) - w^-\big(\sum_{j=1}^{k} p_{o(j)} + \sum_{j=1}^{i-1} p_{\beta(j)}\big)\Big) v(f_{\beta(i)}) \\
&+\sum_{\substack{i=k+1 \\ o(i)\notin\tilde{A}}}^{n} \Big(w^+\big(\sum_{\substack{j=i \\ o(j)\notin\tilde{A}}}^{n} p_{\beta(j)}\big) - w^+\big(\sum_{\substack{j=i+1 \\ o(j)\notin\tilde{A}}}^{k} p_{\beta(j)}\big)\Big) v(f_{\beta(i)}). \qquad (24)
\end{aligned}$$

Hence,

$$\begin{aligned}
& g_A f \succ h_A f \\
\Leftrightarrow\;& V[g_A f] > V[h_A f] \\
\Leftrightarrow\;& \Big(w^-\big(\sum_{j=1}^{k-1} p_{o(j)}\big) - w^-\big(\sum_{j=1}^{k-2} p_{o(j)}\big)\Big)\big(v(h_{o(k-1)}) - v(g_{o(k-1)})\big) \qquad (25) \\
& +\Big(w^-\big(\sum_{j=1}^{k} p_{o(j)} + \sum_{j=1}^{\eta-1} p_{\beta(j)}\big) - w^-\big(\sum_{j=1}^{k-1} p_{o(j)} + \sum_{j=1}^{\eta-1} p_{\beta(j)}\big)\Big)\big(v(h_{o(k)}) - v(g_{o(k)})\big) < 0.
\end{aligned}$$

Conditional preferences are not represented by $E^e_L[\cdot|s \in A]$ if (17) holds. Combining (25) and

(17), we have that preferences are not represented by $E_L^P[\cdot|s \in A]$ if

$$\frac{w^-\big(\sum_{j=1}^{k} p_{o(j)} + \sum_{j=1}^{\eta-1} p_{\beta(j)}\big) - w^-\big(\sum_{j=1}^{k-1} p_{o(j)} + \sum_{j=1}^{\eta-1} p_{\beta(j)}\big)}{w^-\big(\sum_{j=1}^{k-1} p_{o(j)}\big) - w^-\big(\sum_{j=1}^{k-2} p_{o(j)}\big)} < \frac{v(g_{o(k-1)}) - v(h_{o(k-1)})}{v(h_{o(k)}) - v(g_{o(k)})} \leq \frac{w^-\big(\sum_{j=1}^{k} p_{o(j)}\big) - w^-\big(\sum_{j=1}^{k-1} p_{o(j)}\big)}{w^-\big(\sum_{j=1}^{k-1} p_{o(j)}\big) - w^-\big(\sum_{j=1}^{k-2} p_{o(j)}\big)}. \tag{26}$$

As above, the functions $w^-(\cdot)$ and $v(\cdot)$ can be chosen such that both inequalities in (26) are satisfied. Therefore, preferences are not represented as in statement (a) of the theorem.

Suppose finally that $f_{\beta(1)} \succ x_*$ and there does not exist $\hat{x}$ such that $f_{\beta(1)} \succ \hat{x} \succ x_*$. Then, since there are at least four outcomes in the gain domain, there exists x'' such that $0 \succ x'' \succ f_{\beta(1)}$. Therefore, we can choose h and g such that $h_{o(k)} \succsim g_{o(k)} \succ f_{\beta(1)}$ and $g_{o(k-1)} = f_{\beta(1)} \succ h_{o(k-1)}$. If there exists $\tilde{x}$ in the loss domain such that $f_{\beta(\eta)} \succ \tilde{x} \succ f_{\beta(1)}$, we can let $f_{\beta(\eta)} \succsim h_{o(k)}$. Then the utility of $h_A f$ is given by (23) and the utility of $g_A f$ is given by (24). Therefore, preferences are not represented by $E_L^e[\cdot|s \in A]$ if the inequalites in (26) hold, and again it is possible to choose the functions $w^-(\cdot)$ and $v(\cdot)$ such that both inequalities are satisfied.

If there does not exist $\tilde{x}$ in the loss domain such that $f_{\beta(\eta)} \succ \tilde{x} \succ f_{\beta(1)}$, we can, again because the loss domain consists of at least four outcomes, let $g_{o(k)} = f_{\beta(\eta)}$ and $h_{o(k)} \succ f_{\beta(\eta)}$. Define $\rho \equiv \min\{j|f_{\beta(j)} \succ f_{\beta(\eta)}\}$. Then

$$\begin{aligned}
V[h_A f] \;=\;& w^-\big(\sum_{j=1}^{k-2} p_{o(j)}\big)v(h_{o(k-1)}) + \Big(w^-\big(\sum_{j=1}^{k-1} p_{o(j)}\big) - w^-\big(\sum_{j=1}^{k-2} p_{o(j)}\big)\Big)v(h_{o(k+1)}) \\
&+\Big(w^-\big(\sum_{j=1}^{k-1} p_{o(j)} + \sum_{j=1}^{\eta-1} p_{\beta(j)}\big) - w^-\big(\sum_{j=1}^{k-1} p_{o(j)}\big)\Big)v(f_{\beta(1)}) \\
&+\Big(w^-\big(\sum_{j=1}^{k-1} p_{o(j)} + \sum_{j=1}^{\rho-1} p_{\beta(j)}\big) - w^-\big(\sum_{j=1}^{k-1} p_{o(j)} + \sum_{j=1}^{\eta-1} p_{\beta(j)}\big)\Big)v(f_{\beta(\eta)}) \\
&+\Big(w^-\big(\sum_{j=1}^{k} p_{o(j)} + \sum_{j=1}^{\rho-1} p_{\beta(j)}\big) - w^-\big(\sum_{j=1}^{k-1} p_{o(j)} + \sum_{j=1}^{\rho-1} p_{\beta(j)}\big)\Big)v(h_{o(k)}) \\
&+\sum_{i=\rho}^{|\tilde{A}|}\Big(w^-\big(\sum_{j=1}^{k} p_{o(j)} + \sum_{j=1}^{i} p_{\beta(j)}\big) - w^-\big(\sum_{j=1}^{k} p_{o(j)} + \sum_{j=1}^{i-1} p_{\beta(j)}\big)\Big)v(f_{\beta(i)}) \\
&+\sum_{\substack{i=k+1 \\ o(i)\notin\tilde{A}}}^{n}\Big(w^+\big(\sum_{\substack{j=i \\ o(j)\notin\tilde{A}}}^{n} p_{\beta(j)}\big) - w^+\big(\sum_{\substack{j=i+1 \\ o(j)\notin\tilde{A}}}^{k} p_{\beta(j)}\big)\Big)v(f_{\beta(i)})
\end{aligned}$$

and

$$\begin{aligned}
V[g_A f] \;\; = \;\; & w^-\big(\sum_{j=1}^{k-2} p_{o(j)}\big) v(h_{o(k-1)}) + \Big(w^-\big(\sum_{j=1}^{k-1} p_{o(j)}\big) - w^-\big(\sum_{j=1}^{k-2} p_{o(j)}\big)\Big) v(g_{o(k+1)}) \\
& + \Big(w^-\big(\sum_{j=1}^{k-1} p_{o(j)} + \sum_{j=1}^{\eta-1} p_{\beta(j)}\big) - w^-\big(\sum_{j=1}^{k-1} p_{o(j)}\big)\Big) v(f_{\beta(1)}) \\
& + \Big(w^-\big(\sum_{j=1}^{k-1} p_{o(j)} + \sum_{j=1}^{\rho-1} p_{\beta(j)}\big) - w^-\big(\sum_{j=1}^{k-1} p_{o(j)} + \sum_{j=1}^{\eta-1} p_{\beta(j)}\big)\Big) v(f_{\beta(\eta)}) \\
& + \Big(w^-\big(\sum_{j=1}^{k} p_{o(j)} + \sum_{j=1}^{\rho-1} p_{\beta(j)}\big) - w^-\big(\sum_{j=1}^{k-1} p_{o(j)} + \sum_{j=1}^{\rho-1} p_{\beta(j)}\big)\Big) v(g_{o(k)}) \\
& + \sum_{i=\rho}^{|\tilde{A}|} \Big(w^-\big(\sum_{j=1}^{k} p_{o(j)} + \sum_{j=1}^{i} p_{\beta(j)}\big) - w^-\big(\sum_{j=1}^{k} p_{o(j)} + \sum_{j=1}^{i-1} p_{\beta(j)}\big)\Big) v(f_{\beta(i)}) \\
& + \sum_{\substack{i=k+1 \\ o(i)\notin\tilde{A}}}^{n} \Big(w^+\big(\sum_{\substack{j=i \\ o(j)\notin\tilde{A}}}^{n} p_{\beta(j)}\big) - w^+\big(\sum_{\substack{j=i+1 \\ o(j)\notin\tilde{A}}}^{k} p_{\beta(j)}\big)\Big) v(f_{\beta(i)}).
\end{aligned}$$

Therefore,

$$\begin{aligned}
& g_A f \succ h_A f \\
\Leftrightarrow \;\; & V[g_A f] > V[h_A f] \\
\Leftrightarrow \;\; & \Big(w^-\big(\sum_{j=1}^{k-1} p_{o(j)}\big) - w^-\big(\sum_{j=1}^{k-2} p_{o(j)}\big)\Big)\big(v(h_{o(k-1)}) - v(g_{o(k-1)})\big) \\
& + \Big(w^-\big(\sum_{j=1}^{k} p_{o(j)} + \sum_{j=1}^{\rho-1} p_{\beta(j)}\big) - w^-\big(\sum_{j=1}^{k-1} p_{o(j)} + \sum_{j=1}^{\rho-1} p_{\beta(j)}\big)\Big)\big(v(h_{o(k)}) - v(g_{o(k)})\big) < 0.
\end{aligned}$$

Combining this with (17) we have that preferences are not represented by $E_L^e[\cdot|s \in A]$ if

$$\begin{aligned}
& \frac{w^-\big(\sum_{j=1}^{k} p_{o(j)} + \sum_{j=1}^{\rho-1} p_{\beta(j)}\big) - w^-\big(\sum_{j=1}^{k-1} p_{o(j)} + \sum_{j=1}^{\rho-1} p_{\beta(j)}\big)}{w^-\big(\sum_{j=1}^{k-1} p_{o(j)}\big) - w^-\big(\sum_{j=1}^{k-2} p_{o(j)}\big)} \\
& < \frac{v(g_{o(k-1)}) - v(h_{o(k-1)})}{v(h_{o(k)}) - v(g_{o(k)})} \leq \frac{w^-\big(\sum_{j=1}^{k} p_{o(j)}\big) - w^-\big(\sum_{j=1}^{k-1} p_{o(j)}\big)}{w^-\big(\sum_{j=1}^{k-1} p_{o(j)}\big) - w^-\big(\sum_{j=1}^{k-2} p_{o(j)}\big)}.
\end{aligned} \tag{27}$$

Again, the functions $w^-(\cdot)$ and $v(\cdot)$ can be chosen such that both inequalities in (27) are satisfied. Therefore, preferences are not represented as in statement (a) of the theorem. ■