



# 15.401 Finance Theory I

---

Alex Stomper

MIT Sloan School of Management

## *Lecture 10: Capital Budgeting*

- \_ NPV rule
- \_ Cash flows from capital investments
- \_ Discount rates
- \_ Project interaction
- \_ Alternative capital budgeting rules
- \_ Real options

**Readings:**

- \_ Brealey, Myers and Allen, Chapters 6, 7, 23

A firm's business involves capital investments (capital budgeting), e.g., the acquisition of real assets. The objective is to increase the firm's current market value. Decision reduces to valuing real assets, i.e., their cash flows.

Let the cash flow of an investment (a project) be

$$\{CF_0, CF_1, \dots, CF_t\}$$

Its current market value is

$$NPV = CF_0 + \frac{CF_1}{1+r_1} + \dots + \frac{CF_t}{(1+r_t)^t}$$

This is the increase in the firm's market value by the project.

### Investment Criteria:

- \_ For a single project, take it if and only if its NPV is positive.
- \_ For many independent projects, take all those with positive NPV.
- \_ For **mutually exclusive** projects, take the one with positive and highest NPV.

In order to compute the NPV of a project, we need to analyze

1. Cash flows
2. Discount rates
3. Strategic options.

**Main Points:**

1. Use cash flows, not accounting earnings.
2. Use after-tax cash flows.
3. Use cash flows attributable to the project (compare firm value with and without the project):
  - Use incremental cash flows.
  - Forget sunk costs: bygones are bygones.
  - Include investment in working capital as capital expenditure.
  - Include opportunity costs of using existing facilities.

In what follows, all cash flows are attributable to the project.

$$\begin{aligned} \text{CF} &= [\text{Project Cash Inflows}] - [\text{Project Cash Outflows}] \\ &= [\text{Operating Revenues}] \\ &\quad - [\text{Operating Expenses without depreciation}] \\ &\quad - [\text{Capital Expenditures}] \\ &\quad - [\text{Taxes}] \end{aligned}$$

Defining operating profit by

$$\begin{aligned} \text{Operating Profit} &= \text{Operating Revenues} \\ &\quad - \text{Operating Expenses w/o Depreciation} \end{aligned}$$

Let  $\tau$  be the "effective" tax rate. The income taxes are

$$[\text{Taxes}] = (\tau)[\text{Operating Profit}] - (\tau) \times [\text{Depreciation}]$$

Accounting depreciation affects CF because it reduces firm's tax bill.

$$\begin{aligned} \text{CF} &= (1 - \tau)[\text{Operating Profits}] - [\text{Capital Expenditures}] \\ &\quad + (\tau)[\text{Depreciation}] \end{aligned}$$

**Example.** Accounting earnings vs. cash flows. A machine purchased for \$1,000,000 with a life of 10 years generates annual revenue of \$300,000 and operating expense of \$100,000. Assume that the machine will be depreciated over 10 years using straight-line depreciation. The corporate tax rate is 40%.

Date	Accounting Earnings Before Tax	Accounting Earnings After Tax	Cash Flow After-tax
0	0	0	- 1,000,000
1	$300,000 - 100,000 - 100,000 =$ 100,000	$(1-0.4)(100,000) =$ 60,000	$(1-0.4) (300,000-100,000) +$ 40,000 = 160,000
2	100,000	60,000	160,000
3	100,000	60,000	160,000
4	100,000	60,000	160,000
5	100,000	60,000	160,000
6	100,000	60,000	160,000
7	100,000	60,000	160,000
8	100,000	60,000	160,000
9	100,000	60,000	160,000
10	100,000	60,000	160,000

Accounting earnings may not accurately reflect the actual CF timing.

**Example.** Use after-tax cash flows. Consider the following project (the cash flow is in thousands of dollars and tax rate is 50%):

Year	0	1	2	3	4	5
Invest	500					
Operating CF		0	100	300	300	300
Depreciation		100	100	100	100	100
Income		-100	0	200	200	200
Tax		-50	0	100	100	100
After-tax CF	-500	50	100	200	200	200
PV at 10%	-500	45.45	82.64	150.26	136.60	124.18

$$NPV = +39.13.$$



Typically, there are timing differences between the accounting measure of earnings (Sales - Cost of Goods Sold) and cash flows.

$$\text{Working Capital (WC)} = \text{Inventory} + \text{A/R} - \text{A/P}$$

### Changes in Working Capital

- \_ Inventory: Cost of goods sold includes only the cost of items sold. When inventory is rising, the cost of goods sold understates cash outflows. When inventory is falling, cost of goods sold overstates cash outflows.
- \_ Accounts Receivable (A/R): Accounting sales may reflect sales that have not been paid for. Accounting sales understate cash inflows if the company is receiving payment for sales in past periods.
- \_ Accounts Payable (A/P) -- conceptually the reverse of A/R.

**Example.** You run a chain of stores that sell sweaters. This quarter, you buy 1,000,000 sweaters at a price of \$30.00 each. For the next two quarters, you sell 500,000 sweaters each quarter for \$60.00 each. The corporate tax rate is 40%.

In million dollars, your cash flows are

Date	After Tax Profit	Inventory	Cash Flow
0	0	$(1)(30) = 30$	-30
1	$(0.5)(60-30)(1-0.4) = 9$	$(0.5)(30) = 15$	$(0.5)(60) - (0.5)(60-30)(0.4) = 24$
2	$(0.5)(60-30)(1-0.4) = 9$	0	$(0.5)(60) - (0.5)(60-30)(0.4) = 24$

**Note:**

$$\text{Cash flow} = \text{Profit (after tax)} - \text{Change in Inventory}$$

**So far, we have shown that:**

- \_ A project's discount rate (required rate of return or cost of capital) is the expected rate of return demanded by investors for the project
- \_ Discount rate(s) in general depend on the timing and risk of the cash flow(s)
- \_ Discount rate is usually different for different projects
- \_ It is in general incorrect to use a company-wide “cost of capital” to discount cash flows of all projects

**What is the required rate of return on a project?**

- \_ Simple case: single discount rate can be used for all cash flows of a project (the term structure of discount rates is flat)
- \_ General case: different discount rates for different cash flows
  - the term structure of discount rates is not flat
  - different pieces of cash flow at a given time have different risks

## Use CAPM to estimate cost of capital (discount rate)

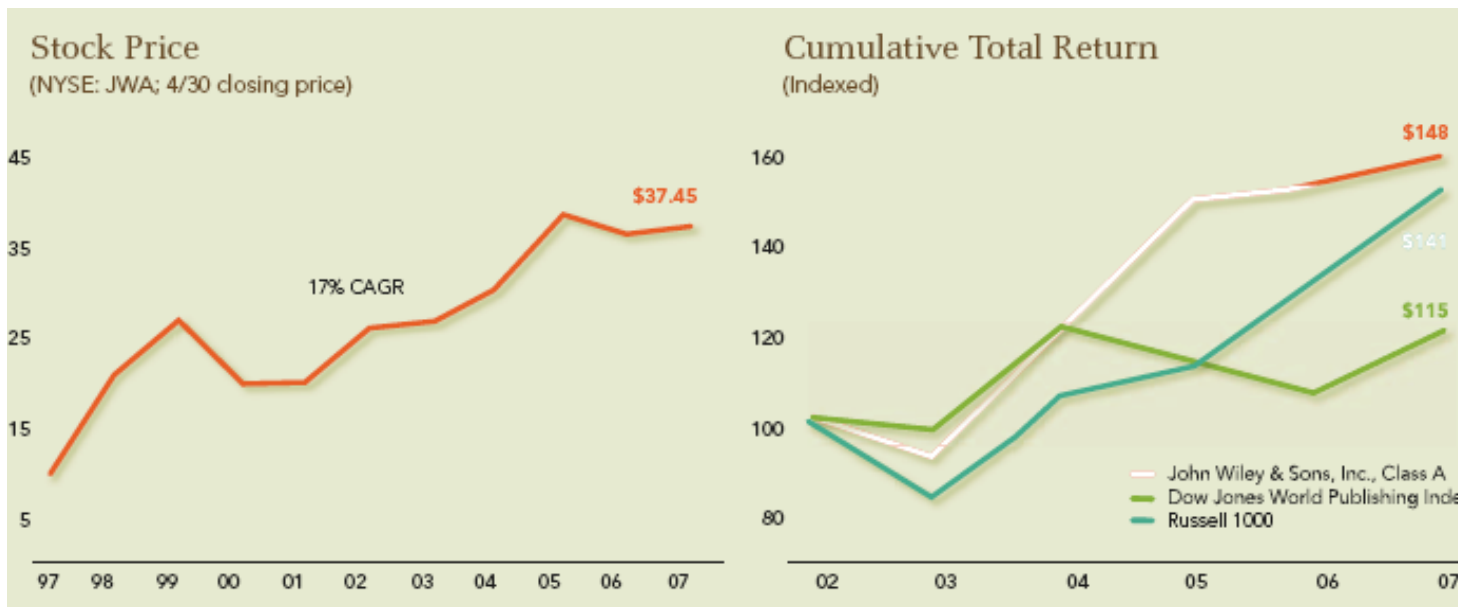
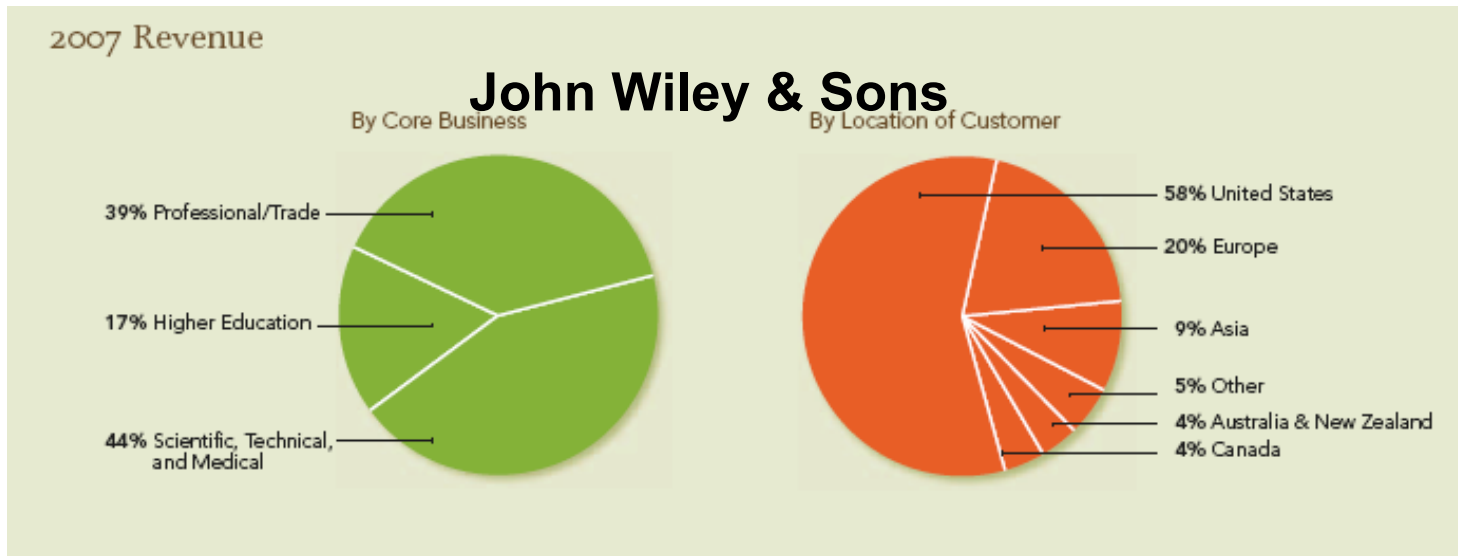
- \_ Project's required rate of return is determined by the **project beta**:

$$\bar{r}_{\text{project}} = r_F + \beta_{\text{project}} (\bar{r}_M - r_F)$$

- \_ What matters is the project beta, not the company beta!
- \_ What if project beta is unknown?
  - \_ Find comparable “pure-play” company and use its beta
  - \_ Find comparable projects and use its cash flows to estimate beta
  - \_ Use fundamental analysis and judgment to guesstimate beta

**Example.** Bloomberg, a provider of financial data and analytics, is considering entering the publishing business (Bloomberg Press), and must evaluate the NPV of the estimated cashflows from this business. What cost of capital should it use for these NPV calculations?

- \_ Bloomberg should not use its own beta to discount Bloomberg Press cash flows
  
- \_ Bloomberg should use the beta of a publishing company (e.g., John Wiley & Sons)
  
- \_ What about using McGraw-Hill's beta?



**Example (cont):**

\_ Beta of JW/A (from <http://finance.yahoo.com>): 1.29

\_ Risk-free rate: 5%

\_ Market risk premium: 6%

$$\begin{aligned}\bar{r}_{\text{project}} &= r_F + \beta_{\text{project}}(\bar{r}_M - r_F) \\ &= 0.05 + 1.29 \times 0.06 = 12.7\%\end{aligned}$$

\_ Use judgment in interpreting and adjusting these estimates

\_ Estimates are merely approximations!

\_ How good is the approximation?

**Example.** MSW Inc. is considering the introduction of a new product: Turbo-Widgets (TW).

- \_ TW were developed at an R&D cost of \$1M over past 3 years
- \_ New machine to produce TW would cost \$2M
- \_ New machine lasts for 15 years, with salvage value of \$50,000
- \_ New machine can be depreciated linearly to \$0 over 10 years
- \_ TW need to be painted; this can be done using excess capacity of the painting machine, which currently runs at a cost of \$30,000 (regardless of how much it is used)
- \_ Operating cost: \$40,000 per year
- \_ Sales: \$400,000, but cannibalization would lead existing sales of regular widgets to decrease by \$20,000
- \_ Working Capital (WC): \$250,000 needed over the life of the project
- \_ Tax rate: 34%
- \_ Opportunity cost of capital: 10%.

Should MSW go ahead to produce TW?



1. Initial investment includes capital expenditure and WC
2. R&D expense is a sunk cost
3. Depreciation is  $\$2\text{M}/10 = \$0.2\text{M}$  for first 10 years
4. Project should not be charged for painting-machine time
5. Project should be charged for cannibalization of regular widget sales
6. Salvage value is fully taxable since the book value at the end of year 10 is  $\$0$  (the machine cost has been fully depreciated).

The cash flows (in thousand dollars) are

Year	Cash Flow
0	$-(2000+250) = -2250$
1-10	$(400-40-20)(1-0.34) + (200)(0.34) = 292.4$
11-14	$(400-40-20)(1-0.34) = 224.4$
15	$224.4 + (50)(1-0.34) + 250 = 507.4$

NPV = - \$57,617.

Often we have to decide on more than one project.

- \_ For mutually independent projects, apply NPV rule to each project.
- \_ For projects dependent of each other (e.g., mutually exclusive), we have to compare their NPVs.

**Example.** Potential demand for your product is projected to increase over time. If you start the project early, your competitors will catch up with you faster, by copying your idea. Your opportunity cost of capital is 10%. Denoting by FPV the project's NPV at the time of introduction,

we have:

Year to Start	FPV	% Change in FPV	NPV
1	100	–	91
2	120	20	99
3	138	15	104
4	149	8	102

Before year 4, the return to waiting is larger than the opportunity cost of capital, 10%. As long as the growth rate of FPV remains below 10% after year 4, it is best to wait and introduce at the end of year 3.

In practice, investment rules other than NPV are also used:

- \_ Payback Period
- \_ Profitability Index (PI)
- \_ Internal Rate of Return (IRR)
- \_ ...

Firms use these rules because they were used historically and they may have worked (in combination with common sense) in the particular cases encountered by these firms.

These rules sometimes give the same answer as NPV, but in general they do not. We should be aware of their shortcomings and use NPV whenever possible.

The bottom line: The NPV rule dominates the alternative rules.

**Payback period** is the minimum length of time  $s$  such that the sum of net cash flows from a project becomes positive

$$CF_1 + CF_2 + \cdots + CF_s \geq -CF_0 = I_0$$

### Decision Criterion Using Payback Period

- \_ For independent projects: Accept if  $s$  is less than or equal to some fixed threshold  $t^*$ .
- \_ For mutually exclusive projects: Among all the projects having  $s \leq t^*$ , accept the one that has the minimum payback period.

**Example.** Let  $t^* = 3$ . Consider the two independent projects with the following cash flows (in thousands):

	$CF_0$	$CF_1$	$CF_2$	$CF_3$	$CF_4$	$CF_5$	$CF_6$	$t^*$
Project 1	-100	20	40	30	10	40	60	4
Project 2	-100	10	10	80	5	10	10	3

Decision: Accept Project 2.

- \_ Payback period rule ignores cash flows after the payback period.
- \_ It ignores discounting.

**Example.** (Cont) Suppose that the appropriate discount rate is a constant 10% per period. Then

$$NPV_1 = 39,315 \quad \text{and} \quad NPV_2 = -7,270$$

But we accepted project 2 and not project 1!

Taking into account appropriate discounting, we have the **discounted payback period**, which is the minimum  $s$  so that

$$\frac{CF_1}{1+r} + \frac{CF_2}{(1+r)^2} + \cdots + \frac{CF_s}{(1+r)^s} \geq -CF_0$$

where  $r$  is the discount rate (cost of capital). (It still ignores the cash flows after the discounted payback period.)

A project's **internal rate of return** (IRR) is the number that satisfies

$$0 = CF_0 + \frac{CF_1}{(1 + IRR)} + \frac{CF_2}{(1 + IRR)^2} + \cdots + \frac{CF_t}{(1 + IRR)^t}$$

### Decision Criterion Using IRR

- \_ For independent projects: Accept a project if its IRR is greater than some fixed  $IRR^*$ , the threshold rate.
- \_ For mutually exclusive projects: Among the projects having IRR's greater than  $IRR^*$ , accept one with the highest IRR.

**Example.** Consider the following mutually exclusive projects:

	$CF_0$	$CF_1$	$CF_2$	$CF_3$	$CF_4$	$CF_5$	$CF_6$
Project 1	-100	20	40	30	10	40	60
Project 2	-100	10	10	80	5	10	10

Then,  $IRR_1 = 21\%$  and  $IRR_2 = 7\%$ .

IRR rule leads to the same decisions as NPV if

1. Cash outflow occurs only at time 0
2. Only one project is under consideration
3. Opportunity cost of capital is the same for all periods
4. Threshold rate is set equal to opportunity cost of capital.

## 1. Non-existence of IRR

	CF <sub>0</sub>	CF <sub>1</sub>	CF <sub>2</sub>
Project 1	-105	250	-150
Project 2	105	-250	150

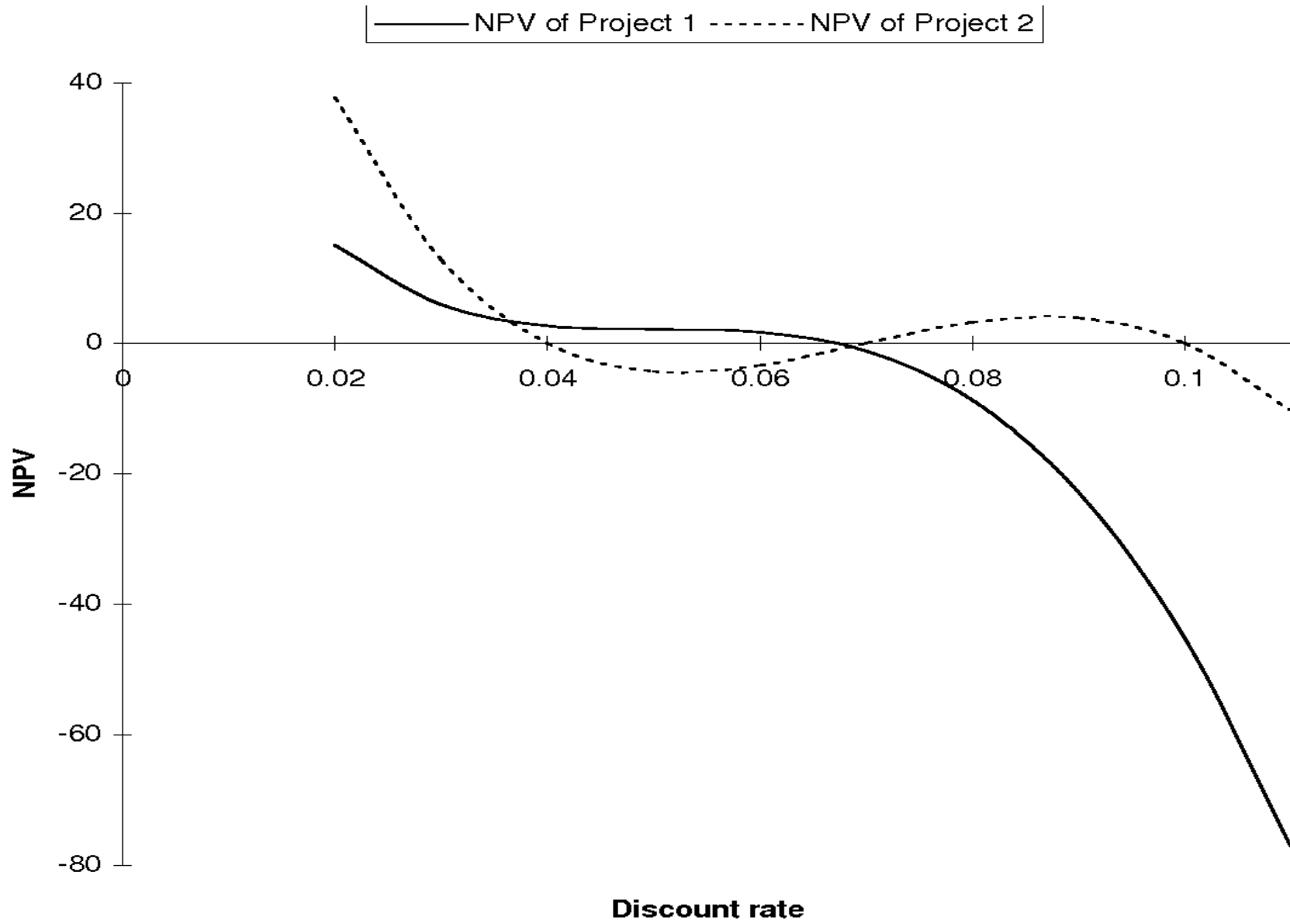
No IRR exists for these two projects.

## 2. Multiple IRR's

	CF <sub>0</sub>	CF <sub>1</sub>	CF <sub>2</sub>	CF <sub>3</sub>
Project 1	-500,000	1,575,000	-1,653,750	578,815
Project 2	-500,000	1,605,000	-1,716,900	612,040

$$\text{IRR}_1 = 7\% \quad \text{and} \quad \text{IRR}_2 = \begin{cases} 4\% \\ 7\% \\ 10\% \end{cases}$$





## 3. Project ranking using IRR for mutually exclusive projects:

## a) Projects of different scales:

	CF <sub>0</sub>	CF <sub>1</sub>	IRR	NPV at 10%
Project 1	-10,000	20,000	100%	8,181.82
Project 2	-20,000	36,000	80%	12,727.27

A way around this problem is to use **incremental** CF:

- \_ See if lower investment (project 1) is a good idea
- \_ See if incremental investment (project 2) is a good idea.

	CF <sub>0</sub>	CF <sub>1</sub>	IRR	NPV at 10%
Project 1	-10,000	20,000	100%	8,181.82
Project 2	-20,000	36,000	80%	12,727.27
Project 2-1	-10,000	16,000	60%	4,545.45

## b) Projects with different time patterns of cash flows:

CF <sub>t</sub>	0	1	2	3	4	5	...	IRR	NPV at 10%
Project 1	-90	60	50	40	0	0	...	33.0%	35.92
Project 2	-90	18	18	18	18	18	...	20.0%	90.00
Project 2-1	0	-42	-32	-22	18	18	...	15.6%	54.08

**Profitability index** (PI) is the ratio of the present value of future cash flows and the initial cost of a project:

$$PI = \frac{PV}{-CF_0} = \frac{PV}{I_0}$$

### Decision criterion using PI

- \_ For independent projects: Accept all projects with PI greater than one (this is identical to the NPV rule)
- \_ For mutually exclusive projects: Among the projects with PI greater than one, accept the one with the highest PI.

## Problems with PI

PI gives the same answer as NPV when

- \_ There is only one cash outflow, which is at time 0
- \_ Only one project is under consideration.

PI scales projects by their initial investments. The scaling can lead to wrong answers in comparing mutually exclusive projects.

	CF <sub>0</sub>	CF <sub>1</sub>	IRR	NPV at 10%	PI at 10%
Project 1	-1,000	2,000	100%	818.18	1.82
Project 2	-2,000	3,600	80%	1,272.73	1.64
Project 2-1	-1,000	1,600	60%	454.55	1.45

Comparison of Methods Used by Large U.S. and Multinational Firms

	Large U.S. Firms	Multinationals	
	Percentage Using Each Method	Use as Primary Method	Use as Secondary Method
Payback Period	80.3%	5.0%	37.6%
IRR	65.5	65.3	14.6
NPV	67.6	16.5	30.0
Other	-	2.5	3.2

Historical Comparison of Primary use of Various Capital Budgeting Techniques

	1959	1964	1970	1975	1977	1979	1981
Payback Period	34%	24%	12%	15%	9%	10%	5.0%
IRR	19	38	57	37	54	60	65.3
NPV	-	-	-	26	10	14	16.5
IRR or NPV	19	38	57	63	64	74	81.8

Source: S. Ross, R. Westerfield, and B. Jordon, *Essentials of Corporate Finance*, Irwin, 1996.

1. Competitive Response:
  - CF forecasts should consider responses of competitors.
  
2. Capital Rationing.
  
3. Sources of Positive-NPV Projects:
  - Short-run competitive advantage (right place at right time)
  - Long-run competitive advantage
    - Patent
    - Technology
    - economies of scale, etc.
  - Noise.

In capital investment decisions, we often face situations involving strategic options.

Common and important options in capital investments include:

- \_ The option to wait before investing
- \_ The option to make follow-on investments
- \_ The option to abandon a project
- \_ The option to vary output or production methods.

Two key elements in strategic options and their valuation:

1. New information arrives over time
2. Decisions can be made after receiving new information.

**Example.** Real options in follow-up projects. In 1990, MC Inc. considers entering PC business:

- \_ R&D has come up with model-A --- a new PC model
- \_ CFs of model-A, if introduced, are as follows

	1990	1991	1992	1993	1994	1995
Investment (\$M) (R&D, plant, WC)	-450	-50	-100	-100	125	125
Operating CF (\$M)		140	159	259	185	
Net CF	-450	90	59	159	310	125

NPV at 20% is -\$46 million. However,

- \_ Development and production of model-A would allow MC Inc. to introduce model-B in 1993.
- \_ Expected CFs from model-B are twice that of model-A.
- \_ In expectation, model-B is a loser too.
- \_ But there are scenarios in which model-B really pays off.



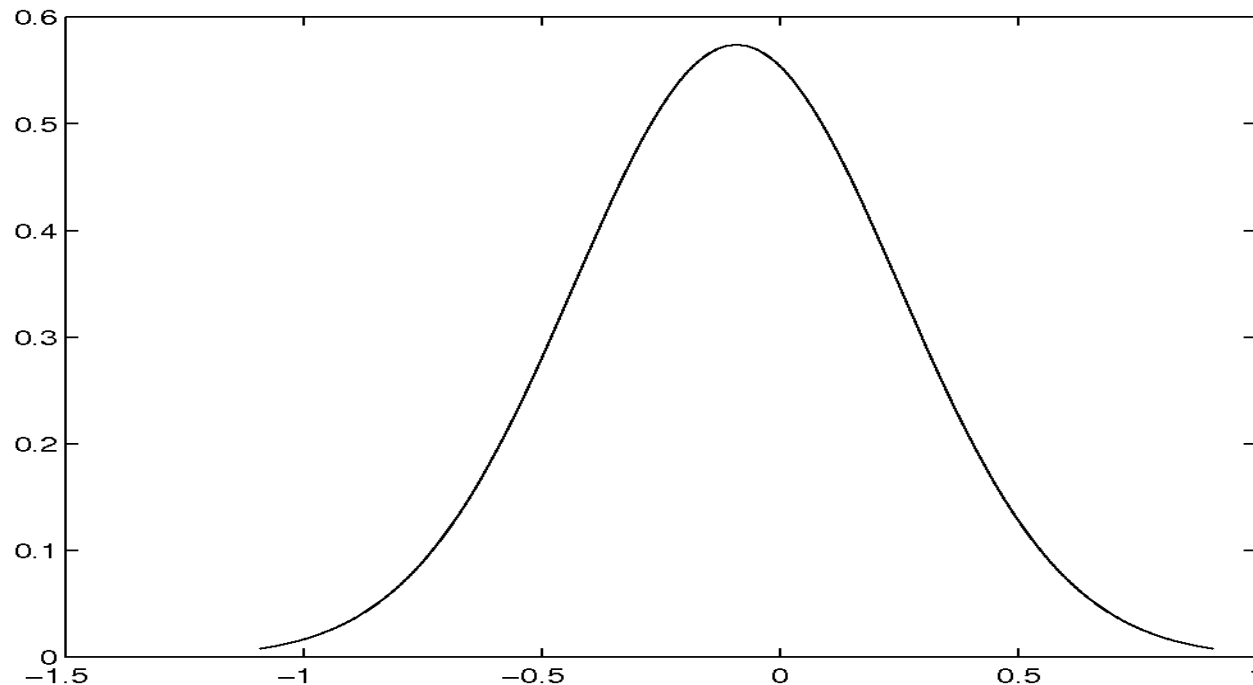
---

Different Scenarios	PV of model-B (\$M)
Benchmark scenario	-92
Initial Investment reduced by 30%	178
Sales increase by 40%	368
Profit margin increases by 50%	302

Should MC Inc. start model-A?

The expected value of model-B is -\$92 million. Could this prospect justify the \$46 million sacrifice to enter the market with model-A?

Probability Distribution of NPV in 1993 for Model-B (in billion dollars)



- \_ Starting model-B in 1993 is an option.
- \_ So long as MC can abandon the business in 1993, only the right-hand-side of the distribution is relevant.
- \_ NPV of the right-hand-side is huge even if the chance of ending up there is less than 50%.

Assume:

- \_ Model-B decision has to be made in 1993
- \_ Entry in 1993 with Model-A is prohibitively expensive
- \_ MC has the option to stop in 1993 (possible loss limited)
- \_ Investment needed for model-B is \$900M (twice that of A)
- \_ PV of operating profits from model-B is \$468 million in 1990
- \_ PV evolves with annual standard deviation of 35%.

The opportunity to invest in model-B is like a 3-year call option on an asset worth \$468 million now with exercise price \$900 million!

Using Black-Scholes formula:

Value of Call = \$55 million

Total NPV of model-A (\$M):

	A	A+B
DCF	-46	-99
Option value	55	
Total value	9	

- Naive DCF analysis tends to under-estimate the value of strategic options:
  - Timing of projects is an option (American call)
  - Follow-on projects are options (American call)
  - Termination of projects are options (American put)
  - Expansion or contraction of production are options (conversion options).
- It is difficult to apply DCF for option valuation.
- Options can be valued (sometimes).

Think of strategic planning as a process of

- 1.Acquiring and disposing of options
- 2.Exercising options optimally.

- \_ NPV rule
- \_ Cash flows from capital investments
- \_ Discount rates
- \_ Project interaction
- \_ Alternative capital budgeting rules
- \_ Real options