Part B Valuation

We have learned that:

- Business decisions often reduce to valuation of assets/CFs
- Two elements are important in valuing a CF: time and risk
- Value of CFs is determined in financial markets

From the market, we can learn

- How to value time --- time value of money
- How to value risk --- risk premium

In particular,

- Prices in the bond market gives the time value of money
- Prices in the stock market gives the risk premium

In this part of the course, we study the valuation of bonds, stocks, forwards, futures and options.

15.401 Finance Theory I

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Lecture 3: Fixed-Income Securities

Key concepts

_ Fixed-income securities
_ Overview of fixed-income markets
_ Term structure of interest rates
_ Discount bonds and coupon bonds
_ Forward interest rates
_ Interest rate risk
_ Inflation risk
_ Default risk

Readings:

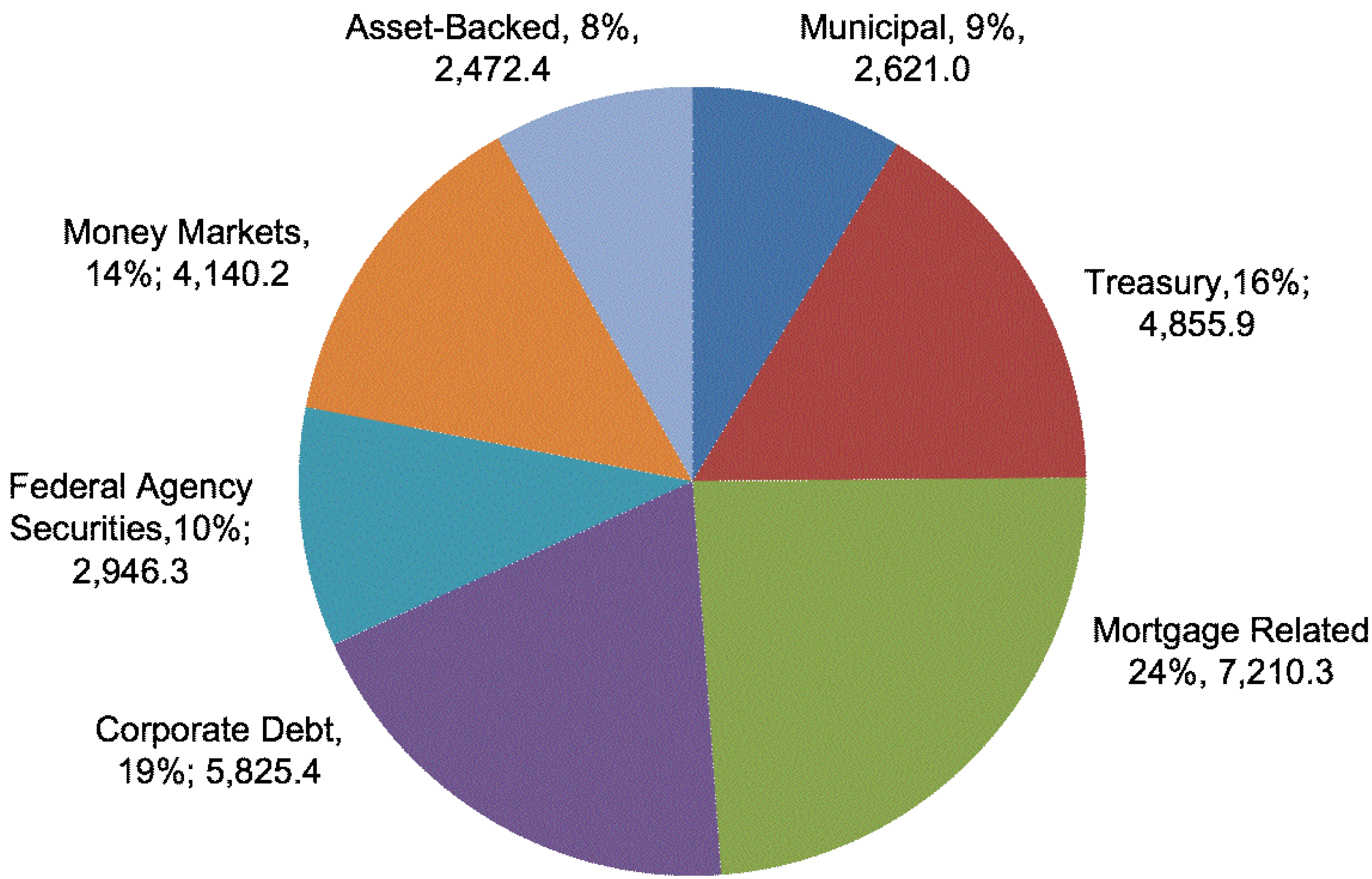
_ Brealey, Myers and Allen, Chapters 4, 24
_ Bodie, Kane and Markus, Chapters 14, 15, 16
_ Salomon Brothers, "Understanding Duration and Volatility"

Fixed-income securities are financial claims with promised cash flows of fixed amount paid at fixed dates.

Classification of Fixed-Income Securities:

1. Treasury Securities:
   - U.S. Treasury securities (bills, notes, bonds)
   - Bunds, JGBs, U.K. Gilts …
2. Federal Agency Securities:
   - Securities issued by federal agencies (FHLB, FNMA …)
3. Corporate securities:
   - Commercial paper (CP)
   - Medium-term notes (MTNs)
   - Corporate bonds …
4. Municipal securities (Munies)
5. Mortgage-backed securities (MBS)
6. Asset backed securities (ABS), …

U.S. bond market debt 2007 ($billions)



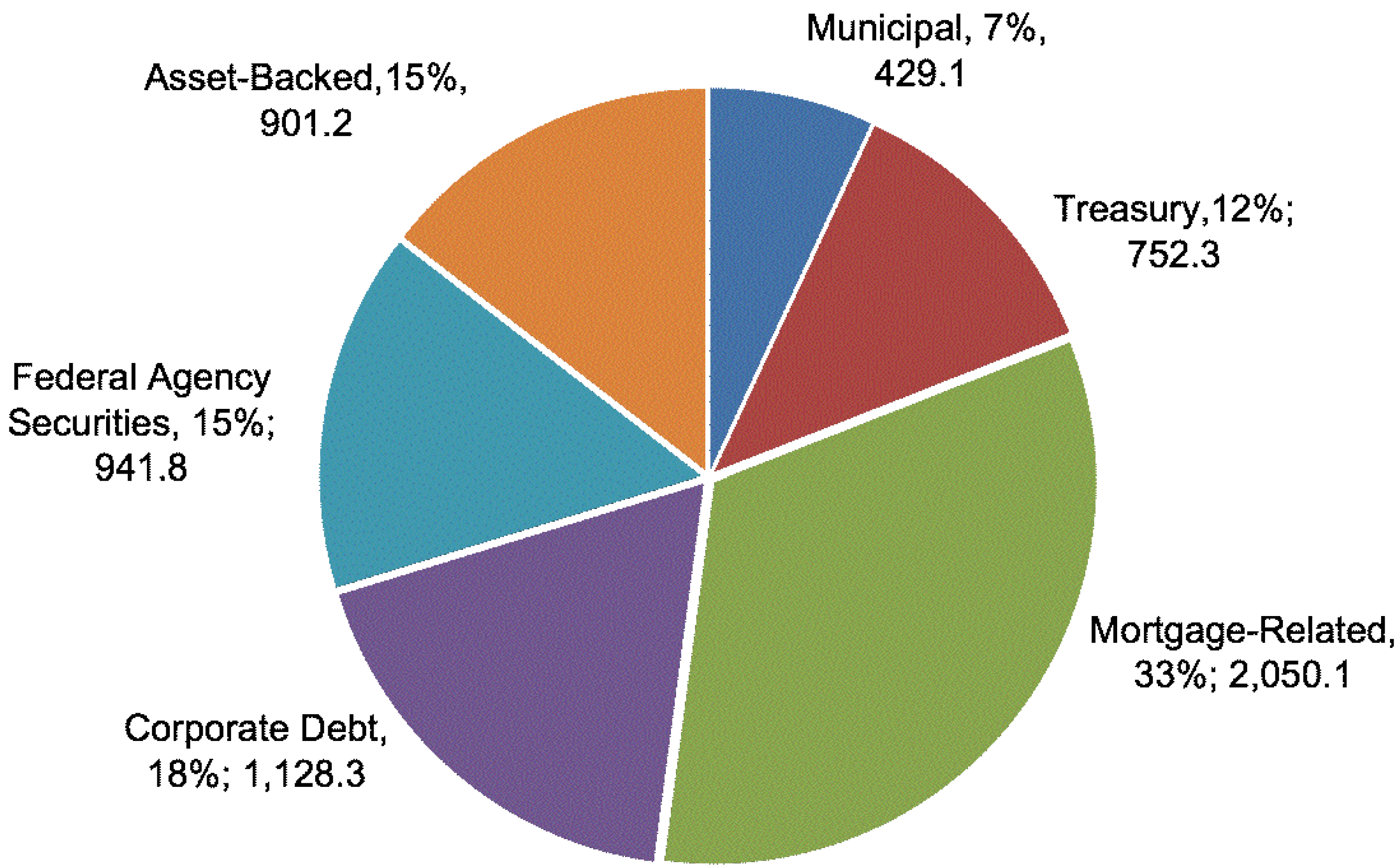
Sources: U.S. Department of Treasury, Federal Reserve System, Federal Agencies, Thomson Financial, Bloomberg, SIFMA

Outstanding U.S. bond market debt ($ billions)

°°	Municipal	Treasury	Mortgage Related	Corporate Debt	Federal Agency Securities	Money Markets	Asset-Backed	Total
1998	1,402.7	3,542.8	2,955.2	2,708.5	1,300.6	1,977.8	731.5	14,619.1
1999	1,457.1	3,529.5	3,334.2	3,046.5	1,620.0	2,338.8	900.8	16,226.9
2000	1,480.5	3,210.0	3,565.8	3,358.4	1,854.6	2,662.6	1,071.8	17,203.7
2001	1,603.6	3,196.6	4,127.6	3,836.4	2,149.6	2,587.2	1,281.1	18,782.1
2002	1,763.0	3,469.2	4,686.4	4,099.5	2,292.8	2,545.7	1,543.3	20,399.9
2003	1,900.7	3,822.1	5,238.6	4,458.4	2,636.7	2,519.9	1,693.7	22,270.1
2004	2,030.9	4,257.2	5,455.8	4,785.1	2,745.1	2,904.2	1,827.8	24,006.1
2005	2,226.0	4,517.3	5,915.6	4,960.0	2,613.8	3,433.7	1,955.2	25,621.6
2006	2,403.2	4,689.8	6,492.4	5,365.0	2,660.1	4,008.8	2,130.4	27,749.7
2007	2,621.0	4,855.9	7,210.3	5,825.4	2,946.3	4,140.2	2,472.4	30,071.5
2008Q1	2,657.0	4,995.8	7,397.0	5,905.6	2,984.2	4,125.9	2,480.3	30,545.8

Sources: U.S. Department of Treasury, Federal Reserve System, Federal Agencies, Thomson Financial, Bloomberg, SIFMA

U.S. bond market issuance 2007 ($billions)



Sources: U.S. Department of Treasury, Federal Reserve System, Federal Agencies, Thomson Financial, Bloomberg, SIFMA

U.S. bond market issuance ($ billions)

°°	Municipal	Treasury	Mortgage Related	Corporate Debt	Federal Agency Securities	Asset-Backed	Total
1998	286.8	438.4	1,143.90	610.7	596.4	286.6	3,362.70
1999	227.5	364.6	1,025.40	629.2	548	287.1	3,081.80
2000	200.8	312.4	684.4	587.5	446.6	337	2,568.70
2001	287.7	380.7	1,671.30	776.1	941	383.3	4,440.10
2002	357.5	571.6	2,249.20	636.7	1,041.50	469.2	5,325.70
2003	382.7	745.2	3,071.10	775.8	1,267.50	600.2	6,842.50
2004	359.8	853.3	1,779.00	780.7	881.8[4]	869.8	4,642.60
2005	408.2	746.2	1,966.70	752.8	669	1,172.10	5,715.00
2006	386.5	788.5	1,987.80	1,058.90	747.3	1,253.10	6,222.10
2007	429.1	752.3	2,050.10	1,128.30	941.8	901.2	6,202.80
2008Q1	85.1	203.8	386.7	213.1	429.5	58.1	1,376.30

Sources: U.S. Department of Treasury, Federal Reserve System, Federal Agencies, Thomson Financial, Bloomberg, SIFMA.

Organization of FI markets

Issuers:

1. Governments
2. Corporations
3. Commercial Banks
4. States
5. Municipalities
6. SPVs
7. Foreign Institutions

Intermediaries:

1. Primary Dealers
2. Other Dealers
3. Investment Banks
4. Credit-rating Agencies
5. Credit Enhancers
6. Liquidity Enhancers

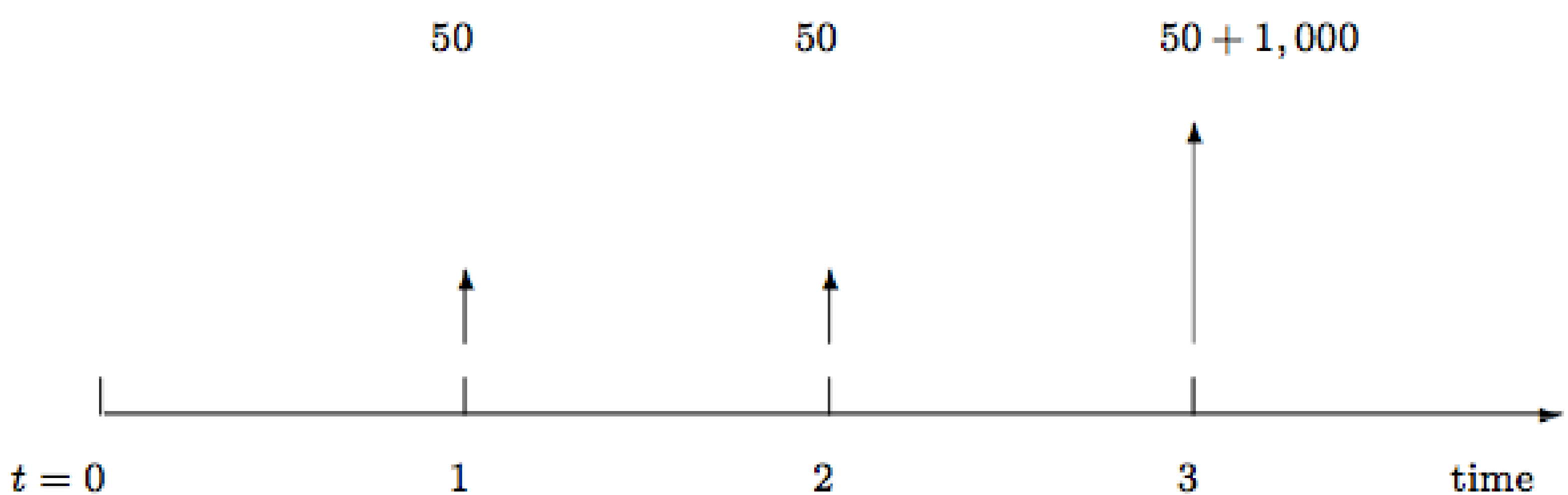
Investors:

1. Governments
2. Pension Funds
3. Insurance Companies
4. Commercial Banks
5. Mutual Funds
6. Hedge Funds
7. Foreign Institutions
8. Individuals

Cash flow:

1. Maturity
2. Principal
3. Coupon

Example. A 3-year bond with principal of $1,000 and annual coupon payment of 5% has the following cash flow:



Valuation:

1. Time value
   - Interest rates
2. Risks:
   - Inflation
   - Credit
   - Timing (callability)
   - Liquidity
   - Currency …

Our objective here is to value riskless cash flows

_ Given the rich set of fixed-income securities traded in the market, their prices provide the information needed to value riskless cash flows at hand

In the market, this information on the time value of money is given in many different forms:

1. Spot interest rates
2. Prices of discount bonds (e.g., zero-coupon bonds)
3. Prices of coupon bonds
4. Forward interest rates

Spot interest rate r_t is the current (annualized) interest rate for maturity date t

- r_t is for payments only on date t
- r_t is different for each different date t

Example. Spot interest rates on 2005.08.01:

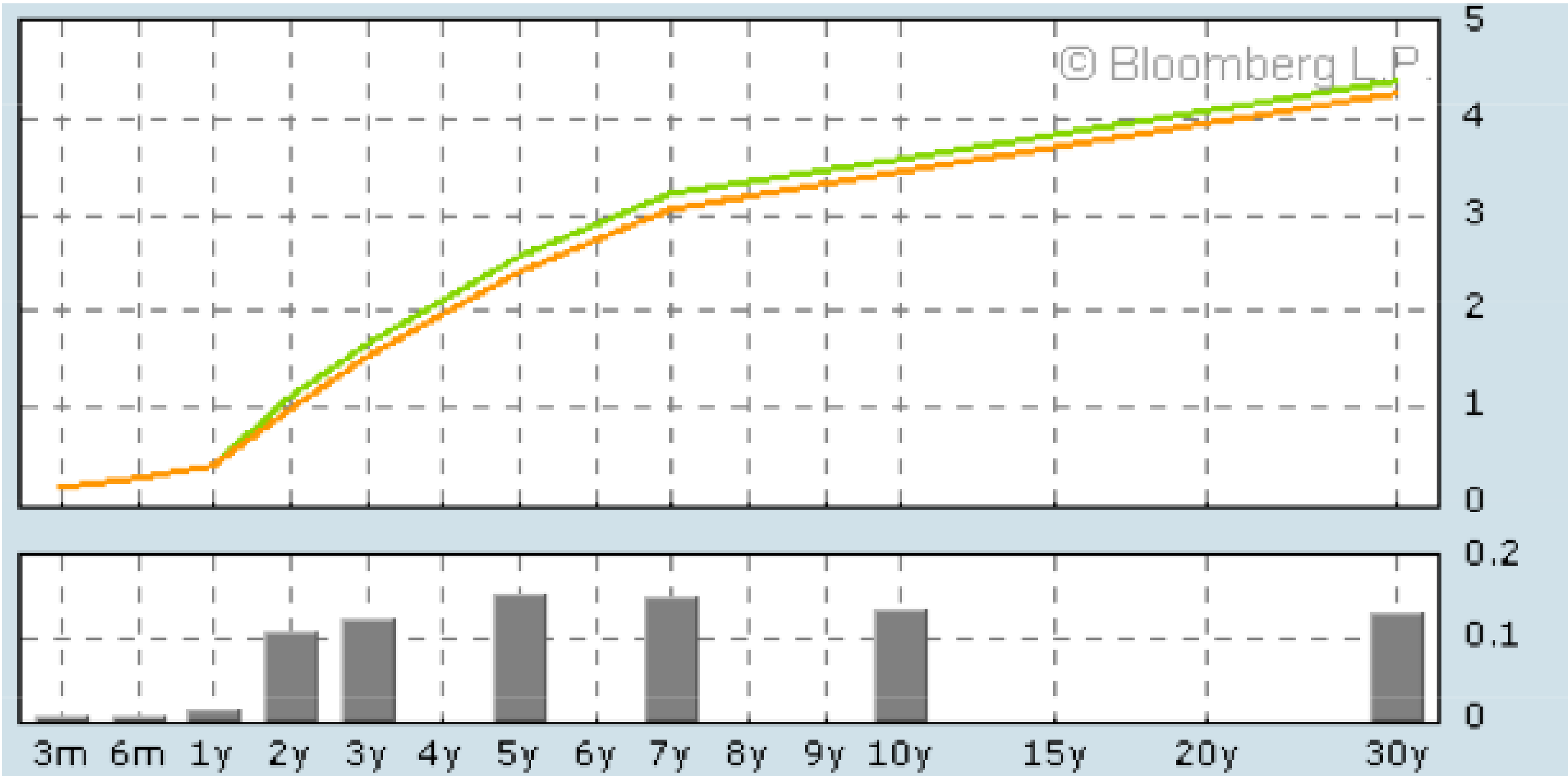
Maturity (year)	1/4	1/2	1	2	5	10	20	25	25.5	(longest)
Interest Rate (%)	3.29	3.61	3.87	3.97	4.06	4.41	4.65	4.57	4.61	

The set of spot interest rates for different maturities

$$\{r_1, r_2, \ldots, r_t, \ldots.\}$$

gives the term structure of interest rates, which refers to the relation between spot rates and their maturities

Sept 5, 2008 (Bloomberg)



http://online.wsj.com/mdc/page/marketsdata.html?mod=topnav_0_0002

History of U.S. term structure of interest rates

http://fixedincome.fidelity.com/fi/FIHistoricalYield

A discount bond (zero coupon bond) with maturity date t is a bond which pays $1 only at t.

Example. STRIPS* are traded at the following prices:

Maturity (year)	1/4	1/2	1	2	5	10	30
Price	0.991	0.983	0.967	0.927	0.797	0.605	0.187

For the 5-year STRIPS, we have

$$0.797 = \frac{1}{(1+r_5)^5} \Rightarrow r_5 = 4.64\%$$

* Separate Trading of Registered Interest and Principal Securities

Let B_t denote the current price (time 0) of a discount bond maturing at t. Then

$$B_t = \frac{1}{(1+r_t)^t} \Leftrightarrow r_t = \frac{1}{B_t^{1/t}} - 1$$

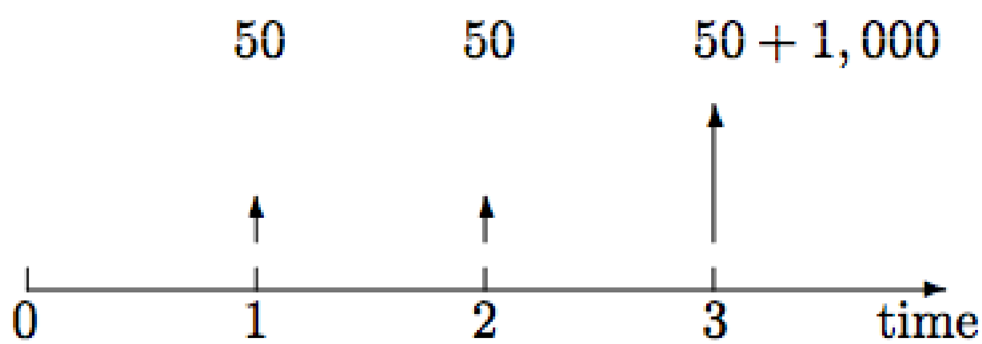
Prices of discount bonds provide information about spot interest rates and vise versa.

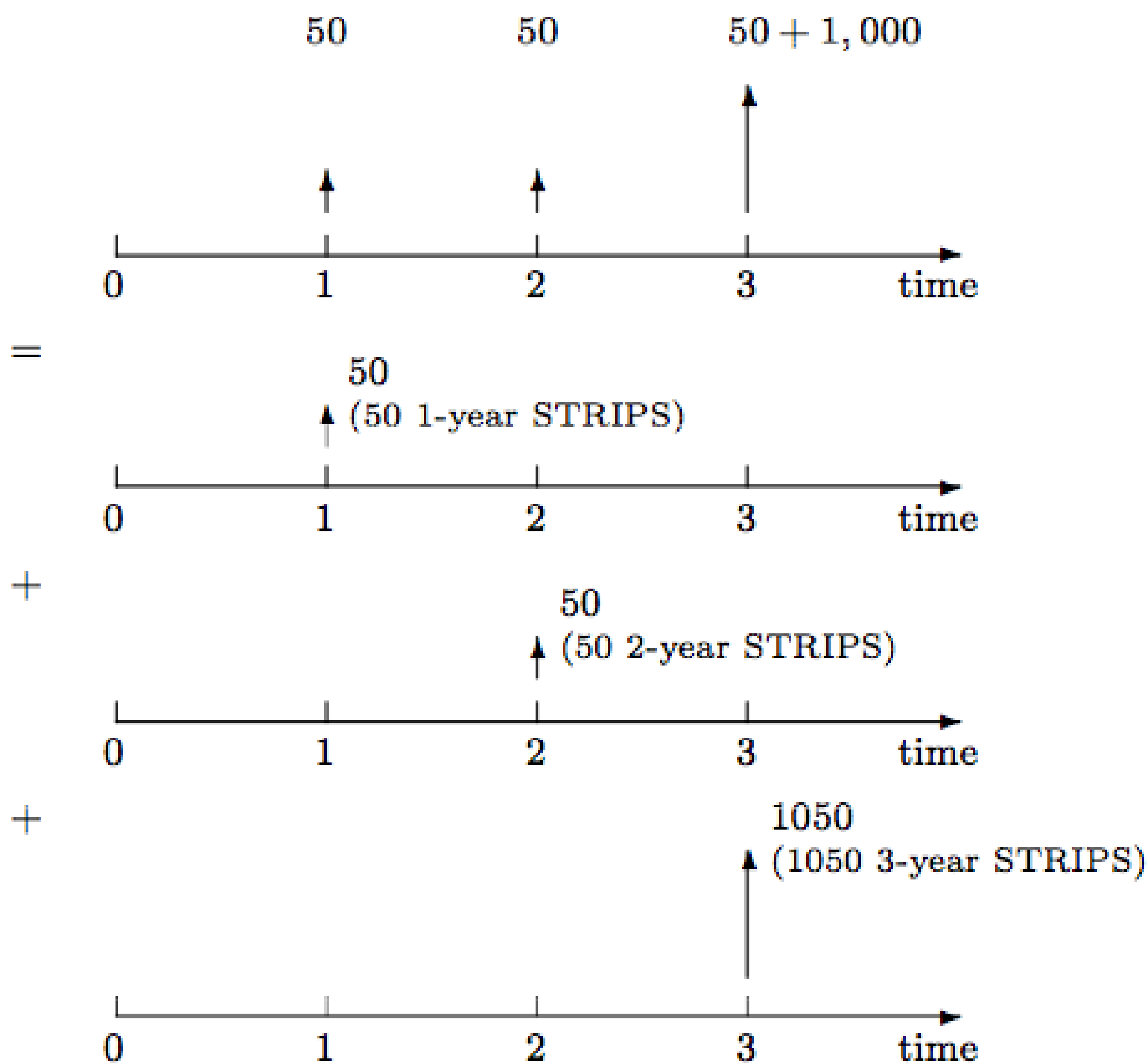
$$\begin{aligned}
B_1 &= \frac{1}{(1+r_1)} \quad \leftrightarrow \quad r_1 \\
B_2 &= \frac{1}{(1+r_2)^2} \quad \leftrightarrow \quad r_2 \\
B_3 &= \frac{1}{(1+r_3)^3} \quad \leftrightarrow \quad r_3 \\
&\vdots \\
B_T &= \frac{1}{(1+r_T)^T} \quad \leftrightarrow \quad r_T
\end{aligned}$$

A coupon bond pays a stream of regular coupon payments and a principal at maturity.

A coupon bond is a portfolio of discount bonds.

Example. A 3-year bond of $1,000 par and 5% annual coupon.




50
50
50 + 1,000
0
1
2
3
time
=
50
(50 1-year STRIPS)
0
1
2
3
time
+
50
(50 2-year STRIPS)
0
1
2
3
time
+
1050
(1050 3-year STRIPS)
0
1
2
3
time

Suppose that the discount bond prices are as follows

t	1	2	3	4	5
B_t	0.952	0.898	0.863	0.807	0.757

What should the price of the coupon bond be?

$$B = 50\ 0.952 + 50\ 0.898 + 1050\ 0.863 = 998.65$$

What if not?

The price of a coupon bond is given by

$$B = \sum_{t=1}^{T} CF_t B_t = \sum_{t=1}^{T} \frac{CF_t}{(1+r_t)^t}$$

Yield-to-maturity of a bond, denoted by y, is given by

$$B = \sum_{t=1}^{T} \frac{CF_t}{(1+y)^t}$$

Given its maturity, the principle and the coupon rate, there is a one to one mapping between the price of a bond and its YTM.

Example. Current 1- and 2-year spot interest rates are 5% and 6%, respectively. The price of a 2-year Treasury coupon bond with par value of $100 and a coupon rate of 6% is

$$B = \frac{6}{1.05} + \frac{106}{1.06^2} = 100.0539$$

Its YTM is 5.9706%:

$$100.0539 = \frac{6}{1+y} + \frac{106}{(1+y)^2}$$

So far, we have focused on spot interest rates: rates for a transaction between today, 0, and a future date, t.

Now, we study forward interest rates: rates for a transaction between two future dates, for instance, t_1 and t_2

For a forward transaction to borrow money in the future:

_ Terms of the transaction are agreed on today, t = 0

_ Loan is received on a future date t_1

_ Repayment of the loan occurs on date t_2

Note:

_ Future spot rates can be different from current corresponding forward rates

Example. As the CFO of a U.S. multinational, you expect to repatriate \$10 M from a foreign subsidiary in 1 year, which will be used to pay dividends 1 year later. Not knowing the interest rates in 1

year, you would like to lock into a lending rate one year from now for a period of one year. What should you do?

The current interest rates are

time to maturity t (years)	1	2
spot interest rate r_t	0.05	0.07

Strategy:

_ Borrow $9.524M now for one year at 5%

_ Invest the proceeds $9.524M for two years at 7%

Outcome (in million dollars):

Year	0	1	2
1-yr borrowing	9.524	−10.000	0
2-yr lending	−9.524	0	10.904
Repatriation	0	10.000	0
Net	0	0	10.904

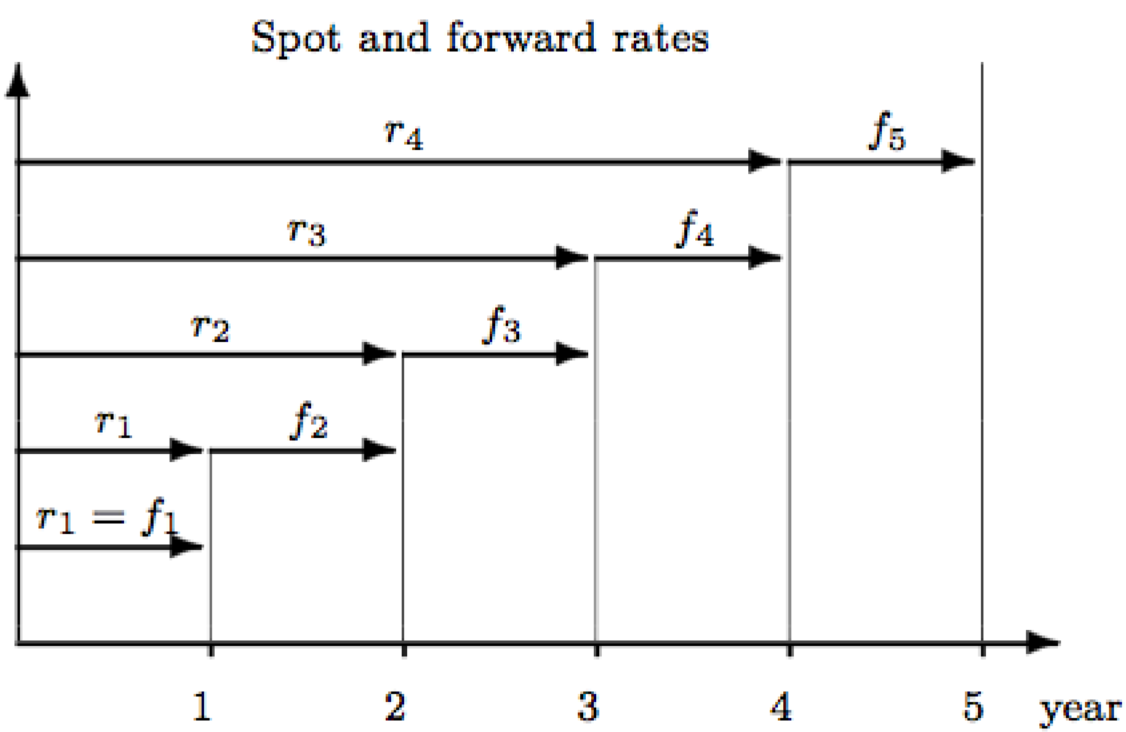
The locked-in 1-year lending rate 1 year from now is 9.04%.

The forward interest rate between time t-1 and t is

$$(1+r_t)^t = (1+r_{t-1})^{t-1}(1+f_t)$$

or

$$f_t = \frac{(1+r_t)^t}{(1+r_{t-1})^{t-1}} - 1$$



Example. Suppose that discount bond prices are as follows:

t	1	2	3	4
B_t	0.9524	0.8900	0.8278	0.7629
r_t	0.05	0.06	0.065	0.07

A customer wants a forward contract to borrow $20M three years from now for one year. Can you (a bank) quote a rate?

$$f_4 = 8.51\%$$

What should you do today to lock-in these cash flows?

1. Buy 20,000,000 of 3 year discount bonds, costing
$$\$20m0.8278 = \$16.556m$$
2. Finance this by selling 4 year discount bonds of amount
$$\$16.556m / 0.7629 = \$21.701m$$

Cash flows from this strategy (in million dollars):

Year	0	1–2	3	4
Purchase of 3-year bonds	-16.556	0	20.000	0
Sale of 4-year bonds	16.556	0	0	-21.701
Total	0	0	20.000	-21.701

The interest for this future investment is given by:

$$\frac{21,701,403}{20,000,000} - 1 = 8.51\%$$

What determines the term structure of interest rates?

1. Expected future spot rates
2. Risk of long bonds

Models of interest rates:

_ Expectations Hypothesis

_ Liquidity Preference

_ Dynamic Models (Vasicek, Cox-Ingersoll and Ross, …)

Expectations Hypothesis: Forward rates predict future spot rates

$$f_t = E[r_1(t)]$$

Implications:

_ The slope of the term structure reflects the market's expectations of future short-term interest rates

Liquidity Preference Hypothesis: Investors regard long bonds as riskier than short bonds

$$f_t = E[r_1(t)] + LP$$

Implications:

_ Long bonds on average receive higher returns than short bonds

_ Forward rate on average ``over-predict'' future short-term rates.

_ Term structure reflects

a) expectations of future interest rates, and

b) risk premium demanded by investors in long bonds

Consider a situation where the short rate is 5%.

Expectations hypothesis:
Suppose that the expected short rate for the following year is 6%.
What is the price of a 2-year zero?

$$(1+r_2)^2 = (1+r_1)(1+E[r_2]) = 1.05\ 1.06$$

$$B_2 = \$1000 / (1+r_2)^2 = \$898.47$$

Suppose that an investor wants to invest for 1 year.
Strategy A: invest into the 1-year zero.
Strategy B: invest into the 2-year zero, and sell it after 1 year.

What is the expected return of strategy B?

If the future short rate is 6%, the future bond price is

$1000/1.06=$943.4.

In this case, the return is ($943.4-$898.47)/$898.47=5%.

So, is the investor indifferent between strategies A and B?

What if the investor requires a discount to hold the 2-year bond? Suppose that the bond trades at $890.

$$B_2 = \$1000 / (1+r_2)^2 = \$890$$

$$r_2 = 6\%$$

$$f_2 = \frac{1.06^2}{1.05} = 7\% = E[r_1] + 1\%$$

The liquidity preference hypothesis is based on the idea that investors require a risk-premium to invest in long-term bonds.

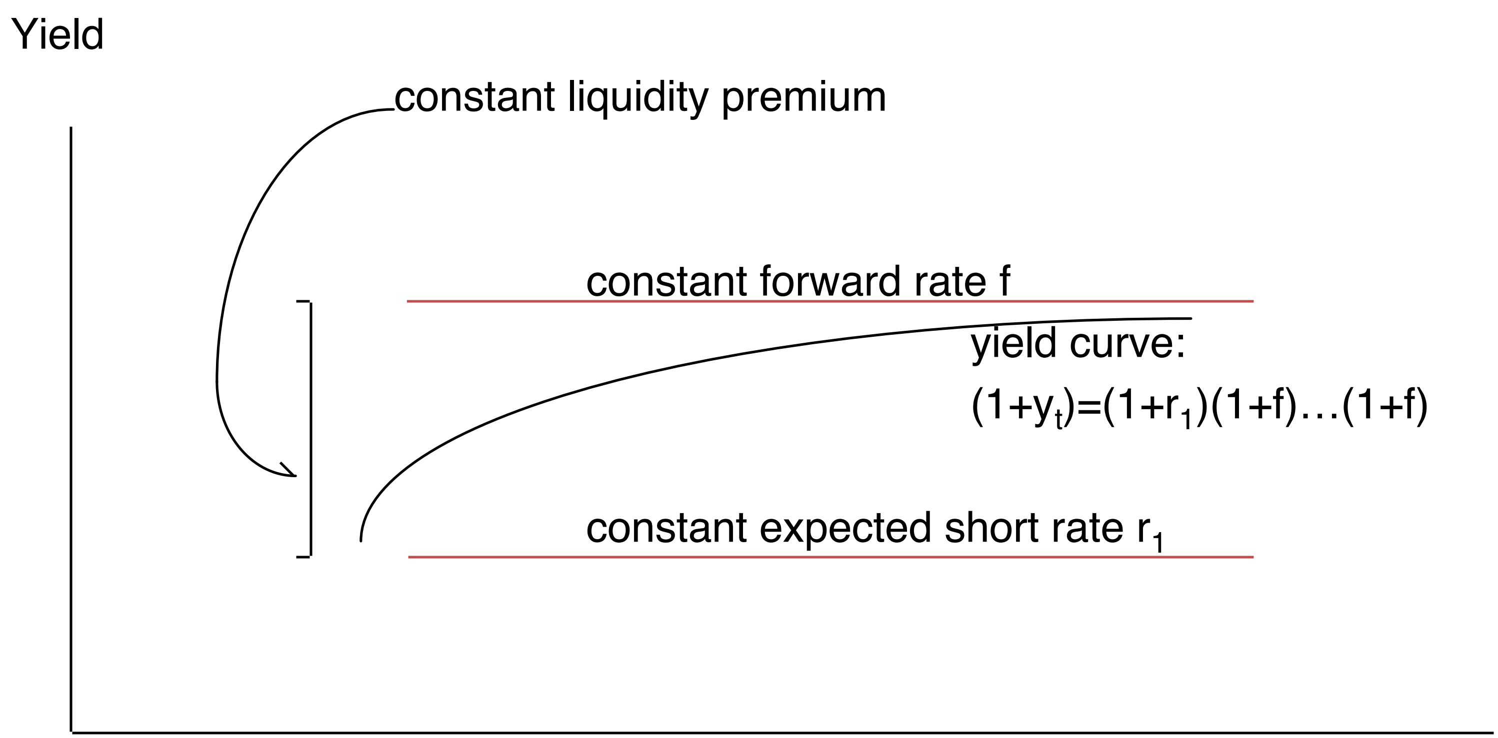
Why?

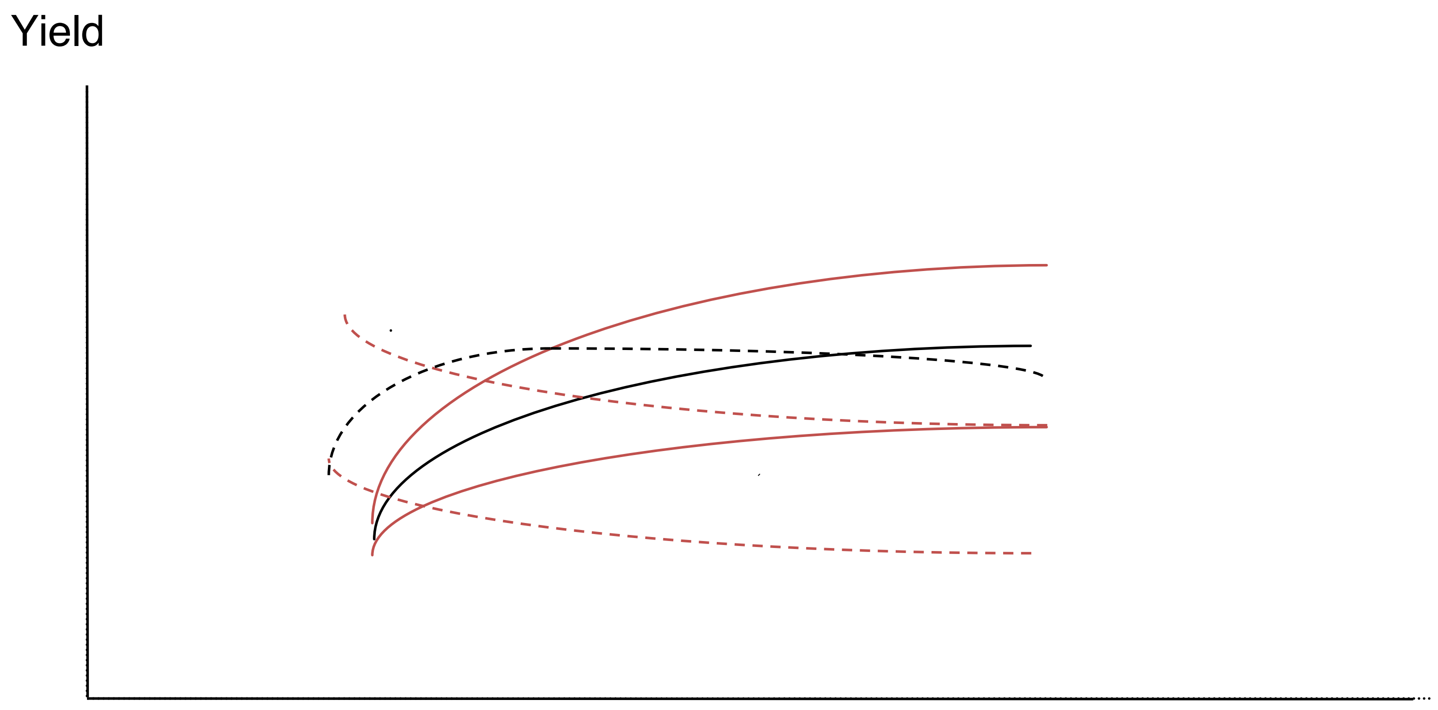
Consider an investor who wants to invest for a 2 year period.

Such an investor bears roll-over risk if she invests in 1 year bonds. To hold 1 year bonds, the investor will require a risk premium...

$$\Rightarrow f_t < E[r_1(t)]$$

Forward rates contain information about
1. expected future short rates and
2. risk premia.


Yield
constant liquidity premium
constant forward rate f
yield curve:
(1+y_t)=(1+r_1)(1+f)…(1+f)
constant expected short rate r_1


Yield

Bond yields

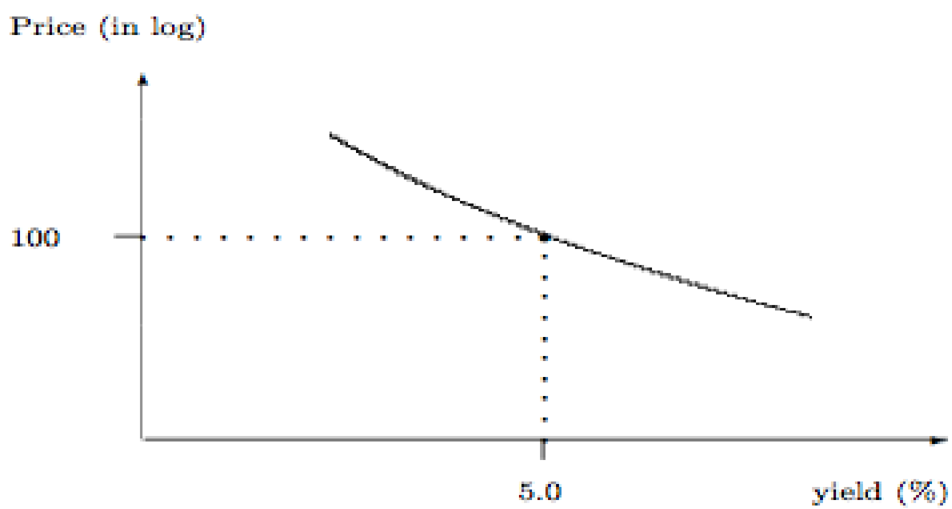
Prices in the bond market contain information about the time-value of money:

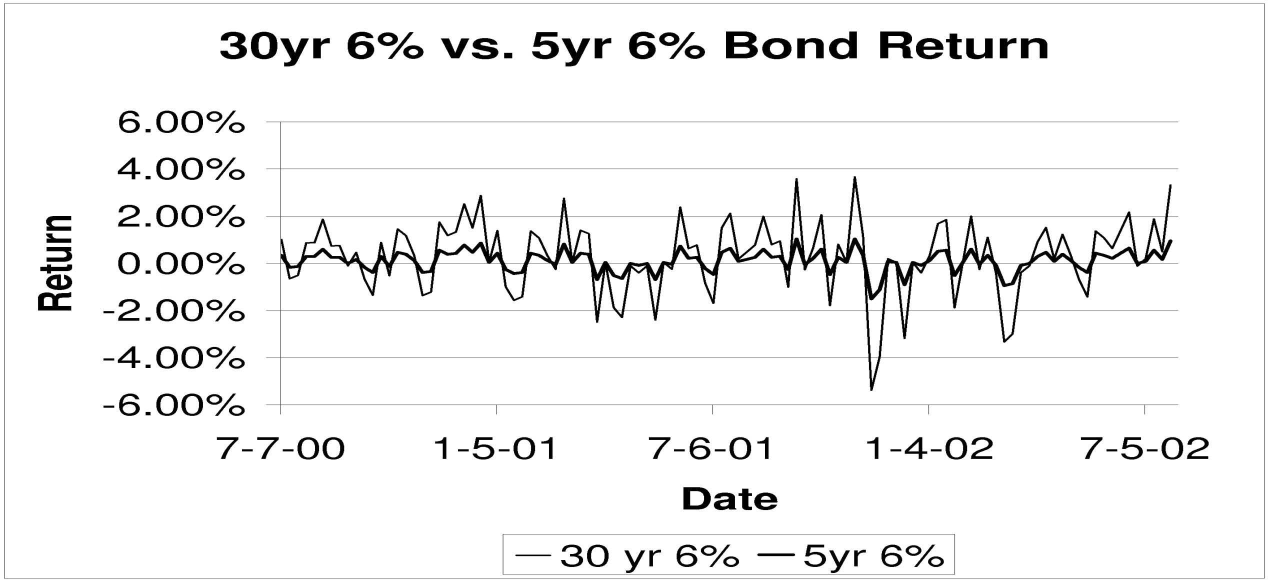
- Spot interest rates
- Forward interest rates

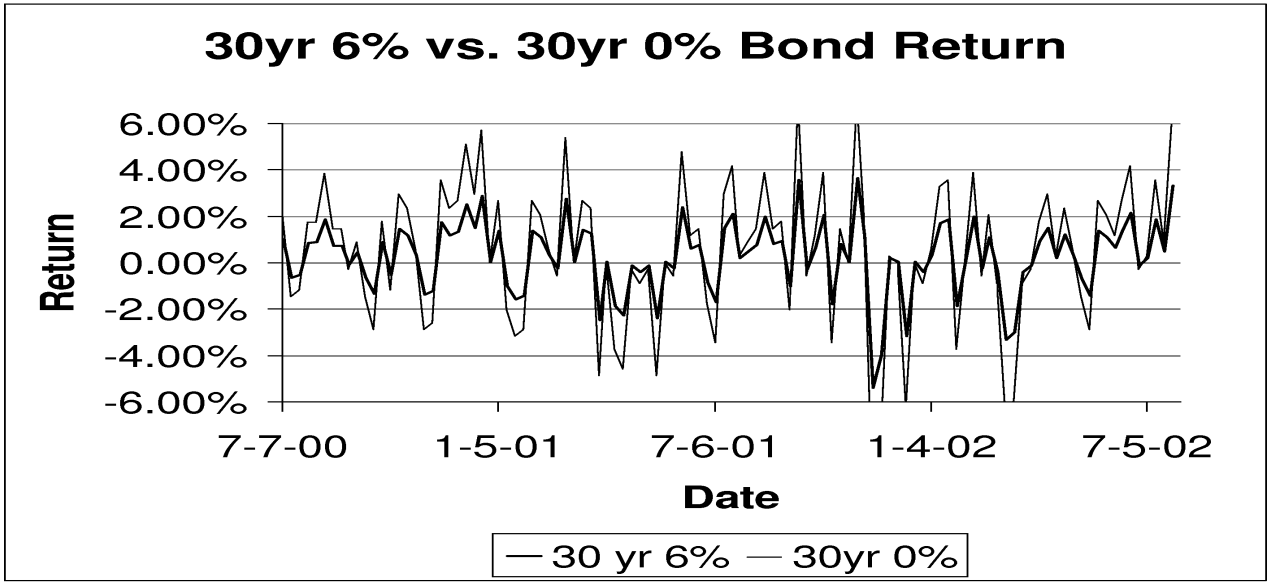
The term structure of interest rates

Next time: managing bond portfolios.

As interest rates change (stochastically) over time, bond prices also change. The value of a bond is subject to interest rate risk.




30yr 6% vs. 5yr 6% Bond Return
6.00%
4.00%
2.00%
0.00%
-2.00%
-4.00%
-6.00%
Return
7-7-00
1-5-01
7-6-01
1-4-02
7-5-02
Date
30 yr 6%
5yr 6%


30yr 6% vs. 30yr 0% Bond Return
6.00%
4.00%
2.00%
0.00%
-2.00%
-4.00%
-6.00%
Return
7-7-00
1-5-01
7-6-01
1-4-02
7-5-02
Date
30 yr 6%
30yr 0%

Duration and Modified Duration (assume a flat term structure at $r_t = y$)

Macaulay duration is the weighted average term to maturity

$$D = \sum_{t=1}^{T} \frac{PV[CF_t]}{B} t = \frac{1}{B} \sum_{t=1}^{T} \frac{CF_t}{(1+y)^t} t$$

A bond's interest rate risk can be measured by its relative price change with respect to a change in yield:

$$MD = -\frac{1}{B}\frac{\Delta B}{\Delta y} = \frac{D}{1+y}$$

This is called a bond's modified duration or volatility.

Example. Consider a 4-year T-note with face value \$100 and 7% coupon, selling at \$103.50, yielding 6%.

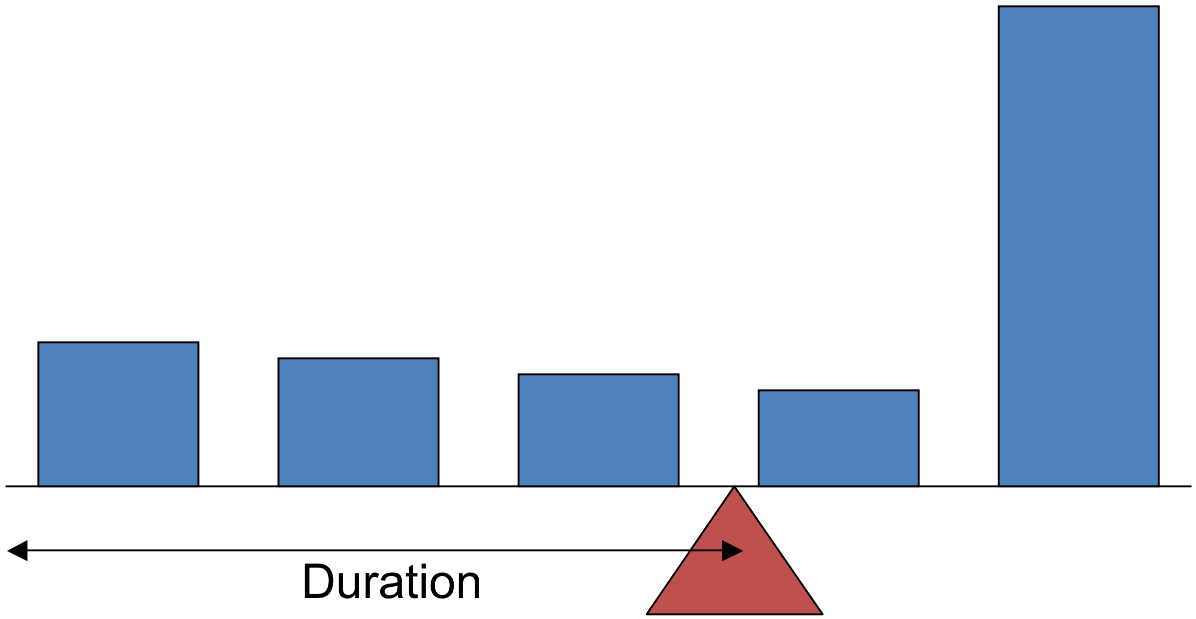
For T-notes, coupons are paid semi-annually. Time count in units of 6 months.

t	CF	PV	$t \cdot \text{PV(CF)}$
1	3.5	3.40	3.40
2	3.5	3.30	6.60
3	3.5	3.20	9.60
4	3.5	3.11	12.44
5	3.5	3.02	15 10
6	3.5	2.93	17.59
7	3.5	2.85	19.92
8	103.5	81.70	653.63
		103.50	738.28

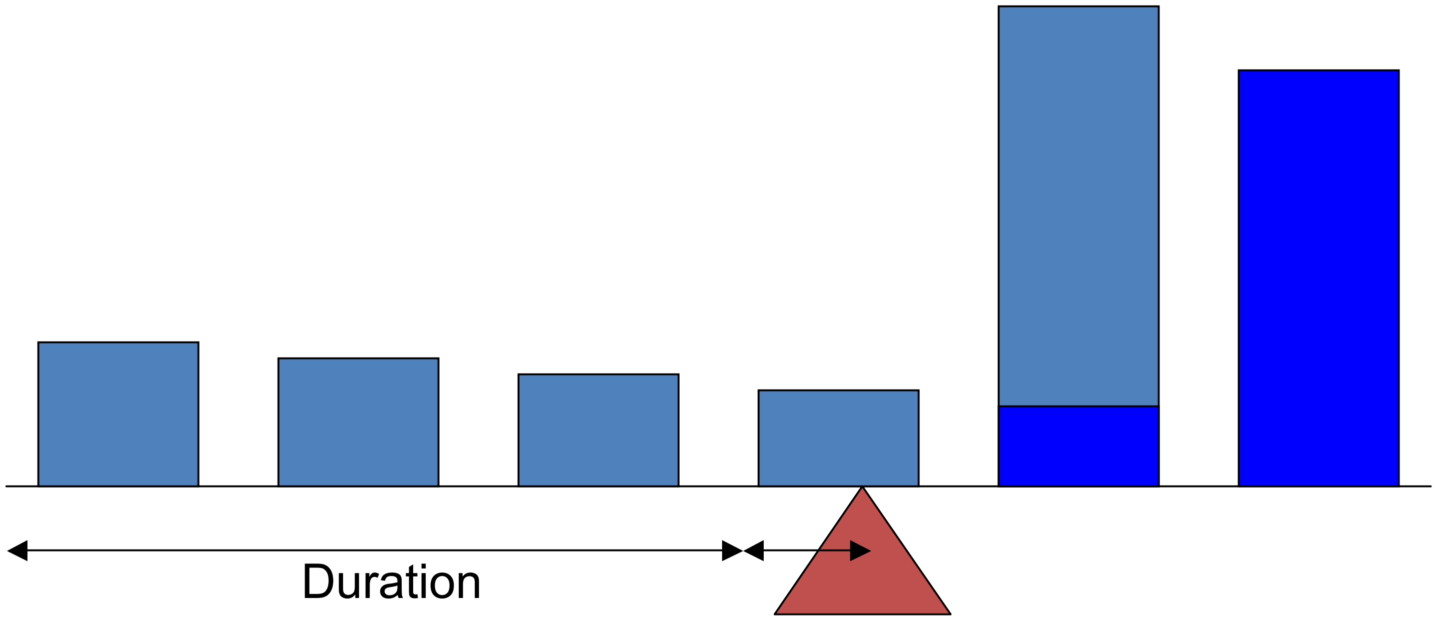
Duration (in 1/2 year units): $D = 738.28 / 103.50 = 7.13$

Modified duration (volatility): $MD = 7.13 / 1.03 = 6.92$

If the semi-annual yield moves up by 0.1%, the bond price decreases roughly by 0.692%.

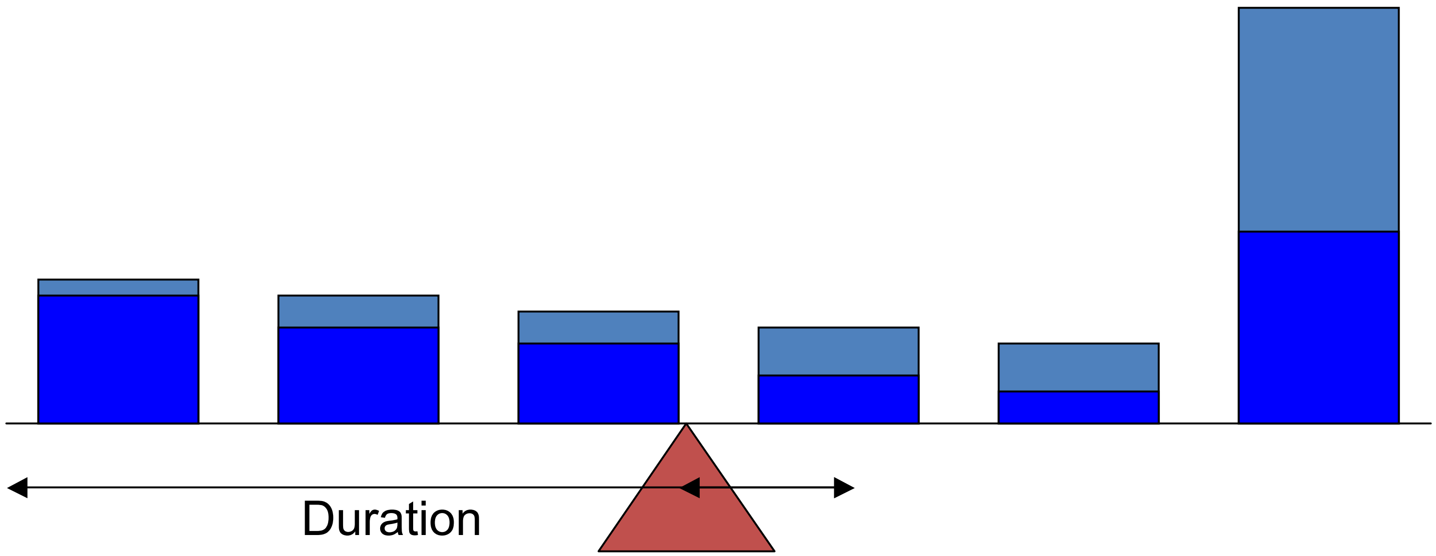


How does duration depend on bond maturity?
What happens if the maturity increases?



Par and premium bonds: duration increases.
Discount bonds: duration can decrease!

How does the duration depend on the bond yield?
What happens if the yield increases?



Negative relation between the duration and the bond yield

Example. (Continued) 4-year T-note with 7% coupon and 6% flat yield curve.

- Duration is D=7.13
- Volatility is MD = 6.92

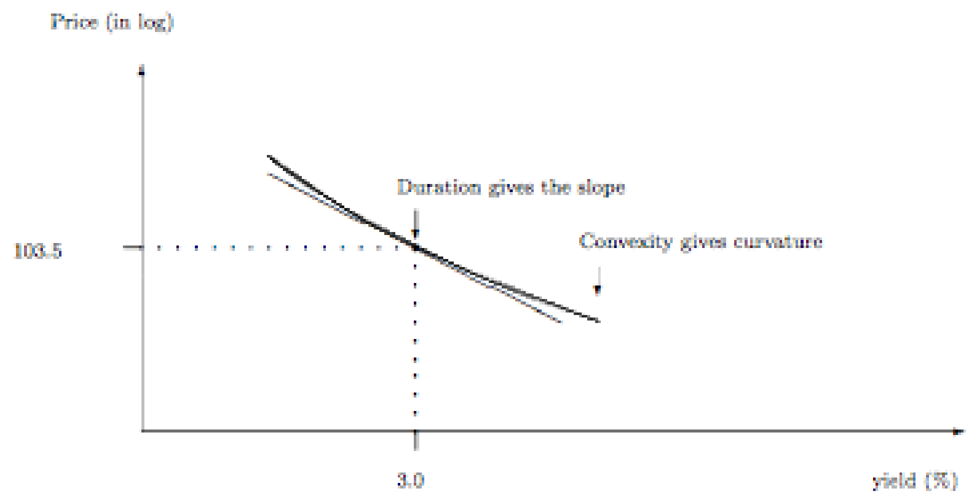
As the yield changes, the bond price also changes:

Yield	Price	Using MD	Difference
0.040	96.63	96.35	0.29
0.035	100.00	99.93	0.07
0.031	102.79	102.79	0.00
0.030	103.50	-	-
0.029	104.23	104.23	0.00
0.025	107.17	107.09	0.08
0.020	110.98	110.67	0.32

0.1%

0.692%

- For small yield changes, pricing by MD is accurate
- For large yield changes, pricing by MD is inaccurate



Bond price is not a linear function of the yield. For large yield changes, the effect of curvature (i.e., nonlinearity) becomes important.

$$\Delta B = -MD\Delta y + CX(\Delta y)^2$$

Convexity, CX, measures the curvature of the bond price (per $) as a function of the yield:

$$CX = \frac{1}{2}\frac{1}{B}\frac{\partial^2 B}{\partial y^2}$$

Most bonds give nominal payoffs. In the presence of inflation risk, real payoffs are risky even when nominal payoffs are safe.

Example. Suppose that inflation next year is uncertain ex ante, with equally possible rate of 10%, 8% and 6%. The real interest rate is 2%.

The 1-year nominal interest rate will be (roughly) 10%.

Consider the return from investing in a 1-year Treasury security:

Year 0 value	Inflation rate (%)	Year 1 nom. payoff	Year 1 real payoff
1000	0.10	1100	1000
1000	0.08	1100	1019
1000	0.06	1100	1038

Fixed-income securities have promised payoffs of fixed amount at fixed times. Excluding government bonds, other fixed-income securities, such as corporate bonds, carry the risk of failing to pay off as promised.

Default risk (credit risk) refers to the risk that a debt issuer fails to make the promised payments (interest or principal).

Bond ratings by rating agencies (e.g., Moody's and S\&P) provide indications of the likelihood of default by each issuer.

Description	Moody's	S&P
Gilt-edge	Aaa	AAA
Very high grade	Aa	AA
Upper medium grade	A	A
Lower medium grade	Baa	BBB
Low grade	Ba	BB

_ Investment grade: Aaa -- Baa by Moody's or AAA -- BBB by S&P
_ Speculative (junk): Ba and below by Moody's or BB and below by S&P

Example. Suppose all bonds have par value $1,000 and

- 10-year Treasury strip is selling at $463.19, yielding 8%
- 10-year zero issued by XYZ Inc. is selling at $321.97
- Expected payoff from XYZ's 10-year zero is $762.22

The XYZ bond:

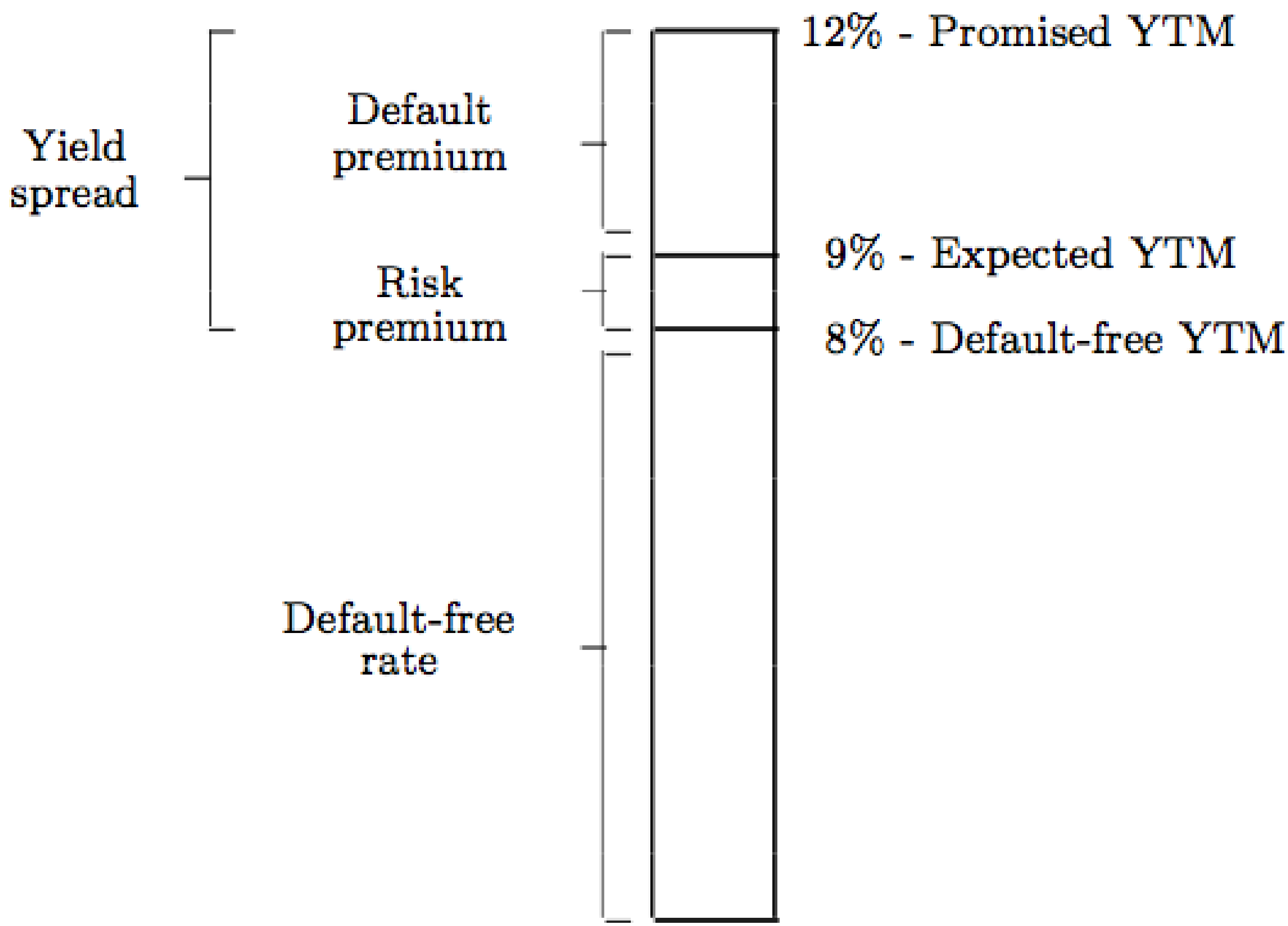
$$\text{Promised YTM} = (1000 / 321.97)^{1/10} - 1 = 12\%$$

$$\text{Expected YTM} = (762.22 / 321.97)^{1/10} - 1 = 9\%$$

$$\text{Default premium} = 12\% - 9\% = 3\%$$

$$\text{Risk premium} = 9\% - 8\% = 1\%$$

- Promised YTM: the yield if default does not occur
- Expected YTM: the probability-weighted average of all possible yields
- Default premium: the difference between promised yield and expected yield
- Bond risk premium: the difference between the expected yield on a risky bond and the yield on a risk-free bond of similar maturity and coupon rate


12% - Promised YTM
Yield spread
Default premium
9% - Expected YTM
Risk premium
8% - Default-free YTM
Default-free rate

Key concepts

_ Fixed-income securities
_ Overview of fixed-income markets
_ Term structure of interest rates
_ Discount bonds and coupon bonds
_ Forward interest rates
_ Interest rate risk
_ Inflation risk
_ Default risk