15.401 Finance Theory I

Alex Stomper

MIT Sloan School of Management

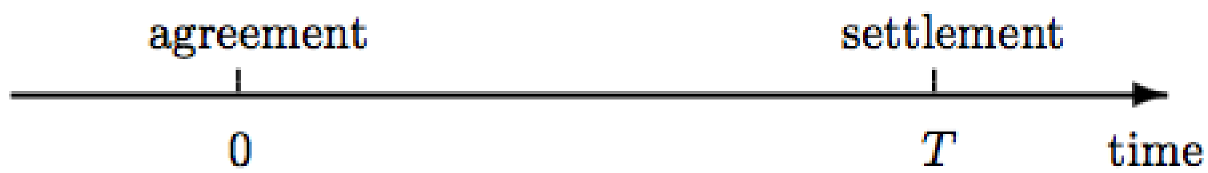
Lecture 5: Forwards and Futures

- Forward contracts
- Futures contracts
- Mark to market
- Forward and futures prices
- Commodity futures
- Financial futures
- Hedging with forwards/futures

Readings:

- Brealey, Myers and Allen, Chapter 27.3
- Bodie, Kane and Markus, Chapters 22, 23.1 - 23.2, 23.6

A forward contract is a commitment to buy (sell) at a future date a given amount of a commodity or an asset at a price agreed on today.



_ The price fixed now for future exchange is the forward price.

_ The buyer obtains a ``long position" in the asset/commodity.

Example. Tofu manufacturer needs 100,000 bushels of soybeans in 3 months. Current price of soybeans is $12.50/bu but may go up

_ Wants to make sure that 100,000 bushels will be available

_ Enter 3-month forward contract for 100,000 bushels of soybeans at $1350/bu

_ Long side buy 100,000 bushels from short side at $13.50/bu in 3 months

Features of forward contracts:

_ Traded over the counter (not on exchanges)

_ Custom tailored

_ No money changes hands until maturity

Advantages of forward contracts:

_ Full flexibility

Disadvantages of forward contracts

_ Illiquidity

_ Non-trivial counter party risk

A futures contract is an exchange-traded, standardized, forward-like contract that is marked to the market daily. Futures contract can be used to establish a long (or short) position in the underlying commodity/asset.

Features of futures contracts:

- Standardized contracts:
  - underlying commodity or asset
  - quantity
  - maturity
- Traded on exchanges
- Guaranteed by the clearing house --- little counter-party risk
- Gains/losses settled daily---marked to market
- Margin account required as collateral to cover losses

Example. NYMEX crude oil (light sweet) futures with delivery in Dec. 2008 were traded at a price of $101.18/barrel on Sept 12, 2008.

_ Each contract is for 1,000 barrels

_ Tick size: $0.01 per barrel, $10 per contract

_ Initial margin: $4,050

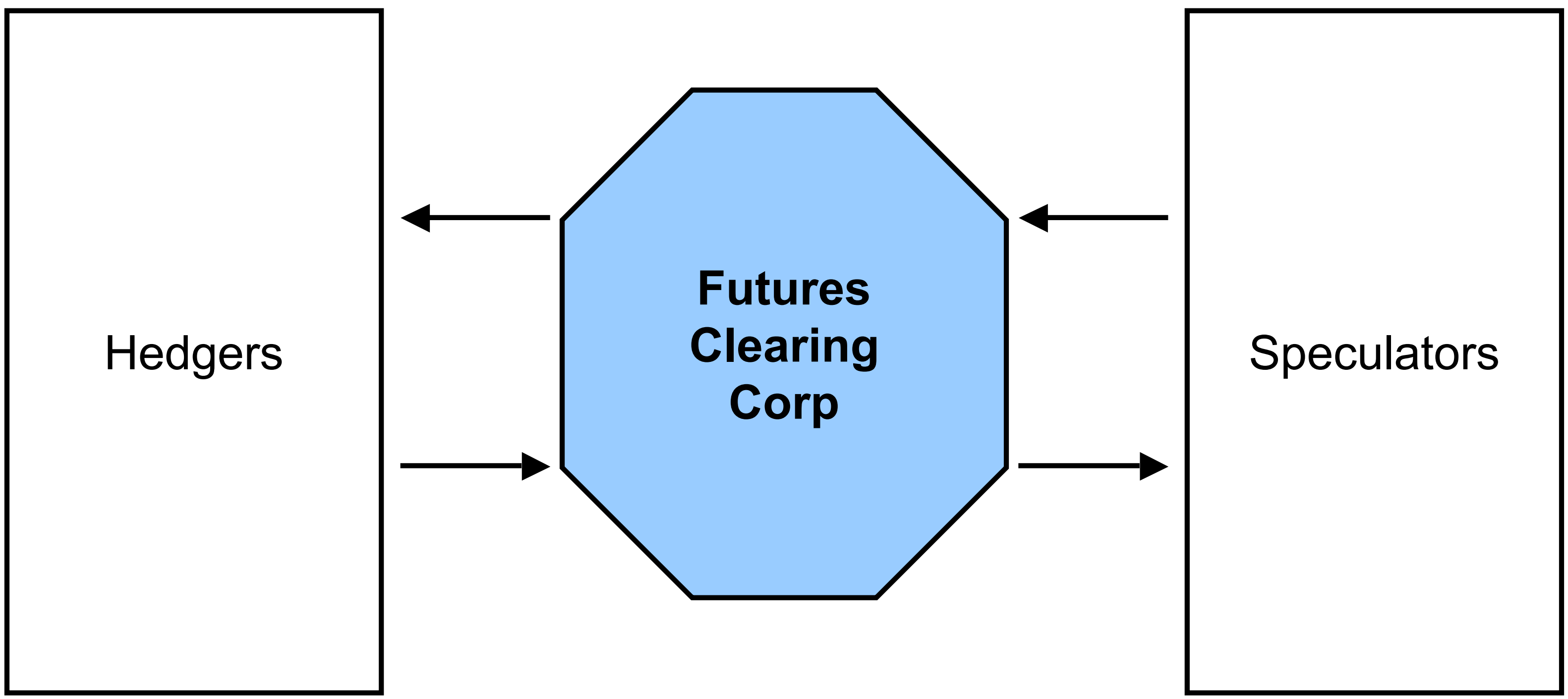
_ Maintenance margin: $3,000

_ No cash changes hands today (contract price is $0)

_ Buyer has a "long" position (wins if prices go up)

_ Seller has a "short" position (wins if prices go down)

Futures clearing house reduces counter party risk and improves liquidity



Example. Yesterday, you bought 10 December live-cattle contracts on the CME, at a price of $0.7455/lb.

_ Contract size 40,000 lb

_ Agreed to buy 400,000 pounds of live cattle in December

_ Value of position yesterday:

$$(0.7455)(10)(40,000) = \$298,200$$

_ No money changed hands

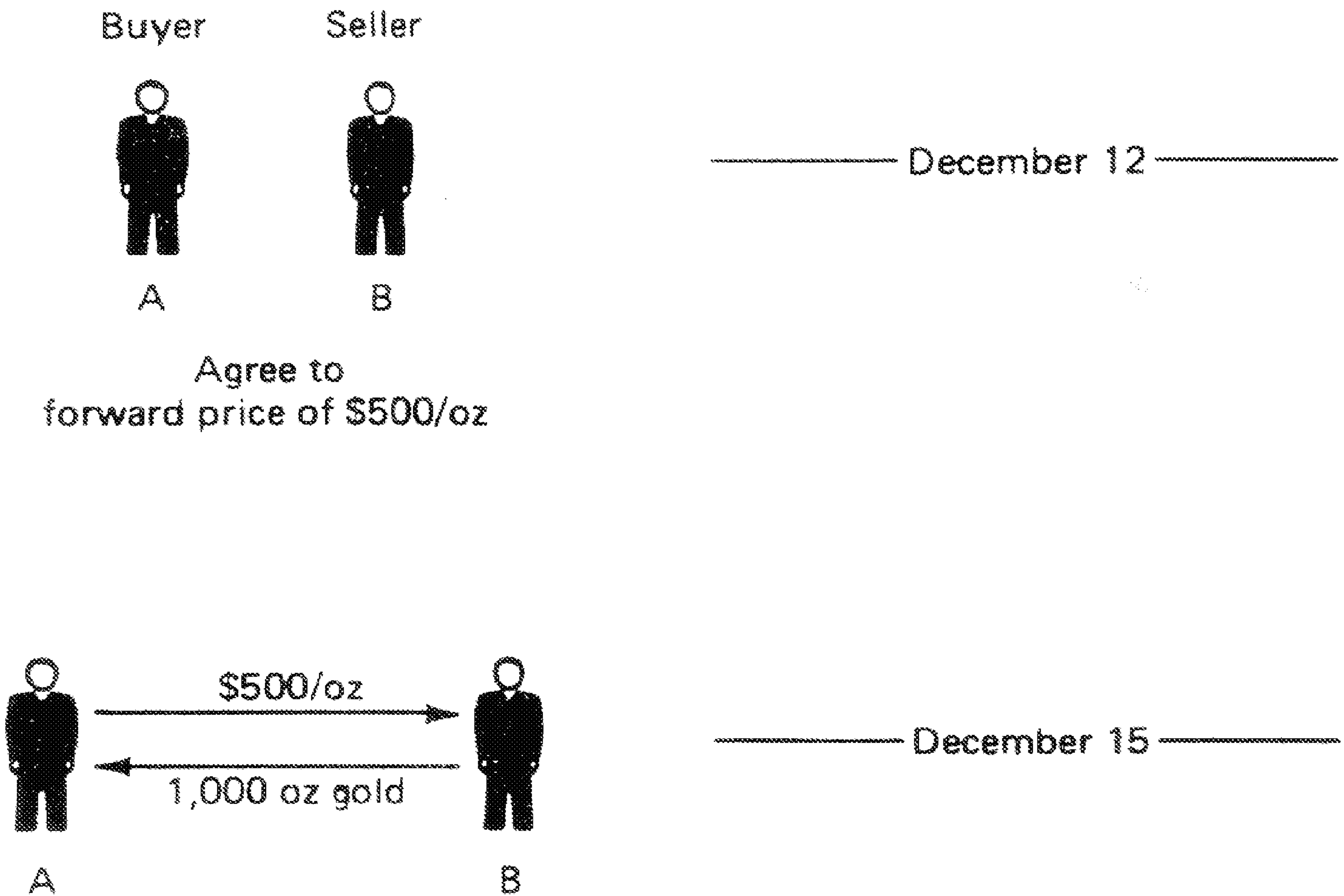
_ Initial margin required (5%–20% of contract value)

Today, the futures price closes at $0.7435/lb, 0.20 cents lower. The value of your position is

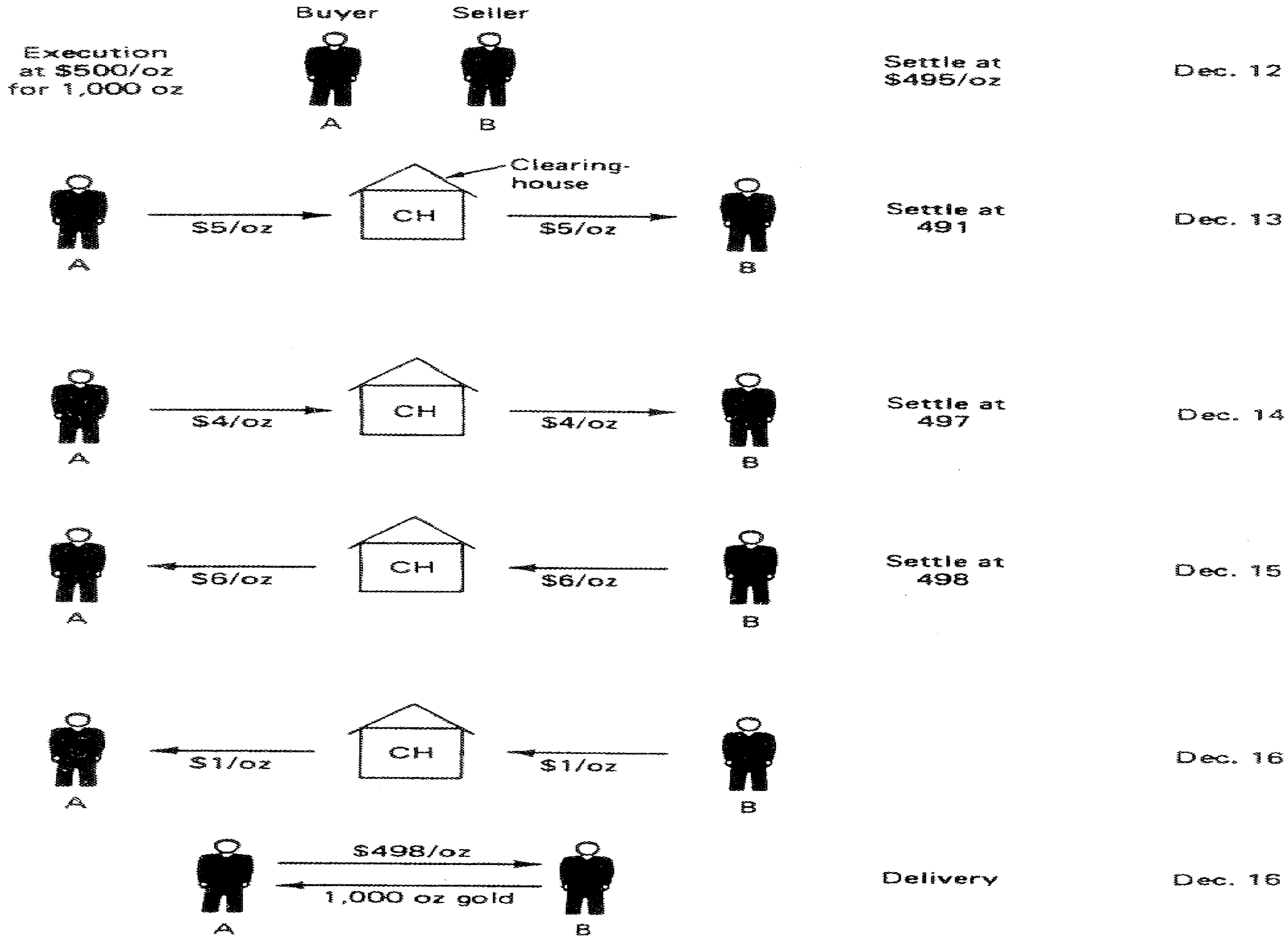
$$(0.7435)(10)(40,000) = \$297,400$$

which yields a loss of $800.

A forward contract

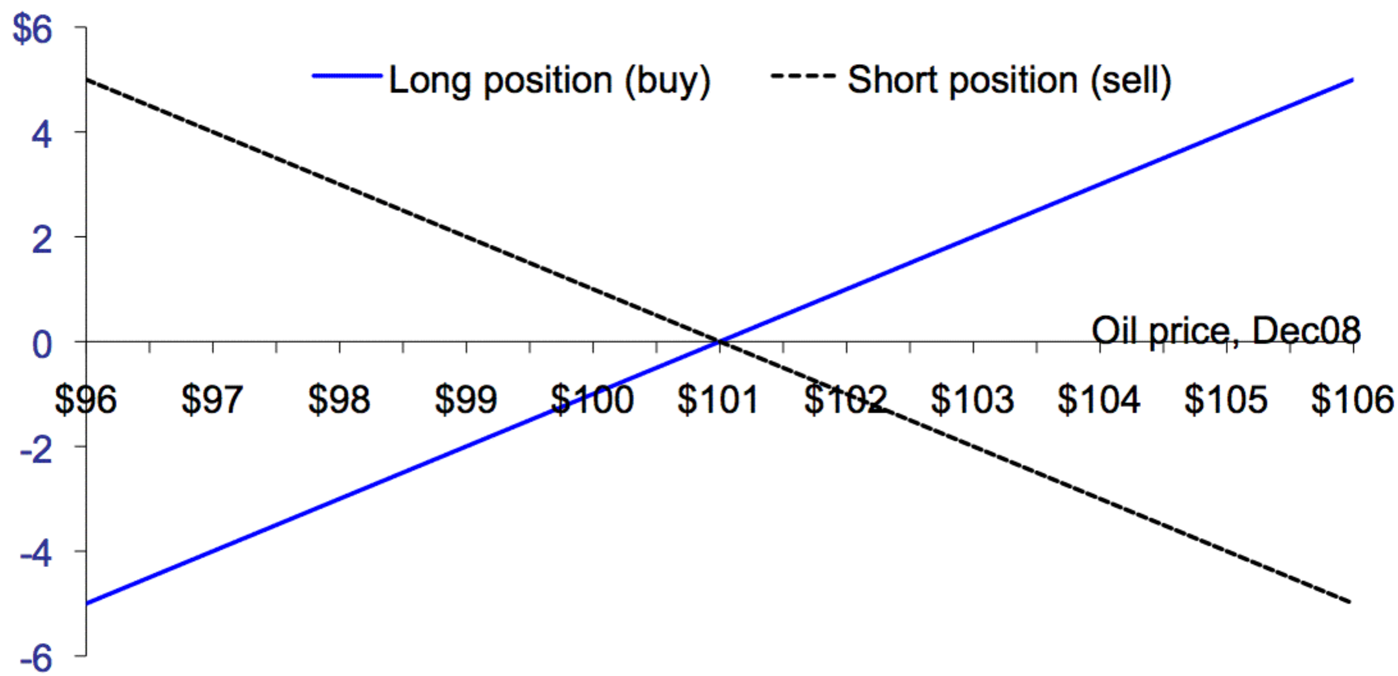


A futures contract



- Forward and futures are derivative securities
  - Payoffs tied to prices of underlying assets/commodities
  - Zero net supply (aggregate positions add to zero)
- Payoffs are linear in underlying asset/commodity price: S(T) - F

Payoff Diagram



What Determines Forward and Futures Prices?

_ Forward/futures prices ultimately linked to spot prices

_ Notation:

Contract	Spot now	Spot at T	Forward	Futures
Price	S	$S(T)$	F_T	H_T

_ Ignore differences between forward and futures prices for now

$$F_T \quad \approx \quad H_T$$

_ Two ways to buy the underlying asset for date-T delivery

1. Buy a forward or futures contract with maturity date T
2. Buy the underlying asset and store it until T

Date	Forward Contract	Outright Asset Purchase
0	▪ Pay \$0 for contract with forward price F_T	▪ Borrow S ▪ Pay S for asset
T	▪ Pay F_T ▪ Own asset	▪ Pay back $S(1+r)^T$ ▪ Pay storage costs (if any) ▪ Deduce "convenience yield" (if any) ▪ Own asset
Total cost at T	**\$$F_T$**	**\$$S(1+r)^T$ + net storage costs**

By "No Free Lunch" Principle:

$$
\begin{aligned}
F_T \approx H_T &= (1+r)^T S + \mathrm{FV}_T(\text{net storage costs}) \\
&= (1+r)^T S - \mathrm{FV}_T(\text{net convenience yield})
\end{aligned}
$$

Gold

- Held for long-run investments
- Easy to store---negligible cost of storage
- No dividends or benefits

Two ways to buy gold at T :

- Buy now for S and hold until T
- Buy forward, pay F and take delivery at T

No-arbitrage requires that $F \approx H = S_0(1+r)^T$

Example. Gold quotes on 2007.08.30 are

- Spot price $665.57/oz
- 2007 February futures (CMX) $681.20/oz

The implied 6-month interest rate is r = 4.75%.

2009.09.02: spot $ 976.6, 2010 Feb futures $ 979.8, implied 5m r=0.79%

Oil

_ Not held for long-term investment (unlike gold), but future use

_ Costly to store

_ Additional benefits (convenience yield) for holding physical commodity (over holding futures)

Let the percentage holding cost be c and convenience yield be y.

$$\begin{aligned} F \approx H &= S\left[1+r-(y-c)\right]^T \\ &= S(1+r-\widehat{y})^T \end{aligned}$$

where $\widehat{y} = y - c$ is the net convenience yield.

Example. Prices on 2009.09.02 are

_ Spot oil price 68.50/barrel (light sweet)

_ Dec 09 oil futures price 69.33/barrel (NYMEX)

_ 3-month interest rate 0.33% (LIBOR)

Annualized net convenience yield: $\widehat{y} = -4.61\%$

For commodity futures:

1. Contango: Futures prices increase with maturity

2. Backwardation: Futures prices decrease with maturity

Another definition is one that adjusts for the time-value of money:

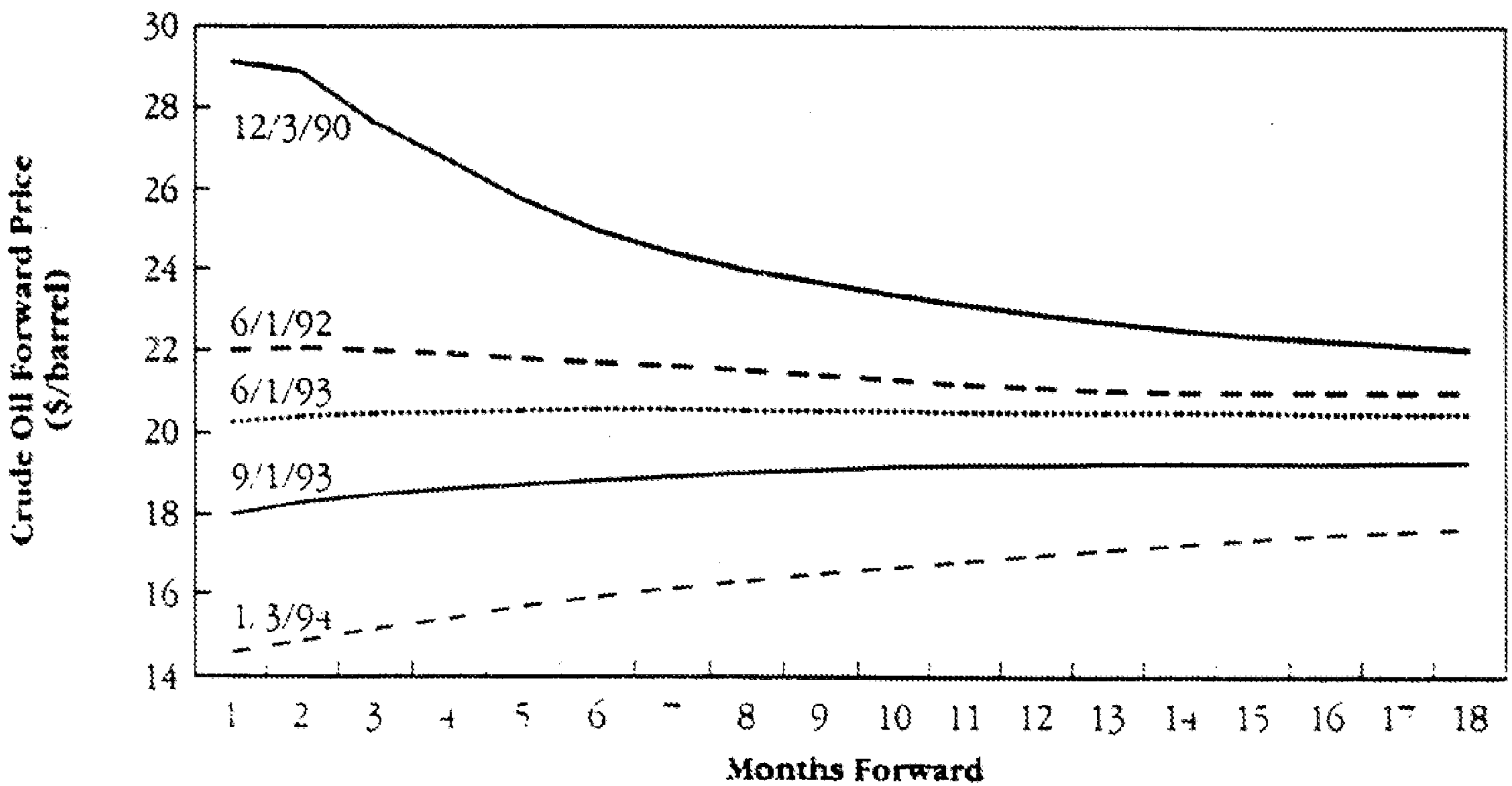
1. Contango: $H > S\ (1+r)^T$

2. Backwardation: $H < S\ (1+r)^T$

Backwardation occurs if net convenience yield exceeds interest rate:

$$\widehat{y} - r = y - c - r > 0$$

Crude oil forward price curves for selected dates



For financial futures, the underlying are financial assets.

_ No cost to store

_ Dividend or interest on the underlying asset

Examples:

- Stock index futures, e.g., S&P, Nikkei,…
  - Underlying: baskets of stocks
- Interest rate futures
  - Underlying: fixed income instruments (T-bonds,…)
- Currency futures

Let the dividend (interest) yield be d. Then the following relation between the forward/futures price and spot price holds:

$$F = S_0(1+r-d)^T \approx H$$

Stock index futures

_ Futures settled in cash (no delivery)

_ Underlying asset (basket of stocks) pay dividends

Example. Prices on 2009.09.02 are

_ S&P 500 closed at 994.75

_ S&P futures maturing in December closed at 989.70

_ 3-month interest rate 0.33%

$$\begin{aligned} d &= \left[1 + r - (F/S)^{12/3}\right] \\ &= \left[1 + (0.0033) - (989.70/994.75)^{4}\right] = 2.35\% \end{aligned}$$

The annual dividend yield is: d =2.35%.

- Since the underlying asset is a portfolio in the case of stock index futures, trading in the futures market is easier than trading in cash market when trading portfolios
- Thus, futures prices may react more quickly to macro-economic news than the index itself
- Index futures are very useful to market makers, investment bankers, stock portfolio managers:
  - hedging market risk in block purchases & underwriting
  - creating synthetic index fund
  - Implementing portfolio insurance

Example. You have $1 million to invest in the stock market and you have decided to invest in a diversified portfolio. S&P seems a good candidate. How would you do this?

a) One approach is to buy S&P in the cash market:
   - Buy the 500 stocks
   - Weights proportional to their market capitalization

b) Another way is to buy S&P futures:
   - Put the money in your margin account
   - Assuming S&P is at 1000 now and the each contract assigns $250 to each index point (see, e.g., WSJ), the number of contracts to buy is:

$$\frac{1000000}{(250)(1000)} = 4$$

Example. (Cont'd)

As the S&P index fluctuates, the future value of your portfolio (in $M) would look as follows (ignoring interest payments and dividends):

S&P	Portfolio (a)	Portfolio (b)
900	0.90	0.90
1000	1.00	1.00
1100	1.10	1.10

Interest rate futures

- Underlying assets are riskless or high grade bonds
- Delivery is required (allowing substitutes)

Example. Consider a T-bond with annul coupon rate of 7% (with semi-annual coupon payments) that is selling at par. Suppose that the current short rate is 5% (APR). What should be the 6-month forward price of the T-bond?

Coupon yield on the bond:

$$y = (7\%)/2 = 3.5\%$$

Forward price:

$$F = S\,(1 + r - y)^T = (100)(1+2.5\%-3.5\%) = \$99.00$$

Hedging with Forwards

Hedging with forward contracts is simple, because one can tailor the contract to match maturity and size of position to be hedged.

Example. Suppose that you, the manager of an oil exploration firm, have just struck oil. You expect that in 5 months time you will have 1 million barrels of oil. You are unsure of the future price of oil and would like to hedge the oil price risk.

Using a forward contract, you could hedge your position by selling forward 1 million barrels of oil. Let S(t) be the spot oil price at t (in months). Then,

Position	Value in 5 months (per barrel)
Long position in oil	$S(5)$
Short forward position	$F - S(5)$
Net payoff	F

One problem with using forwards to hedge is that they are illiquid.

If after 1 month you discover that there is no oil, then you no longer need the forward contract. In fact, holding just the forward contract you are now exposed to the risk of oil-price changes.

In this case, you would want to unwind your position by buying back the contract. Given the illiquidity of forward contracts, this can be difficult and expensive.

To avoid problems with illiquidity of forwards, one may use futures contracts.

Example. (Cont'd) In the above example, you can sell 1 million barrels worth of futures. Suppose that the size of each futures contract is 1,000 barrels. The number of contracts you want to short is

$$\frac{1,000,000}{1,000} = 1,000$$

Example. We have \$10 million invested in government bonds and are concerned with volatile interest rates over the next six months. Use the 6-month T-bond futures to protect investment value.

- Duration of the bond portfolio is 6.80 years
- Current futures price is \$93 2/32 (for face value of \$100)
  - The T-bond to be delivered has a duration of 9.20 years
  - Each contract delivers \$100,000 face value of bonds
  - Futures price for the total contract is \$93,062.50
  - 6-month interest rate is 4%

Should we short or long the futures. Short. (Why?)

How many contracts to short?

Match duration: (# of contracts)(93,062.50)(9.20) = (10,000,000)(6.80)

Thus:

$$(\#\text{ of contracts}) = \left(\frac{10,000,000}{93,062.50}\right)\left(\frac{6.80}{9.20}\right) = 79.42$$

Since futures contracts are standardized, they may not perfectly match your hedging need. The following mismatches may arise when hedging with futures:

_ Maturity

_ Contract size

_ Underlying asset

Thus, a perfect hedge is available only when

1. the maturity of futures matches that of the cash flow to be hedged
2. the contract has the same size as the position to be hedged
3. the cash flow being hedged is linearly related to the futures'

In the event of a mismatch between the position to be hedged and the futures contract, the hedge may not be perfect.

Key concepts

- Forward contracts
- Futures contracts
- Mark to market
- Forward and futures prices
- Commodity futures
- Financial futures
- Hedging with forwards/futures