Lecture Notes

Valuation

WS 2009
Dr. Alex Stomper
Introduction
What does this course offer?

- Scientific tools and techniques for valuing financial assets
- We focus on:
  - Valuation of publicly traded firms
    - Value of equity
    - Value of total company (debt+equity)
  - Valuation of investment projects

Valuation

Alex Stomper
Relevance of valuation

- Valuation and portfolio management
  - Central role in fundamental analysis
  - Useful input for technical analysis

- Valuation and corporate finance
  - Corporate objective: value maximization
  - Capital budgeting
  - Mergers and acquisitions
  - Other corporate restructurings
Valuation and market efficiency

In an “efficient“ market, the market price is the best estimate of the true value of an asset.

Deviations of market price from true value are random.

- Weak form: the current price reflects all information in past prices (i.e., past prices do not help to identify under or over value stocks)
- Semi-strong form: the current price reflects all public information
- Strong form: current price reflects all information, public as well as private.
Valuation and market efficiency (2)

- Strong-form market efficiency is theoretically impossible: Grossman and Stiglitz (1980).
- However, the market is usually smarter than you think.
- Therefore, compare your valuation results with the market price whenever possible
  - Do you know something that the market does not know?
  - Or do you make a mistake?
Approaches to valuation

- **Discounted cashflow valuation**, relates the value of an asset to the present value of expected future cash flows on that asset.

- **Relative valuation**, estimates the value of an asset by looking at the pricing of 'comparable' assets relative to a common variable like earnings, cash flows, book value or sales.

- **Real options approach to valuation**, quantifies the value of managerial flexibility using option pricing models.
1. Discount cash flow valuation
Generic DCF valuation formula

- The value of an asset is determined by the present value of expected future cash flows generated by the asset.

\[
V = \sum_{t=1}^{N} \frac{E(CF_t)}{(1 + r)^t}
\]

where \( CF_t \) is the cash flow in period \( t \), \( r \) is the appropriate discount rate.

- Underlying principle: valuation is additive!
Key components in DCF valuation

- **Relevant cash flows**
  - All cash flows from and to investors (inflows and outflows)
  - Difference between cash flow and profit/loss

- **Appropriate discount rate**
  - Account for the time value of cash flows (earlier cash flows are more valuable)
  - Account for the uncertainty (risk) of cash flows

- **Matching principle**: different valuation models use different combinations of cash flows and discount rates.
DCF valuation models

- Free cash flow valuation model
- Capital cash flow valuation model
- Adjusted present value model
- Dividend discount model
- Economic-profit-based valuation model
Key steps in FCF valuation

- Estimating current free cash flows
- Estimating growth rate
- Estimating cost of capital
- Estimating residual value (company value at the end of the explicit forecast period)

\[
V_0 = \sum_{t=1}^{n} \frac{E(FCF_t)}{(1 + WACC)^t} + \frac{RV}{(1 + WACC)^n}
\]
## I. From earnings to free cash flows

<table>
<thead>
<tr>
<th>EBIT (earnings before interest and taxes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>- Corporate tax rate* EBIT</td>
</tr>
<tr>
<td>= NOPLAT (net operating profit less adjusted taxes)</td>
</tr>
<tr>
<td>+ accounting deductions that did not involve a cash outflow (depreciation, amortization)</td>
</tr>
<tr>
<td>- accounting income that did not involve a cash inflow</td>
</tr>
<tr>
<td>= Gross Cash Flow</td>
</tr>
<tr>
<td>- Gross investments (net investment + depreciation)</td>
</tr>
<tr>
<td>= Free Cash Flow</td>
</tr>
<tr>
<td>- (1-Corporate tax rate)* Interest expense</td>
</tr>
<tr>
<td>- Repayment of principal</td>
</tr>
<tr>
<td>= Free Cash Flow to Shareholders</td>
</tr>
</tbody>
</table>
Firm value vs equity value

- Value of firm:

\[ V = \sum_{t=1}^{N} \frac{E(FCF_t)}{(1 + WACC)^t} \]

- Value of equity

\[ E = \sum_{t=1}^{N} \frac{E(FCFE_t)}{(1 + k_e)^t} \]
Taxes in FCF valuation

- FCF are calculated as if firms are all-equity financed
- Marginal corporate tax rate is applied to EBIT, without taking into account that interest payments are tax deductible
- Debt tax shields are recognized through the discount rate
- Alternatively, one can include the debt tax shields in cash flows, then the discount rate should be the pretax discount rate (Capital cash flow valuation model, see later).
Investment expenditures

- Gross investment
  = capital expenditures + change in working capital
- Working capital = current assets (inventory, cash and account receivable) - current liabilities (account payable, short-term debt)
- Capital expenditures minus depreciation is called net capital expenditures
- Gross investment minus depreciation is called net investment
- If depreciation is the only noncash expense/income then
  FCF = NOPLAT - net investment
# Estimating current FCF: example

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>EBIT</td>
<td>1500</td>
</tr>
<tr>
<td>Tax rate</td>
<td>40%</td>
</tr>
<tr>
<td>NOPLAT</td>
<td>900</td>
</tr>
<tr>
<td>Depreciation</td>
<td>300</td>
</tr>
<tr>
<td>Gross cash flow</td>
<td>1200</td>
</tr>
<tr>
<td>Capital expenditure</td>
<td>500</td>
</tr>
<tr>
<td>Change in working capital</td>
<td>100</td>
</tr>
<tr>
<td>FCF</td>
<td>600</td>
</tr>
<tr>
<td>Interest expense</td>
<td>120</td>
</tr>
<tr>
<td>repayment of principal</td>
<td>150</td>
</tr>
<tr>
<td>FCF to shareholder</td>
<td>378</td>
</tr>
</tbody>
</table>
II. Estimating growth rate

- Look at historical growth rate
- Look at forecasts by analysts
- Look at fundamental drivers of growth rate
  - Reinvestment rate:
    \[ IR = \frac{\text{Net investment}}{\text{NOPLAT}} \]
  - Return on invested capital (ROIC)
    \[ \text{ROIC} = \frac{\text{NOPLAT}}{\text{Invested capital}} \]
    \[ IR \times \text{ROIC} = \frac{\text{Net investment}}{\text{Invested capital}} \]
    \[ \Rightarrow IR \times \text{ROIC} = \text{capital growth rate} \]
Invested capital

- Invested capital = total assets – excess cash – marketable securities – noninterest bearing short term liabilities
- Excess cash and marketable securities are excluded because it is easier to value them separately
- Noninterest bearing short term liabilities are excluded because they are financed by suppliers and their costs may have already been reflected in NOPLAT
# ROIC: example

<table>
<thead>
<tr>
<th>NOPLAT in year t</th>
<th>900</th>
</tr>
</thead>
<tbody>
<tr>
<td>total asset at the end of year t-1</td>
<td>3000</td>
</tr>
<tr>
<td>excess cash (t-1)</td>
<td>20</td>
</tr>
<tr>
<td>marketable security (t-1)</td>
<td>100</td>
</tr>
<tr>
<td>noninterest-bearing liabilities (t-1)</td>
<td>150</td>
</tr>
<tr>
<td>Invested capital at the end of year t-1</td>
<td>2730</td>
</tr>
<tr>
<td>ROIC in year t</td>
<td>0.3</td>
</tr>
</tbody>
</table>
Determinants of ROIC

\[ ROIC = (1 - t) \times \frac{EBIT}{\text{Revenues}} \times \frac{\text{Revenues}}{\text{Invested Capital}} \]

- Profit margin
- Asset turnover
Fundamental growth rate

- The case of constant ROIC
  \[ g_{\text{NOPLAT}} = IR_t \times \text{ROIC} \]

- Return on new invested capital (RONIC) ≠ ROIC
  \[ g_{\text{NOPLAT}} = IR_t \times \text{RONIC} \]

- The case of changing ROIC
  \[ g_{\text{NOPLAT}} = IR_t \times \text{ROIC}_{t+1} + \left(\text{ROIC}_{t+1} - \text{ROIC}_t\right)/\text{ROIC}_t \]
### Forecasting FCFs: example

<table>
<thead>
<tr>
<th></th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
<th>average</th>
</tr>
</thead>
<tbody>
<tr>
<td>invested capital (beginning)</td>
<td>400.00</td>
<td>450.00</td>
<td>530.00</td>
<td></td>
</tr>
<tr>
<td>NOPLAT</td>
<td>150.00</td>
<td>200.00</td>
<td>205.00</td>
<td></td>
</tr>
<tr>
<td>ROIC</td>
<td>0.38</td>
<td>0.44</td>
<td>0.39</td>
<td>0.40</td>
</tr>
<tr>
<td>Net investment</td>
<td>50.00</td>
<td>80.00</td>
<td>70.00</td>
<td></td>
</tr>
<tr>
<td>IR</td>
<td>0.33</td>
<td>0.40</td>
<td>0.34</td>
<td>0.36</td>
</tr>
<tr>
<td>FCF</td>
<td>100.00</td>
<td>120.00</td>
<td>135.00</td>
<td></td>
</tr>
</tbody>
</table>
Forecasting FCFs: example

- Forecast FCFs in the next three years, assuming alternatively that
  
  1. IR and ROIC are the same as in 2006
  2. IR and ROIC are the same as in 2006, RONIC equals to 0.3
  3. IR and ROIC equal to the average levels in 2004-2006

Valuation
Forecasting FCFs: solution
III. Estimating cost of capital

- The expected FCFs to the firm are discounted using the **weighted average cost of capital** (WACC) to get the firm value.
- The expected FCFs to equityholders are discounted using **cost of equity** to get the equity value.
- The weight of each financing form is defined as the ratio between the **market value** of that financing form to the total market value of the firm.
- Noninteresting-bearing liabilities are not considered when computing WACC.
- The tax advantage of debt financing is reflected in WACC.
WACC

\[ WACC = k_d (1 - t) \frac{D}{V} + k_p \frac{P}{V} + k_e \frac{E}{V} \]

where
- \( k_d \) = pre-tax cost of debt
- \( k_p \) = cost of preferred stock
- \( k_e \) = cost of equity
- \( t \) = corporate tax rate
- \( D/V \) = target debt ratio using market values
- \( P/V \) = target preferred stock ratio using market values
- \( E/V \) = target equity ratio using market values
- \( V \) = market value of the firm (D+P+S)
Weights in WACC

- The target weights instead of the current weights should be used.
- Weights should be calculated using market values.
- The market value may not exist, especially for the debt.

Possible estimation procedure:
- Identify all payment obligations to debt holders
- Estimate the credit risk of debt-type financing instruments
- Find market-traded instruments that have similar credit risk and time to maturity
- Use the market returns to discount the outstanding payments to debt holders
Cost of debt/preferred stocks

- Cost of debt = expected return on debt *(1-t)
  - Expected return on debt ≠ coupon rate
  - Expected return on debt ≠ promised yield
- Investment-grade debt (debt rated at BBB or better): use yield to maturity of the company’s long-term, option-free bonds
- If the bond rarely trades, use the average yield to maturity on a portfolio of long term bonds with the same credit rating
- Below-investment-grade debt: use CAPM to estimate the expected return
- Adjust for interest tax shields
- Preferred stocks: preferred dividend divided by the market price of preferred stocks
Cost of equity: CAPM

\[ k_e = r_f + \beta [E(r_m) - r_f] \]

where

- \( r_f \) = risk-free rate
- \( \beta \) = the sensitivity of the stock return to market return
- \( E(r_m) \) = expect return of the market portfolio
- \( E(r_m) - r_f \) = market risk premium
CAPM: implementation (1)

- Risk-free rate: use long-term government bond
- Market risk premium: historical data
  - Use the longest period possible: short-term estimates are very noisy (annual standard deviation of stock returns 20%).
  - Use geometric average instead of arithmetic average
  - Adjust for survivorship bias.
  - Normally used numbers: 4.5-5.5%
CAPM: implementation (2)

- Beta: estimated from the market model

\[ r_{it} = \alpha + \beta r_{mt} + \varepsilon_{it} \]

  - Normally five-year monthly data are used
  - Adjustment for low trading frequency (Dimson(1979))

- Market portfolio

  - In theory, all assets must be included
  - In practice, well-diversified stock indexes are used as a proxy: S&P 500, MSCI world index, MSCI Europe index, etc
Beyond CAPM: APT model

\[ r_i = \alpha + \sum_{k=1}^{n} \beta_{ik} F_k + \varepsilon \]

\[ E(r_i) = r_f + \sum_{k=1}^{n} \beta_{ik} \lambda_k \]

where \( F_k \) = the k-th systematic factor that drives security return,
\( \lambda_k \) = risk premium of the k-th factor

- **Difficulty in implementation: not clear**
  - What are the factors?
  - How to measure them?
Beyond CAPM: Fama-French model

\[ E(r_i) = r_f + \beta_1[E(r_m) - r_f] + \beta_2[E(r_S) - E(r_B)] + \beta_3[E(r_H) - E(r_L)] \]

where \( \beta_1, \beta_2, \beta_3 \) are exposures to the market portfolio, size portfolio and book-to-market portfolio respectively, \( r_m, r_S, r_B, r_H, r_L \) are returns on the market portfolio, small stock portfolio, large stock portfolio, high book-to-market stock portfolio, low book-to-market stock portfolio respectively.

- An empirical model designed to capture the size and book-to-market effects in stock return
- Theoretical fundation still not clear
Capital structure and cost of capital

- Modigliani and Miller theorem: In a perfect market without tax, capital structure has no impact on either the firm value or the cost of capital.
- An easy way to understand this fundamental result in corporate finance: In a perfect market without tax, capital structure has no impact on the expected cash flows to the firm.
- In the MM world, when the more expensive equity is substituted by the less expensive debt, the cost of equity increases according, leaving the weighted cost of capital unchanged.
- In a world with tax, debt increases the cash flows to the firm by reducing taxes. This is not reflected in FCFs, therefore it must be reflect in WACC.
- In the real world (with both tax and market imperfections), an optimal capital structure is determined by the trade-off between the tax advantage of debt and the cost of high leverage.
MM world

Valuation
MM world with corporate tax

\[ k_e \]

\[ k_d(1-t) \]

\[ \text{WACC} \]

\[ \text{D/E} \]
Real world with tax and other frictions

\[ \text{WACC} \]

\[ k_e \]

\[ k_d(1-t) \]

\[ (D/E)^* \]

\[ D/E \]
Leverage and equity beta

- Industry beta is often used to improve the estimation of company beta.
- However, firms in the same industry may have different leverage ratio.
- Procedure for inferring beta from comparable firms:
  - Estimate beta for each comparable firm
  - Back out the unlevered beta
  - Calculate the relevered beta using the target leverage ratio
Two alternative leverage policies

- The relation between levered- and unlevered-beta depends on the assumed leverage policy.
- MM assumption (Modigliani and Miller 1963): constant debt value
- ME assumption (Miles-Ezzell 1980): constant debt ratio
- Note that ME assumption is different from MM assumption even if expected growth rate is zero.
- Failing to recognize this difference has led to confusion even among experts (Fernandez 2004, Cooper and Nyborg 2006)
Unlevered beta: constant debt level

If the debt value is constant, then interest tax shields should be discounted by cost of debt, therefore

\[ V_L = E + D = V_U + V_{TS} = V_U + tD \]

\[ \Rightarrow \beta_C = \beta_E \frac{E}{V_L} + \beta_D \frac{D}{V_L} = \beta_U \frac{V_U}{V_L} + \beta_D \frac{tD}{V_L} \]

\[ \Rightarrow \beta_U = \beta_E \frac{E}{V_U} + \beta_D \frac{(1-t)D}{V_U} \]

\[ = \beta_E \frac{E}{E + (1-t)D} + \beta_D \frac{(1-t)D}{E + (1-t)D} \]

\[ = \beta_E \frac{1}{1 + (1-t)D/E} + \beta_D \frac{(1-t)D/E}{1 + (1-t)D/E} \]

\[ \Rightarrow \beta_E = \beta_U + (\beta_U - \beta_D) \frac{D}{E}(1-t) \]
Unlevered beta: constant debt level (2)

- If $\beta_D = 0$, then we have
  - Hamada (1972) formula
    \[ \beta_U = \beta_E \frac{1}{1 + (1 - t)D/E} \]
  - Relevered beta
    \[ \beta_E = \beta_U [1 + (1 - t) \frac{D}{E}] \]
Unlevered (levered) cost of equity

Unlevered cost of equity can be derived either using unlevered beta or directly from the following formula:

\[ k_u = k_e \frac{1}{1 + (1-t)D/E} + k_d \frac{(1-t)D/E}{1 + (1-t)D/E} \]

\[ \Rightarrow k_e = k_u + (k_u - k_d) \frac{D}{E} (1-t) \]

\[ \Rightarrow WACC_L = k_u (1 - \frac{tD}{E + D}) \]
Unlevered beta: example

- A privately-held company has a leverage ratio (D/E) of 60%.
- A comparable publicly-traded company with a leverage ratio of 40% has an equity beta of 1.2.
- The comparable firm is assumed to maintain the current debt level (in value) in the future.
- The corporate tax rate is 35%.
- What is the beta of equity for the private company?
Unlevered beta: solution
Unlevered beta: constant debt ratio

If the debt ratio is constant, then interest tax shields should be discounted using unlevered cost of equity, therefore

\[ V_L = E + D = V_U + V_{TS} \]

\[ \Rightarrow \beta_C = \beta_E \frac{E}{V_L} + \beta_D \frac{D}{V_L} = \beta_U \frac{V_U}{V_L} + \beta_U \frac{V_{TS}}{V_L} = \beta_U \]

\[ \Rightarrow \beta_U = \beta_E \frac{E}{D + E} + \beta_D \frac{D}{D + E} \]

\[ \Rightarrow k_u = k_E \frac{E}{D + E} + k_D \frac{D}{D + E} = \text{pretax WACC} \]
Unlevered beta: constant debt ratio

(2)

- Relevered beta

\[ \beta_e = \beta_u + \frac{D}{E} (\beta_u - \beta_d) \]

- Levered cost of equity

\[ k_e = k_u + \frac{D}{E} (k_u - k_d) \]

- WACC of the levered firm

\[ WACC_L = k_u - \frac{tk_d D}{E + D} \]
Constant leverage ratio: example

- Redo the previous exercise assuming that the comparable firm is going to maintain the current debt ratio in the future
IV. Estimating residual value

- Method 1
  \[ RV_T = \frac{NOPLAT_{T+1}(1 - g / RONIC)}{WACC - g} \]

- Method 2
  \[ RV_T = \frac{FCF_{T+1}}{WACC - g} \]

- Since \( FCF_{T+1} = NOPLAT_{T+1} \times (1 - IR_{T+1}) = NOPLAT \times (1 - g / RONIC) \), these two methods are equivalent.
- Since the reinvestment rate in the residual period may be different from that in the explicit forecast period, \( FCF_{T+1} \) may not equal \( FCF_T \times (1 + g) \).
- Method 1 automatically takes this into account.
Residual value: example

- $ \text{FCF}_T = 100, \text{NOPLAT}_T = 200$
- From year $T+1$ on, $g = 5\%$, $\text{RONIC} = 12\%$
- $WACC = 10\%$
Residual value: other methods

- If RONIC = WACC, then method 1 reduces to the convergence formula

\[ RV = \frac{NOPLAT_{T+1}}{WACC} \]

This method assumes that new investment in the residual period does not create any value.

- Liquidation value: only if liquidation is very likely
- Replacement cost: no good economic reason
Other DCF valuation model

- Capital cash flow valuation model
- Adjusted present value model
- Economic-profit-based valuation model
- Discount dividend model
Capital cash flow valuation model

- CCF = FCF + interest tax shield
- Firm value is derived by discounting the CCF using unlevered cost of equity
- Given that interest tax shield are discounted using unlevered cost of equity, it follows that unlevered cost of equity equals pretax WACC.
- Advantage: no need to adjust discount rate for changes in financial structure.
CCF valuation model: example

- Example: Current FCF of a firm is 200, interest expense is 50, tax rate is 40%. The firm has a pretax WACC of 10% and an expected growth rate of 5%. Value this firm using CCF valuation model.
Adjusted present value model

- Valuation by components approach:

\[ V_L = V_U + V_{TS} \]

where \( V_U \) is the expected FCF discounted using unlevered cost of equity, \( V_{TS} \) is the present value of interest tax shields

- Possible discount rates for interest tax shields:
  - Pretax cost of debt: varying or risky debt
  - Unlevered cost of equity: costant debt ratio (in this case, APV model is equivalent to CCF valuation model)

- Advantage:
  - No need to adjust discount rate for changes in capital structure
  - Can easily be combined with a variety of valuation models
## APV model: example

<table>
<thead>
<tr>
<th></th>
<th>year1</th>
<th>year2</th>
<th>year3</th>
<th>year4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unlevered cash flows</td>
<td>100</td>
<td>100</td>
<td>1000</td>
<td>1000</td>
</tr>
<tr>
<td>debt</td>
<td>0</td>
<td>0</td>
<td>2000 (at 8%)</td>
<td>2000 (at 8%)</td>
</tr>
<tr>
<td>unlevered cost of equity</td>
<td></td>
<td></td>
<td>0.14</td>
<td></td>
</tr>
<tr>
<td>tax rate</td>
<td></td>
<td></td>
<td>0.34</td>
<td></td>
</tr>
</tbody>
</table>
Economic-profit-based valuation model

- Economic profit (or Economic Value Added)
  \[ \text{Economic profit} = \text{Invested capital} \times (\text{ROIC} - \text{WACC}) \]
  \[ = \text{NOPLAT} - \text{invested capital} \times \text{WACC} \]

- Firm value and economic profit

\[
V_0 = \text{Invested capital}_0 + \sum_{t=1}^{\infty} \frac{E(\text{Economic profit}_t)}{(1 + \text{WACC})^t}
\]

- Important message: a project creates value for shareholders iff its return is higher than its cost of capital
Discount dividend model

- General model

\[ P_0 = \sum_{t=1}^{\infty} \frac{E(DPS_t)}{(1 + k_e)^t} \]

where DPS is dividend per share, \( k_e = \) cost of equity

- Gordon growth model: for stocks with a stable growth rate

\[ P_0 = \frac{E(DPS_1)}{k_e - g} \]

where \( E(DPS_1) \) is expected dividend next period, \( g \) is growth rate in dividends forever
Gordon growth model

- Works best for companies
  - in stable growth
  - in stable leverage
  - pays out dividend regularly

- Limitations
  - Extremely sensitive to the input for growth rate
  - Extremely simple growth pattern
Two-stage growth model

- Assumption: high growth rate \((g)\) in the first \(n\) periods and normal growth rate \((g_n)\) in the rest periods

\[
P_0 = \sum_{t=1}^{n} \frac{E(DPS_t)}{(1 + k_e)^t} + \frac{P_n}{(1 + k_e)^n}
\]

\[
E(DPS_1)[(1 - \frac{1 + g}{1 + k_e})^n] = \frac{E(DPS_{n+1})}{k_e - g} + \frac{E(DPS_{n+1})}{(1 + k_e)^n (k_e - g_n)}
\]
2. Relative valuation
How does it work?

- In relative valuation, you try to figure out the value of the firms being analyzed by looking at the market values of similar or comparable firms.

- Steps in relative valuation
  - Identify comparable firms
  - Calculate the "multiples"
  - Compare the multiples and control for factors that might affect the multiples

- Implicit assumption: market is on average right
Most popular multiples

- **Earnings multiples**
  - Price/earnings ratio and variants
  - Value/EBITDA
  - Value/FCF

- **Book value multiples**
  - Price/book value (PBV, or market-to-book equity)
  - Value/book value
  - Value/replacement cost (Tobin’s Q)

- **Revenues multiples**
  - Price/sales
  - Value/sales
Price / Earnings ratio

PE = market price per share / Earnings per share

Price can be
• Current price (most of the time)
• Average price for the year

Earnings per share (EPS) can be
• EPS in most recent financial year
• EPS in trailing 12 months (trailing PE)
• Forecast EPS next year (forward PE)
Distribution of PE ratio: US stocks

source: Damodaran 2002
# PE ratio across countries: July 2000

## Developed markets
- **UK**: 22.02
- **Germany**: 26.33
- **France**: 29.04
- **Switzerland**: 19.6
- **Belgium**: 14.74
- **Italy**: 28.23
- **Sweden**: 32.39
- **Netherlands**: 21.1
- **Australia**: 21.69
- **Japan**: 52.25
- **United States**: 25.14
- **Canada**: 26.14

**source:** Damodaran 2002

## Emerging markets
- **Argentina**: 14
- **Brazil**: 21
- **Chile**: 25
- **Hong Kong**: 20
- **India**: 17
- **Indonesia**: 15
- **Malaysia**: 14
- **Mexico**: 19
- **Pakistan**: 14
- **Peru**: 15
- **Phillipines**: 15
- **Singapore**: 24
- **South Korea**: 21
- **Thailand**: 21
- **Turkey**: 12
- **Venezuela**: 20
Determinants of PE ratio

\[ PE = \frac{P_0}{EPS_0} = \frac{DPS_0(1 + g)}{(k_e - g)EPS_0} = \frac{\text{Payout ratio} (1 + g)}{k_e - g} \]

- Other things equal, PE ratio is higher for firms with
  - High growth potential
  - High payout ratio
  - Low cost of equity (low equity risk, low risk free rate)
PE ratio: regression analysis

- Advantage of regression analysis: the information in the entire cross-section instead of a few comparable firms can be used.
- Problem: the coefficients may be unstable.
- Example: regression results for Compustat sample (Damodaran 2002)

<table>
<thead>
<tr>
<th>Year</th>
<th>Regression</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1987</td>
<td>$\text{PE} = 7.1839 + 13.05 \text{PAYOUT} - 0.6259 \text{BETA} + 6.5659 \text{EGR}$</td>
<td>0.9287</td>
</tr>
<tr>
<td>1988</td>
<td>$\text{PE} = 2.5848 + 29.91 \text{PAYOUT} - 4.5157 \text{BETA} + 19.9143 \text{EGR}$</td>
<td>0.9465</td>
</tr>
<tr>
<td>1989</td>
<td>$\text{PE} = 4.6122 + 59.74 \text{PAYOUT} - 0.7546 \text{BETA} + 9.0072 \text{EGR}$</td>
<td>0.5613</td>
</tr>
<tr>
<td>1990</td>
<td>$\text{PE} = 3.5955 + 10.88 \text{PAYOUT} - 0.2801 \text{BETA} + 5.4573 \text{EGR}$</td>
<td>0.3497</td>
</tr>
<tr>
<td>1991</td>
<td>$\text{PE} = 2.7711 + 22.89 \text{PAYOUT} - 0.1326 \text{BETA} + 13.8653 \text{EGR}$</td>
<td>0.3217</td>
</tr>
</tbody>
</table>
PEG ratio

- PEG = PE / Expected growth rate in earnings
- A simple way to control for the influence of growth rate on PE ratio
- But not completely neutralize it since PE is not a linear function of expected growth rate

$$PEG = \frac{Payout \ ratio \ (1 + g)}{g(k_e - g)}$$

- No standard time frame for measuring expected growth rate
Value multiples

- \( \frac{V}{EBITDA} = \frac{E + D}{EBITDA} \)
- \( \frac{V}{FCF} = \frac{E + D}{FCF} \)
- \( FCF = EBIT \times (1-t) - (CAP \ EX - D&A) - \Delta \text{working capital} \)
  = \( (EBITDA - D&A)(1-t) - (CAP \ EX - D&A) - \Delta \text{working capital} \)
  = \( EBITDA(1-t) + t \times D&A - CAP \ EX - \Delta \text{working capital} \)

Advantages

- Less firms with negative EBITDA than firms with negative earnings
- Not influenced by difference in depreciation schemes
- Not influenced by differences in capital structure
Determinants of V/FCF ratio

- **Stable growth case**

\[
\frac{V_0}{FCF_0} = \frac{FCF_0(1+g)}{FCF_0(WACC - g)} = \frac{1+g}{WACC - g}
\]

- **Two stage growth case**

\[
\frac{V_0}{FCF_0} = \frac{(1+g)(1 - \frac{(1+g)^n}{(1+WACC)^n})}{WACC - g} + \frac{(1+g)^n(1+g_n)}{(1+WACC)^n(WACC - g_n)}
\]
Value multiples: Example

- Consider a firm with the following characteristics
  - Tax rate = 33%
  - Capital Expenditure/EBITDA = 30%
  - Depreciation & Amortization/EBITA = 20%
  - Cost of capital = 10%
  - No requirement for working capital
  - Stable growth rate = 5%

- Calculate V/EBITDA & V/FCF
Value multiples: solution
Price-to-book ratio

- For a stable growth firm

\[
PBV = \frac{P_0}{BV_0} = \frac{Earnings_1 \times Payout\ ratio}{BV_0(k_e - g)} = \frac{ROE_1 \times Payout\ ratio}{(k_e - g)}
\]

- Since \( g = (1 - \text{Payout ratio}) \times \text{ROE} \), we can further derive

\[
PBV = \frac{ROE - g}{k_e - g}
\]
PBV and ROE: S&P 500 (Damodaran 2002)
Value-to-book ratio

- **Definition**

\[
\frac{Value}{Book \, value} = \frac{market \, value \, of \, equity + market \, value \, of \, debt}{book \, value \, of \, equity + book \, value \, of \, debt}
\]

- **For stable growth firm**

\[
\frac{V_0}{BV_0} = \frac{FCF_1}{BV_0(WACC - g)} = \frac{EBIT_1(1 - t)(1 - g / ROIC)}{BV_0(WACC - g)} = \frac{ROIC - g}{WACC - g}
\]
Value-to-book ratio: example

Example: Consider a stable growth firm with the following characteristics: ROIC=12%, WACC=10%, g=5%. Estimate its Value-to-book ratio.
Tobin’s Q ratio

- Definition

\[
Tobin's\ Q = \frac{\text{Market value of assets in place}}{\text{Replacement cost of assets in place}}
\]

- If Tobin’s Q is smaller than 1, then a firm destroys value; if it is bigger than 1, then it creates value

- Advantage: replacement costs provide a more updated measure of asset value than do book values

- Disadvantage: replacement costs are hard to estimate
Revenue multiples

- **Price-to-sales ratio**
  = market value of equity / total revenues
  - Internally inconsistent, since the market value of equity is divided by the total revenues of the firm.
    => High leverage leads to low price-to-sales ratio

- **Value-to-sales ratio**
  = market value of firm/ total revenues

- **Advantages**
  - Available even for young or troubled firms
  - Not heavily influenced by accounting rules
  - Relatively stable
Determinants of revenue multiples

- For a stable growth firm

\[
\frac{P_0}{Sales} = \frac{Earnings_1 \times Payout\ ratio}{Sales(k_e - g)} = \frac{Net\ margin \times Payout\ ratio}{k_e - g}
\]

\[
\frac{V_0}{Sales} = \frac{EBIT_1(1-t) \times (1-IR)}{Sales(WACC - g)} = \frac{After\ tax\ operating\ margin \times (1 - IR)}{WACC - g}
\]
Choosing between multiples

- There are many potentially useful multiples
- Which ones to use in valuation?
  - Use a simply average of valuations obtained using different multiples
  - Use a weighted average of valuations obtained using different multiples
  - Rely entirely on one of the multiples
    - Most relevant one
    - Most accurately estimated one
Choosing the comparison firms

- Three possible choices
  - A few very similar firms
  - All firms in the same sector
  - All firms in the market
- Regression analysis is necessary if you choose the second or third approach
- It is recommended to check whether the firm is over or under valued at both the sector and market level.
3. Real options approach to valuation
Managerial flexibility (strategic options)

- Managers react to changes in economic environment
- DCF valuation and relative valuation do not explicitly account for this.
- Real options theory provides an useful framework to quantify the value of flexibility.
- This approach is particularly relevant for the valuation of individual businesses and projects.
Strategic options: examples

- Option to postpone a project
- Option to abandon a project
- Option to temporarily shut down a project
- Option to expand a project
- Option to downsize a project
- Option to change input or output factors

.....
Certainty equivalent method

- Option pricing is based on the Certainty Equivalent Method as opposed to the Risk-Adjusted Discount Rate Method.
- Instead of discounting the expected cash flowes using a risk-adjusted discount rate, the certainty equivalent method discounts the certainty equivalent of future uncertain cash flows at the risk-free rate.

\[ V = \sum_{t=1}^{\infty} \frac{CEQ(CF_t)}{(1 + r_f)^t} \]

- The certainty equivalent of some uncertain payoff is defined as a sure amount of payoff that is considered to be as valuable as the uncertain payoff.
- This alternative method can be very useful even in the absence of strategic options.
Obtaining certainty equivalents

How can we obtain certainty equivalents?

• By looking at prices in the forward or futures market (when forward or futures market exists)

• Expected value minus dollar value of risk premium (when risk premium and risk exposure are known)

• Expected value under the risk-neutral probability (when markets are complete)
Example: two-period gold mine

<table>
<thead>
<tr>
<th>period</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>output</td>
<td>1000</td>
<td>1000</td>
</tr>
<tr>
<td>price</td>
<td>$S_1$</td>
<td>$S_2$</td>
</tr>
<tr>
<td>revenue</td>
<td>$1000S_1$</td>
<td>$1000S_2$</td>
</tr>
<tr>
<td>Costs</td>
<td>300</td>
<td>300</td>
</tr>
<tr>
<td>NCF</td>
<td>$1000S_1$-300</td>
<td>$1000S_2$-300</td>
</tr>
</tbody>
</table>
Example: two-period gold mine (2)

- Suppose that risk free rate is 10%, the current forwards prices are 320 for a one-year contract and 350 for a two-year contract. What is the value of this mine?
Option pricing methods

- Binomial model
- Black-Scholes formula
- Monte Carlo simulation
I. Binomial model

\[ S_u = \max(S_u - X, 0) \]
\[ S_d = \max(S_d - X, 0) \]

\[ C_u = \max(S_u - X, 0) \]
\[ C_d = \max(S_d - X, 0) \]

\( X = \) exercise price, \( C = \) call option value, \( S = \) underlying value,
\( p = \) probability that underlying value goes up (irrelevant for valuation!)
The risk-neutral probability $q$ is given by

$$S_0 (1 + r_f) = qS_u + (1 - q)S_d$$

$$\Rightarrow q = \frac{S_0 (1 + r_f) - S_d}{S_u - S_d}$$

The call option value is thus given by

$$C_0 = \frac{qC_u + (1 - q)C_d}{1 + r_f}$$
II. Black-Scholes formula

For European call and put, Black and Scholes (1973) derive the following formula

\[ C = SN(d_1) - X e^{-r_f T} N(d_2) \]
\[ P = X e^{-r_f T} N(-d_2) - SN(-d_1) \]

\[ d_1 = \frac{\ln \frac{S}{X} + (r_f + \frac{1}{2} \sigma^2) T}{\sigma \sqrt{T}} \]
\[ d_2 = d_1 - \sigma \sqrt{T} \]

S = underlying price, K = Exercise price, \( \sigma \) = annualized volatility of the underlying, T = time to maturity, \( r_f \) = continuously-compounded risk-free rate, N(.) = cumulative standard normal distribution
Monte Carlo simulation can be used to value more complex options.

- Step 1: simulate the distribution of underlying value under the risk neutral probability by generating a large number of underlying price paths following

\[ S_{it} = S_{i,t-1} \tilde{R} = S_{i,t-1} e^{r_f - \frac{1}{2}\sigma^2 + \sigma \tilde{Z}} \]

where \( \tilde{Z} \) is a normally distributed random variable.
Monte Carlo Simulation (2)

- Step 2: calculate the net cash flow in each period on each sample path
  \[ NCF_{it} = \max(S_{it} - X_t, 0) \]
  where \( X_t \) is the production cost in period \( t \).
- Step 3: calculate the option value for each sample path \( i \)
  \[ V_i = \sum_{i=1}^{T} e^{-r_i t} NCF_{it} \]
- Step 4: calculate the average option value over all sample paths
  \[ V = \frac{1}{N} \sum_{t=1}^{N} V_i \]
Example: option to shut down

Value a gold mine with the following characteristics:

- Produces gold in two periods
- Temporary shut-down possible
- Current gold price 300
- Annual gold price volatility 20%
- Annually compounded risk free rate 10%
- Annual production 1000
- Annual Production cost 300
Solution: Binomial model

\[ u = e^{\sigma \sqrt{t}} = 1.2214, \quad d = e^{-\sigma \sqrt{t}} = 0.8187 \]
Solution: Binomial model (2)

\[ q = \frac{1 + r_f - d}{u - d} \]

\[ V_u = \max(S_u - 300, 0) + \frac{qV_{uu} + (1 - q)V_{ud}}{1 + r_f} \]

\[ V_d = \max(S_d - 300, 0) + \frac{qV_{ud} + (1 - q)V_{dd}}{1 + r_f} \]

\[ V_0 = \frac{qV_u + (1 - q)V_d}{1 + r_f} \]
Solution: Black-Scholes

\[ r_f = \ln(1 + 10\%) = 0.0953 \]

\[ V = 1000[300 \times N(d_{11}) - 300e^{-0.0953} \times N(d_{12})] \]
\[ + 1000[300 \times N(d_{21}) - 300e^{-0.0953^{2}} \times N(d_{22})] \]

\[ d_{t1} = \frac{\ln(300/300) + (r_f + 0.5\sigma^2)t}{\sigma\sqrt{t}} \]

\[ d_{t2} = d_{t1} - \sigma\sqrt{t} \]
Solution: Monte Carlo simulation

\[ S_{it} = S_{i,t-1} \tilde{R} \]

\[ \tilde{R} = e^{r_f - \frac{1}{2}\sigma^2 + \sigma \tilde{Z}} \]

\[ \tilde{Z} = \text{NORMSINV}(\tilde{U}) \]

\[ \tilde{U} = \text{RAND}() \]

- Function \( \text{RAND}() \) generates a random realization of a random variable uniformly distributed over the interval \([0,1]\).
- \( \text{NORMSINV}(U) \) generates a random realization of a random variable following a standard normal distribution.
Monte Carlo simulation: a sample path

<table>
<thead>
<tr>
<th>period</th>
<th>t=1</th>
<th>t=2</th>
</tr>
</thead>
<tbody>
<tr>
<td>U</td>
<td>0.2679</td>
<td>0.7208</td>
</tr>
<tr>
<td>Z</td>
<td>-0.6193</td>
<td>0.5853</td>
</tr>
<tr>
<td>R</td>
<td>0.9526</td>
<td>1.2121</td>
</tr>
<tr>
<td>S</td>
<td>285.78</td>
<td>346.40</td>
</tr>
<tr>
<td>Production cost</td>
<td>300</td>
<td>300</td>
</tr>
<tr>
<td>NCF</td>
<td>0</td>
<td>46.40</td>
</tr>
<tr>
<td>PV</td>
<td>0</td>
<td>38350.41</td>
</tr>
<tr>
<td>V</td>
<td>38350.41</td>
<td></td>
</tr>
</tbody>
</table>
Option to delay: example

- Panel A: Invest now
  - good:
    - 10
    - 15
    - 15 per year for ever
  - bad:
    - -100
    - 10
    - 2.5
    - 2.5 per year for ever

- Panel B: wait one year and invest only in good state
  - good:
    - -100
    - 10
    - 15
    - 15 per year for ever
  - bad:
    - 0
    - 0
    - 0
    - 0 per year for ever
Option to delay (2)

- Risk free rate = 5% per year.
- $1 invested in the market portfolio will be worth either $1.3 (when the state is good) or $0.8 (when the state is bad) in one year.
- Should we invest now or should we wait until next year?
- What is the value of the option to wait?
Option to delay: solution
Option to expand: example

- A project can generate the following CFs:

- The firm has the option to double its capacity by investing another 140 in year 1 if the economy looks good.
Option to expand (2)

- Risk free rate 5%.
- Risk neutral probabilities: \( q = 0.6 \) in both periods.
- What is the value of the project without considering the option value?
- What is the value of the project after considering the option value?
- What is the value of the option to expand?
Option to expand: solution