Competition, Market Coverage, and Quality Choice in Interconnected Platforms

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ABSTRACT

We study duopoly competition between two interconnected Internet Service Providers (ISP) that compete in quality and prices for both Content Providers (CP) and consumers. We develop a game theoretic model using a two-sided market framework, where ISP's are modeled as interconnected platforms with quality bottlenecks; a consumer on a low quality network accessing content on a high quality platform experiences low quality. Platforms first pick quality levels and in the subsequent stages compete in prices for both CP's and consumers. CP's are heterogenous in content quality which is uniformly distributed between $[\overline{\gamma}-1, \overline{\gamma}]$. We first establish the existence of a price subgame perfect equilibrium (SPE) given any asymmetric pair of platform quality choices. We show that the higher the asymmetry the more likely the CP market is to be uncovered if the average content quality (represented by $\overline{\gamma}$) is low. In contrast, if $\overline{\gamma}$ is high then the market is always covered. We then show that a SPE for the whole game exists and characterize all the equilibrium choices of the quality game. In particular, we show that the equilibria involve either maximal differentiation or partial differentiation depending on $\overline{\gamma}$. Moreover, we characterize the resulting market configurations in the final stage and show that they depend on the content quality characteristic, $\overline{\gamma}$, and the asymmetry between platforms represented by the ratio of the qualities, $\mathcal{I}$.

1. INTRODUCTION

Consumers and Content providers (CP) base their choice of ISP not only on prices but also on other features such as speed of access, special add-ons like spam blocking and virus protection [2]. These extra features can be abstracted as quality. ISP's have the ability to influence quality through upgrade of infrastructure, offering of enhanced services or even traffic-management [4]. A key question that arises in the provision of network access is what quality profit maximizing ISP's will offer in a competitive environment and its concomitant effect on market coverage.

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In this paper we study this question using a model of duopoly competition in interconnected two-sided-market platforms in the presence of quality choice. We develop a game-theoretic model where ISP's are represented as profit maximizing two-sided interconnected platforms that choose quality levels and then compete in prices for both CP's and consumers. In addition, we model quality of service effects through a bottleneck effect. While there is much work on competition models between two-sided platforms (see for example, [10, 1, 11, 6]), most existing work focuses on determinants of pricing. These works do not address interconnection between platforms, endogenous quality choice by platforms and market coverage. In this paper we consider these effects in tandem. Our objective is to understand what strategic quality choices interconnected platforms make and their effects on market structure.

Our model consists of two interconnected platforms, a heterogenous mass of CP's, and a heterogenous mass of consumers. Platforms provide connection services to consumers and CP's and charge a flat access fee to both. We model the interaction between ISP's and end-users[1] as a six-stage game. In the first stage, platforms simultaneously pick a quality level from a bounded interval. Second, they simultaneously compete in CP prices. Third, the CP's decide which platforms, if any, to connect to. Fourth, the platforms simultaneously compete in consumer prices. In the fifth stage, consumers decide what platforms to join. In the last stage Consumers decide which CP's to patronize.

We first derive results relating to the price competition SPE between the platforms given the quality choices and then use these results to solve for the quality choice SPE. The former results show that given an asymmetric quality pair, a subgame perfect equilibrium (SPE) in both consumer and CP prices exists. In addition, we show the relationship between the ISPs' quality choices and market structure on the content provider side. Specifically, we show that the resulting market configuration depends on the content quality characteristic, $\overline{\gamma}$,[2] of the CP's and the asymmetry they induce

[1]End-users refers to both CP's and Consumers

[2]Viewed as a proxy for the average quality of content providers.

on the platforms. We show that if $\overline{\gamma}$, is low, then an increase in the quality ratio, $\mathcal{I}$, defined as the ratio between platform qualities, increases the platforms market power resulting in an uncovered content provider market. On the other hand, when $\overline{\gamma}$ is high, the relative difference in content quality diminishes and content providers make similar connection decisions regardless of quality ratio levels, i.e., they all join the high quality network resulting in a pre-empted market. The above results suggest that platforms which are highly differentiated in quality pose a barrier to entry for CP's that have low content quality. Indeed, the high asymmetry in quality induces market power on the platforms which enables them to raise prices and thus exclude low quality content CP's from the market.

Our second set of results pertain to the quality choice SPE. We show that if we assume a fixed cost (or costless) quality investment model, then a SPE in quality choice stage exists. Moreover, we characterize all such equilibria and show that depending on $\overline{\gamma}$, one of the following three types of equilibrium exists in the final stage of the game: a)Maximal differentiation equilibrium which involves one platform picking the highest quality and the other the lowest, b)Partial differentiation equilibrium where one platform picks the highest quality and the other chooses a proportion of the highest quality that depends on $\overline{\gamma}$, c)Partial differentiation where one platform picks the lowest quality and the other picks some positive fraction of the highest quality.

Other than the papers cited above on two sided-markets, our paper builds on contributions to the industrial organization literature on price competition and quality choice in vertically differentiated markets, [5, 15, 3, 9, 12]. Somewhat related to our work are the competition models analyzed in Mussachio [et al] [7]. Although they study the effect of a network regime on CP investments and network quality, their neutral and non-neutral models can be independently viewed as modeling competition between network providers where quality choice is endogenous. In both of these models, the providers are assumed to be symmetric. Moreover, consumers and CP's are viewed as homogenous and the user base on both sides of the market is fixed. The resulting equilibrium in network quality is therefore symmetric with all network providers choosing the same quality. Issues of market coverage are also not addressed. The distinguishing feature our work is to consider the strategic interactions between quality-picking platforms and heterogenous end-users and their effect on market entry by the CP's[3]

The rest of this paper is organized as follows. In Section 2 we present the model. In Section 3 we analyze the model solving for the SPE of this game as well as discussing our findings. In Section 4 we conclude. Due to space limitations certain proofs have been omitted and those that are essential have been relegated to the Appendix.

2. MODEL

We consider two platforms denoted by α and β, and a continuum of consumers and content providers with a unit volume. Let y_α and y_β be the quality-of-service chosen by platforms α and β, respectively. We represent the quality of a CP j by the scalar γ_j. We assume γ_j is uniformly distributed with support $[\overline{\gamma}-1, \overline{\gamma}]$ where $\overline{\gamma} \geq 1$. We also assume γ_j are independent identically distributed random variables across the population of content providers. Let $\phi : [0,1] \to \{\alpha, \beta\}$ and $\widehat{\phi} : [0,1] \to \{\alpha, \beta\}$ be mappings from the space of consumers and providers respectively to the set of platforms. A consumer i on a platform $\phi(i) \in \{\alpha, \beta\}$ connecting to a CP j on platform $\widehat{\phi}(j) \in \{\alpha, \beta\}$ receives utility,

$$u_{ij}(y_{\phi(i)}, y_{\widehat{\phi}(j)}, \gamma_j) = \gamma_j + \min\{y_{\phi(i)}, y_{\widehat{\phi}(j)}\},$$

The consumer utility implies that a consumer on a high quality platform, connecting to a content provider present on a high quality platform, receives more utility than if he connected to a content provider of the same quality on the lower quality platform. In essence, utility captures the fact that service quality depends on the bottleneck, [13].

A consumer i on platform $\phi(i)$ connects with CP j if and only if $u_{ij} \geq 0$. Let $F_i(y_{\phi(i)}, r_\alpha, r_\beta)$ be the quality perceived by consumer i when he joins platform $\phi(i)$. Formally,

$$F_i(y_{\phi(i)}, r_\alpha, r_\beta, \overline{\gamma}) = \int_0^1 E\left[\max\{u_{ij}(y_{\phi(i)}, y_{\widehat{\phi}(j)}, \gamma_j), 0\}\right] dj.$$

Here r_α and r_β are the masses of content providers that join platform α and β respectively. We assume that consumers have heterogenous preferences represented by a taste parameter θ_i which is uniformly distributed in the interval [0,1]. A consumer i perceives the quality of platform $\phi(i)$ as his expected utility, $F_i(y_{\phi(i)}, r_\alpha, r_\beta, \overline{\gamma})$. In addition, each consumer has a reservation utility R. The prices charged by the platforms are p_α and p_β for platforms α and β respectively. Each consumer connects to at most one platform but once connected has access to all content due to the interconnection of the platforms. Therefore, the net utility of a consumer i connecting to platform $\phi(i)$ is given by,

$$U_i(\phi(i)) = \max\{R + \theta_i F_i(y_{\phi(i), r_\alpha, r_\beta, \overline{\gamma}}, \cdot) - p_{\phi(i)}, 0\}.$$

Consumers prefer the platform with the higher perceived quality, ceteris paribus.

Platforms also charge a fixed connection fee w_α and w_β to CP's that connect to them. We assume that CP's make revenues by selling advertising. Let q_α and q_β denote the mass of consumers locating on platforms α and β respectively. Without loss of generality we assume $y_\alpha \geq y_\beta$. We also assume that $y_\phi(i) > \epsilon$ for some $\epsilon > 0$, where ϵ is some minimum quality level that platforms have to guarantee. The utility v_j of a CP j is defined to be his profit

$$v_j = V(\gamma_j, y_\alpha, y_\beta, q_\alpha, q_\beta) - w_{\widehat{\phi}(j)},$$

where,

$$V(\cdot) = \begin{cases} g(\gamma_j, y_\alpha)q_\alpha + g(\gamma_j, y_\beta)q_\beta, & \text{if } \widehat{\phi}(j) = \alpha, \\ g(\gamma_j, y_\beta)q_\beta + g(\gamma_j, y_\beta)q_\alpha, & \text{if } \widehat{\phi}(j) = \beta. \end{cases}$$

Here $g(\gamma_j, y_{\widehat{\phi}(i)})$ is a function that represents the advert price and is increasing in both parameters; CP j gets a higher advert price for having a higher content quality and also for locating on a platform with higher quality. The function $V(\gamma_j, y_\alpha, y_\beta, q_\alpha, q_\beta)$ represents the gross revenue earned by

[3]Market entry is proxied by market coverage.

a CP j. This function depends on which platform the CP joins as well as the number of consumers on the other side of the market. In particular, if a CP j joins the high quality platform, it is able to command a higher advert price for connections arising from consumers on that platform. If a CP joins the lower quality platform its advert price is the same for the two platforms, i.e, the advert price depends on the platform that acts as the bottleneck.

Finally we consider the payoff functions of the platforms: we assume that platforms incur no cost(or a fixed cost) in choosing the quality level. The payoff of platform α, which we denote by π_α, is given by

$$\pi_\alpha = p_\alpha q_\alpha + w_\alpha r_\alpha,$$

where q_α is the mass of consumers attached to platform α and r_α is the mass of CP's attached to platform α. The payoff for platform β is similar. The model we have outlined corresponds to a dynamic game with the following timing of events:

1) Quality Choice Stage: Platforms α and β simultaneously choose quality-of-service from the interval $[\epsilon, \overline{y}]$. [4]
2) Pricing Decisions: Platforms simultaneously choose connection fees w_α and w_β.
3) Connection Decisions: CPs decide which platform to join.
4) Pricing Decisions: Platforms simultaneously choose prices p_α and p_β.
5) Connection Decisions: Users decide which platform to join.
6) Consumption Decisions: Consumers decide which CPs to connect.

We solve this game by considering its subgame perfect equilibrium (SPE), which we find using backward induction. Steps 4-6 are similar to a pricing game with vertical differentiation; steps 1-3 are similar to a quality choice and pricing game with vertical differentiation.

3. MODEL ANALYSIS

Let $\mathcal{I} = \{\alpha, \beta, [0,1]_j, [0,1]_i\}$ denote the set of players in the multi-stage game, where α and β are the platforms, $[0,1]_j$ and $[0,1]_i$ are the continuum of content providers and consumers respectively, both with unit volume. We denote the information set at stage k of the game for a player $i \in \mathcal{I}$ by h_i^k. Let the set of actions available to a player i at stage k and information set h_i^k be denoted as $A_i(h_i^k)$.

3.1 Consumption Decisions.

We begin by analyzing the last stage of the game, i.e, the consumption decisions of the consumers. Only the consumers make a move in this stage.The choice set of a consumer $i \in [0,1]_i$ given an information set h_i^k is $A_i(h_i^k) \subset 2^{[0,1]_j}$. A consumer i on a platform $\phi(i) \in \{\alpha, \beta\}$ accessing content of a CP j on platform $\widehat{\phi}(j) \in \{\alpha, \beta\}$ receives utility,

$$u_{ij}(y_{\phi(i)}, y_{\widehat{\phi}(j)}, \gamma_j) = \gamma_j + \min\{y_{\phi(i)}, y_{\widehat{\phi}(j)}\}.$$

As previously discussed, this implies that a consumer connecting to a higher quality platform gets more utility when he accesses content providers on that platform, compared to when he connects to the same content providers while connected to the lower quality platform. Consumer i on platform $\phi(i)$ connects with CP j whenever $u_{ij} \geq 0$ which implies that i connects with CP j if $\gamma_j \geq -\min\{y_{\phi(i)}, y_{\widehat{\phi}(j)}\}$. Since $\gamma_j > 0$, whenever a consumer joins any of the platforms he will connect to all content providers on that platform.

3.2 Consumer Platform Connection Decisions.

In this stage the consumers are the only movers and they decide which platforms to join. The choice set of a consumer i given any h_i^k is $A_i(h_i^k) = \{\alpha, \beta\}$. Through his information set, a consumer has knowledge of the number of content providers on each platform, the prices that the platforms charge and the quality level of each platform. Each consumer i solves the following utility maximization problem,

$$\begin{aligned} \text{maximize} \quad & U_i(\phi(i)) \\ s.t. \quad & \phi(i) \in \{\alpha, \beta\}. \end{aligned}$$

A consumer that does not join any platform receives a utility of zero. We proceed next to give the demand functions faced by each platform based on consumer choices in this stage. We first make the following assumption on the reservation price which we invoke through out the analysis of this paper.

ASSUMPTION 1. *R is large enough that the consumer market is covered.*

Let $y_\alpha > y_\beta$, then it follows that $F_i(y_\alpha, \cdot) > F_i(y_\beta, \cdot)$. If $\tilde{\theta} \equiv \frac{p_\alpha - p_\beta}{F_i(y_\alpha,\cdot) - F_i(y_\beta,\cdot)}$ consumers with a taste parameter $\theta_i \geq \tilde{\theta}$ join the platform with the higher perceived quality, $F_i(y_\alpha, \cdot)$, since $\theta_i F_i(y_\alpha, \cdot) - p_\alpha \geq \theta_i F_i(y_\beta, \cdot) - p_\beta$ if and only if $\theta_i \geq \tilde{\theta}$. Those whose taste parameter $\theta_i < \tilde{\theta}$ will join platform β if and only if $\theta_i \geq \frac{p_\beta - R}{F_i(y_\beta,\cdot)}$. From Assumption 1 we can deduce that if R is large enough then all consumers get a positive utility by participating in the market. In such a market, the demands for the platforms are characterized as follows,

$$\begin{aligned} q_\beta(p_\alpha, p_\beta) &= \left(\frac{p_\alpha - p_\beta}{F_i(y_\alpha, \cdot) - F_i(y_\beta, \cdot)} \right), \\ q_\alpha(p_\alpha, p_\beta) &= \left(1 - \frac{p_\alpha - p_\beta}{F_i(y_\alpha, \cdot) - F_i(y_\beta, \cdot)} \right). \end{aligned}$$

We will show in the next stage that if $y_\alpha = y_\beta$ then any allocation of demand across platforms is possible at the resulting price equilibrium.

3.3 Platform Pricing Decisions for the Consumer Side.

In this stage of the game the platforms are the only movers and they decide what prices to charge to the consumers. The choice set of platform $i \in \{\alpha, \beta\}$, given any h_i^k, is $A_i(h_i^k) = p_i \in \mathbb{R}$. Thus the platforms simultaneously decide what prices p_α and p_β to charge to consumers. Through

[4] ϵ represents a minimum quality that a platform is required to maintain. $\overline{y}$ represents the maximum quality that can be achieved, for instance due to technological limits.

his information set, a platform has knowledge of the number of content providers on each platform and the quality level of each platform. Profit for platform i is given by, $\pi_i = p_i q_i + w_i r_i$, where w_i is the price charged to content providers and r_i is the mass of content providers on platform i. The demand for platform i denoted by q_i is defined by the set of consumers who maximize their utility when they join platform i. The Nash equilibrium in this price subgame depends in the information set h_i^k. In particular, if h_i^k is such that $y_\alpha > y_\beta$ it can be shown that, $p_\beta = \frac{1}{3}(F_i(y_\alpha,\cdot) - F_i(y_\beta,\cdot))$, and $p_\alpha = \frac{2}{3}(F_i(y_\alpha,\cdot) - F_i(y_\beta,\cdot))$, and the consumer demands addressed to the platforms at this equilibrium are $q_\alpha = \frac{2}{3}$ and $q_\beta = \frac{1}{3}$. If h_i^k is such that $y_\alpha = y_\beta$ then $F_i(y_\alpha,\cdot) = F_i(y_\alpha,\cdot)$. A Bertrand competition ensues with $p_\alpha = p_\beta = 0$ the resulting subgame Nash equilibrium. The consumer demands addressed to the platforms at this equilibrium price are indeterminate, i.e any allocation such that $q_\alpha + q_\beta = 1$ is a solution.

3.4 Content Provider Connection Decisions

Given the quality of service offered by platforms y_α and y_β and the prices w_α and w_β, the content providers decide on which platform to locate. The choice set of a CP j given any h_j^k is $A_j(h_j^k) = \{\alpha, \beta\}$. As mentioned in section 3, γ_j is uniformly distributed with a support $[\overline{\gamma}, \overline{\gamma} - 1]$ where $\overline{\gamma} \geq 1$. The following characterizes the utility v_j gained by a content provider when he joins a platform:

$$v_j = \begin{cases} g(\gamma_j, y_\alpha)q_\alpha + g(\gamma_j, y_\beta)q_\beta - w_\alpha, & \text{if } \widehat{\phi}(j) = \alpha, \\ g(\gamma_j, y_\beta)q_\beta + g(\gamma_j, y_\beta)q_\alpha - w_\beta, & \text{if } \widehat{\phi}(j) = \beta. \end{cases}$$

A CP's utility is zero if he doesn't join any platform. In this stage, CP's take the investment(choice) in quality as given. Moreover, they anticipate the mass of consumers on each platform q_α and q_β. Let $g(\gamma_j, y_{\widehat{\phi}(j)}) = \gamma_j y_{\widehat{\phi}(j)}$, a CP j perceives the quality of platform α to be $y_\alpha q_\alpha + y_\beta q_\beta$ and that of platform β to be $y_\beta q_\alpha + y_\beta q_\beta$.

A CP j maximizes the utility v_j and is indifferent between the two platforms if and only if $\gamma_j(y_\alpha q_\alpha + y_\beta q_\beta) - w_\alpha = \gamma_j(y_\beta q_\alpha + y_\beta q_\beta) - w_\beta$. Let $\widetilde{\gamma_j} = \frac{w_\alpha - w_\beta}{q_\alpha(y_\alpha - y_\beta)}$, then the content providers with quality exceeding $\widetilde{\gamma_j}$ join the high quality platform α. Those whose content quality is lower than $\widetilde{\gamma_j}$, but larger than $w_\beta/(y_\beta(q_\beta + q_\alpha))$, join the lower quality platform β. The others do not join any platform. Since $y_\alpha > y_\beta$ there's a possibility of platform α preempting the market with a limit price $w_\alpha = w_\beta + (\overline{\gamma} - 1)(q_\alpha(y_\alpha - y_\beta))$, this is a similar effect to that described by Wauthy [15]. The mass of content providers $r_\alpha(r_\beta)$ is defined by those content providers who maximize v_j when they join platform $\alpha(\beta)$. It follows that given the n-tuple $(\overline{\gamma}, y_\alpha, y_\beta, w_\alpha, w_\beta)$, there are four possible market configurations that may arise depending on the demands addressed to the platforms. We next describe the market configurations of content providers at different CP prices

1. Uncovered Market: $r_\alpha(w_\alpha, w_\beta) < 1$, $r_\beta(w_\alpha, w_\beta) = 0$. We denote this configuration as CI. The demand addressed to platform α is given by $r_\alpha(w_\alpha, w_\beta) = \left(\overline{\gamma} - \frac{w_\alpha}{q_\alpha y_\alpha + q_\beta y_\beta}.\right)$.

2. Uncovered Market: $r_\alpha(w_\alpha, w_\beta) + r_\beta(w_\alpha, w_\beta) < 1$, $0 < r_\alpha(w_\alpha, w_\beta) < 1$, $0 < r_\beta(w_\alpha, w_\beta) < 1$. We denote this configuration as CII. The CP demands addressed to platform α and β are given by $r_\beta(w_\alpha, w_\beta) = \left(\frac{w_\alpha - w_\beta}{q_\alpha(y_\alpha - y_\beta)} - \frac{w_\beta}{y_\beta(q_\alpha + q_\beta)}\right)$, and $r_\alpha(w_\alpha, w_\beta) = \left(\overline{\gamma} - \frac{w_\alpha - w_\beta}{q_\alpha(y_\alpha - y_\beta)}\right)$.

3. Covered market: $r_\alpha(w_\alpha, w_\beta) + r_\beta(w_\alpha, w_\beta) = 1$, $r_\alpha(w_\alpha, w_\beta) > 0$ and $r_\beta(w_\alpha, w_\beta) > 0$. We denote this configuration as $CIII$. The CP demands in this configuration are given by,$r_\beta(w_\alpha, w_\beta) = \left(\frac{w_\alpha - w_\beta}{q_\alpha(y_\alpha - y_\beta)} - (\overline{\gamma} - 1)\right)$ and $r_\alpha(w_\alpha, w_\beta) = \left(\overline{\gamma} - \frac{w_\alpha - w_\beta}{q_\alpha(y_\alpha - y_\beta)}\right)$.

4. Preempted covered market: $r_\alpha(w_\alpha, w_\beta) = 1, r_\beta(w_\alpha, w_\beta) = 0$. We denote this as configuration CIV.

3.5 Platform Pricing Decision for the Content Provider Side

In this stage of the game the platforms are the only movers and they decide what prices to charge to the CPs. The choice set of platform $i \in \{\alpha, \beta\}$ given any h_i^k is $A_i(h_i^k) = w_i \in \mathbb{R}$. Thus the platforms simultaneously decide what prices w_α and w_β to charge to CPs. In the price subgame Nash equilibrium, each platform i maximizes $\pi_i = p_i q_i + r_i w_i$.

We now proceed to show that if $y_\alpha > y_\beta$ given any tuple $(\overline{\gamma}, y_\alpha, y_\beta)$ there exists a pure strategy price subgame Nash equilibrium pair (w_α^*, w_β^*). In addition, we characterize the price equilibria and the market configuration that results. In particular, we show the conditions under which particular market configurations arise depending on the parameters $\overline{\gamma}, y_\alpha$, and y_β.

We prove the existence by a construction argument. We first find possible price equilibrium pairs in each market configuration. We then check to see whether these price equilibrium pairs are indeed Nash equilibria of the price subgame. The rest of this section is organized as follows. In section 3.5.1 we will identify the possible candidates for the equilibrium price pairs under each market configuration. In section 3.5.2, we check to see if the equilibrium price candidates are best replies on the whole domain of strategies: That is, not only are they best responses in their respective market configurations but that they are also best replies if the other market configurations are taken into account. We then give the conditions in terms $\overline{\gamma}, y_\alpha$, and y_β that define what market configurations will result in the price subgame Nash Equilibrium.

3.5.1 Equilibrium Price Pairs: Exogenous Market.

Uncovered Market - CI**.** The condition for an uncovered market where only the high-quality platform participates in the market is, $\frac{w_\beta}{y_\beta} \geq \frac{w_\alpha}{y_\beta q_\beta + y_\alpha q_\alpha} > \overline{\gamma} - 1$. One can show that equilibrium price pairs exist. We denote a price pair in this configuration as $(w_\alpha^{ui}, w_\beta^{ui})$

Uncovered Market - CII**.** The condition for an uncovered market in which both the high-quality and low-quality platforms serve the market is given by, $\overline{\gamma} - 1 < \frac{w_\beta}{y_\beta} < \frac{3(w_\alpha - w_\beta)}{2(y_\alpha - y_\beta)} < \overline{\gamma}$. One can show that their exists a unique equilibrium price pair in this configuration. We denote this pair as (w_α^u, w_β^u).

Covered market- $CIII$ **.** The condition for a covered market in which both the high-quality and low-quality platforms serve the market is, $\frac{w_\beta}{y_\beta} \leq \overline{\gamma} - 1 < \frac{3(w_\alpha - w_\beta)}{2(y_\alpha - y_\beta)} < \overline{\gamma}$. We define two types of solution under this configuration.

Interior Solution. A market is covered with an interior solution in the price subgame if the price charged by the lower quality platform is lower than the value derived by the lowest quality content provider. In this market configuration the lowest quality content provider prefers the lowest quality platform. We denote the unique equilibrium pair in this configuration by $(w_\alpha^{ci}, w_\beta^{ci})$.

Corner solution. We denote the content provider market to be covered with a corner solution in the price subgame if the lower quality platform quotes a price that is just sufficient so that the lowest quality content provider joins the platform. In this case, a corner solution occurs and we get a unique equilibrium price pair denoted by $(w_\beta^{cc}, w_\alpha^{cc})$.

Covered Preempted market CIV **.** The condition for a covered market where only the high-quality platform participates in the market is, $(\overline{\gamma} - 1)(y_\beta q_\beta + y_\alpha q_\alpha) - w_\alpha \geq \max\{0, (\overline{\gamma} - 1)y_\beta - w_\beta\}$ In this configuration one can show that there are many possible price equilibria, we denote a pair in this set as (w_α^p, w_β^p).

3.5.2 Nash Equilibrium in the Price Subgame

In this section, we provide results that show the existence of a pure strategy Nash equilibrium in the price-subgame. We characterize the price subgame equilibria in terms of $\overline{\gamma}$, y_α, and y_β, conditions for their existence, and the resulting market configurations. Our results show that the uncovered market configuration, (CI), does not occur at a subgame price equilibrium. On the other hand, we show that given a tuple $(y_\alpha, y_\beta, \overline{\gamma})$ one of the other configurations, CII, $CIII$ or CIV, will emerge. In doing so, we determine the set of parametric values $(\overline{\gamma}, y_\alpha, \beta)$ for which these different configurations exist and characterize the prices in each configuration using the same parameters. The proofs, which are omitted due to space limitations[5], evaluate the equilibrium price pairs derived in the previous section and determine if they are best replies across all the configurations.

In this section we present the Theorem that summarizes the above results. To this purpose, we define the following sets of prices,

$$\begin{aligned}
\mathcal{R}_{\mathcal{I}} &= \{(w_\alpha, w_\beta) | r_\alpha + r_\beta < 1,\ r_\alpha > 0,\ r_\beta = 0\},\\
\mathcal{R}_{\mathcal{II}} &= \{(w_\alpha, w_\beta) | r_\alpha + r_\beta < 1,\ r_\alpha > 0,\ r_\beta > 0\},\\
\mathcal{R}_{\mathcal{III}} &= \{(w_\alpha, w_\beta) | r_\alpha + r_\beta = 1,\ r_\alpha > 0,\ r_\beta > 0\},\\
\mathcal{R}_{\mathcal{IV}} &= \{(w_\alpha, w_\beta) | r_\alpha + r_\beta = 1,\ r_\alpha = 1,\ r_\beta = 0\}.
\end{aligned}$$

The sets $\mathcal{R}_{\mathcal{I}}$, $\mathcal{R}_{\mathcal{II}}$, $\mathcal{R}_{\mathcal{III}}$ and $\mathcal{R}_{\mathcal{IV}}$ consists of price pairs (w_α, w_β) that result in configuration CI, CII, $CIII$ and CIV respectively. Before proceeding we make the following definition of a subgame price equilibrium.

DEFINITION 1. *A (subgame perfect) Nash price equilibrium pair (w_α^*, w_β^*) is a pair of price strategies such that $\pi_\alpha(w_\alpha^*, w_\beta^*) \geq \pi_\alpha(w_\alpha, w_\beta^*)$ for all $w_\alpha \in \mathbb{R}$ and $\pi_\beta(w_\alpha^*, w_\beta^*) \geq \pi_\beta(w_\alpha^*, w_\beta)$ for all $w_\beta \in \mathbb{R}$.*

[5]The proofs can be found in the LIDS technical report

We next present the Theorem showing that for any tuple $(\overline{\gamma}, y_\alpha, y_\beta)$ a price subgame Nash equilibrium exists and only one market configuration is feasible. In addition, for market configurations CII and $CIII$, the price characterizations are unique.

THEOREM 1. *Let Assumption 1 hold. Given $(\overline{\gamma}, y_\alpha, y_\beta)$ there exists a Nash equilibrium pair (w_α^*, w_β^*) in the price-subgame. Moreover, the resulting market configuration is unique and the following hold:*

1. If $1 < \overline{\gamma} < \frac{5y_\beta + 22y_\alpha}{9(y_\beta + 2y_\alpha)}$, then the equilibrium price pair is unique and $(w_\alpha^*, w_\beta^*) \in \mathcal{R}_{\mathcal{II}}$. In particular, the equilibrium price pair is given by (w_α^u, w_β^u).

2. If $\frac{5y_\beta + 22y_\alpha}{9(y_\beta + 2y_\alpha)} \leq \overline{\gamma} \leq \min\left\{\frac{y_\beta + 8y_\alpha}{3(2y_\alpha + y_\beta)}, \frac{17y_\beta + 10y_\alpha}{3(2y_\alpha + 7y_\beta)}\right\}$ then the equilibrium price pair is unique and $(w_\alpha^*, w_\beta^*) \in \mathcal{R}_{\mathcal{III}}$. In particular, the equilibrium price pair is given by $(w_\alpha^{cc}, w_\beta^{cc})$.

3. If $\frac{17y_\beta + 10y_\alpha}{3(2y_\alpha + 7y_\beta)} < \overline{\gamma} < \frac{7}{6}$ then the equilibrium price pair is unique and $(w_\alpha^*, w_\beta^*) \in \mathcal{R}_{\mathcal{III}}$. In particular, the equilibrium price pair is given by $(w_\alpha^{ci}, w_\beta^{ci})$

4. If $\max\left\{\frac{7}{6}, \frac{y_\beta + 8y_\alpha}{3(2y_\alpha + y_\beta)}\right\} \leq \overline{\gamma} < \infty$ then $(w_\alpha^p, w_\beta^p) \in \mathcal{R}_{\mathcal{IV}}$.

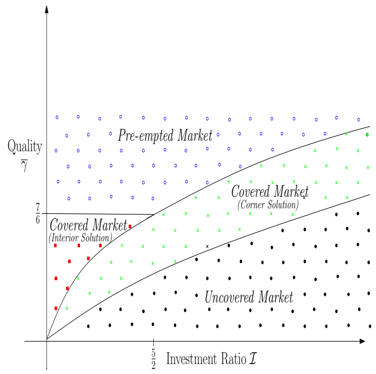

Figure 1: Quality Characteristic $\overline{\gamma}$ versus Investment Ratio $y_\alpha/y_\beta = \mathcal{I}$ and Resulting Market Configurations .

Figure 1 shows the resulting market configurations for different values of the investment ratio and quality characteristic. In particular, given any investment ratio and content quality characteristic, the above figure shows the distinct resulting market configuration. For a fixed $\overline{\gamma}$, when the quality characteristic $\overline{\gamma}$ increases then a preempted solution is more likely. Note that the CPs are relatively close to each other, in terms of content quality, since $\overline{\gamma} \to \infty$ the ratio $\frac{\overline{\gamma}}{\overline{\gamma}-1} \to 1$. Thus CPs are less distinguishable from each other; a decision made by any one CP will be mirrored by the other CPs. As the investment ratio increases the two platforms become more differentiated. This means that the platforms can exert some market power and therefore uncovered market region gets larger and the onset of a preempted market requires a higher value of $\overline{\gamma}$. However, for large $\overline{\gamma}$, the relative likeness of the content providers dominates the differentiation effects of the platforms and a preempted market is realized.

3.6 Quality Choice

In this stage of the game the platforms are the only movers and they decide what quality to set. We assume that once platforms are in operation quality choice is costless. The choice set of platform $i \in \{\alpha, \beta\}$ given any h_i^k is $A_i(h_i^k) = y_i$ where $y_i \in [\epsilon, \overline{y}]$. Thus the platforms simultaneously decide what quality to choose. We find the equilibrium quality choices by considering the best reply responses of the two platforms. We find the set that contains platform β's best replies to platform α's choices and vice versa. We then analyze the points where these sets intersect and show that they indeed are the subgame perfect equilibria. Due to space limitations the analysis is provided in Appendices A.1 and A.2. For ease of presenting the Theorem that characterizes the subgame perfect equilibrium of the quality choice game, and the Corollaries that characterize the resulting market configurations, we make the following classifications: **C.1** $1 < \overline{\gamma} < \frac{7}{6}$, **C.2** $\frac{7}{6} \leq \overline{\gamma} < \frac{24}{18}$ and $\epsilon \geq -\overline{y}\frac{2(3\overline{\gamma}-4)}{3\overline{\gamma}-1}$, **C.3** $\frac{7}{6} \leq \overline{\gamma} < \frac{24}{18}$ and $\epsilon < -\overline{y}\frac{2(3\overline{\gamma}-4)}{3\overline{\gamma}-1}$, **C.4** $\overline{\gamma} \geq \frac{8}{6}$

The above classifications follow from the analysis in the Appendix where we partition the range in which $\overline{\gamma}$ lies into three sections depending on the types of market configurations that are possible in each partition. From a qualitative view, the three partitions represent the ranges in which the average content quality is low, medium or high. In the medium range we make two further classifications depending on the asymmetry of the platforms;**C.2** and **C.3** represents low and high asymmetry respectively. We will now present the theorem that characterizes the results of the quality choice game.

THEOREM 2. *Given $(\overline{\gamma}, \overline{y}, \epsilon)$ there exists a subgame perfect Nash equilibrium (SPE) in the quality choice game. Moreover, the following hold:*

*(i)If **C.1** holds then the SPE entails maximal differentiation where one platform chooses the best quality, $\overline{y}$, and the other platform chooses the lowest quality, ϵ.*

*(ii) If **C.2** or **C.4** holds then the SPE entails partial to maximal differentiation where one platform chooses a quality, $\tilde{y} \in [f(\overline{y}, \epsilon), \overline{y}]$, and the other platform chooses the lowest quality, ϵ.*

*(iii) If **C.3** holds then the SPE entails one platform choosing the highest quality, $\overline{y}$, and the other one choosing a proportion of $\overline{y}$ that depends on the quality characteristic $\overline{\gamma}$. In particular, the low quality platform picks $y_l = -\overline{y}\frac{2(3\overline{\gamma}-4)}{3\overline{\gamma}-1}$.*

In general, the above results suggest that the platforms differentiate in platform quality to soften price competition. The extent of the differentiation depends on the content quality characteristic $\overline{\gamma}$. In particular, when the average content provider quality[6]is low, platforms soften price competition through maximal differentiation. When $\overline{\gamma}$ is in the medium range or the high range partial differentiation occurs.

COROLLARY 1. *If **C.1** holds both platforms enjoy positive market share in the content provider market with the resulting market configuration depending on the investment ratio $\mathcal{I}$ and $\overline{\gamma}$. In particular,*

1. If $1 < \mathcal{I} < -\frac{21\overline{\gamma}-17}{2(3\overline{\gamma}-5)}$ then a covered market with an interior solution is the outcome.

2. If $-\frac{21\overline{\gamma}-17}{2(3\overline{\gamma}-5)} \leq \mathcal{I} < -\frac{9\overline{\gamma}-5}{2(9\overline{\gamma}-11)}$ then a covered market with a corner solution is the outcome.

3. If $-\frac{9\overline{\gamma}-5}{2(9\overline{\gamma}-11)} \leq \mathcal{I} < \Phi$, where $\Phi < \infty$, then an uncovered market is the outcome.

COROLLARY 2. *If **C.2** or **C.3** or **C.4** holds, one platform has all the market share in the content provider market, i.e., a pre-empted market is the outcome. This market configuration is independent of the investment ratio, $\mathcal{I}$.*

When $\overline{\gamma}$ is low, any of the three market configurations can occur depending on the asymmetry of the platforms[7]. When $\mathcal{I}$ is low, a covered market results since the differentiation is small and price competition is intense. In contrast, high values of asymmetry result in an uncovered market since the differentiation is high and price competition is relaxed. For medium to high values of $\overline{\gamma}$ only a preempted market results regardless of the asymmetry between platforms. A similar argument to that in section 3.5.2 applies in this case.

4. CONCLUSIONS

We study duopoly competition between two interconnected ISP's in the presence of quality choice and service bottle neck effects. We show that given two asymmetric platforms (i.e. with different quality levels), a price SPE on both sides of the market exists. Moreover, we show that the higher the asymmetry the more likely the CP market is to be uncovered if $\overline{\gamma}$ is low. This suggests that highly differentiated platforms provide a barrier to entry because of their market power. Our final results show that a SPE exists for the Quality choice game and the equilibrium involves either maximal differentiation,partial differentiation with one platform choosing the highest quality and the other a fraction of the highest quality, or partial differentiation with one platform choosing the lowest quality and the other a positive fraction of the highest quality.

A limitation of our model is the assumption a fixed investment cost for quality (or costless quality choice). Nevertheless we believe that with low marginal costs of investment the effects captured in this model will still hold. Moreover, in certain cases ISP's can increase or lower their quality without cost. For example once an ISP invests a fixed amount on a router it can increase or decrease bandwidth(hence altering quality) for certain traffic types by simply setting parameters at no additional costs.

[6]Represented by the content quality characteristic $\overline{\gamma}$

[7]Characterized by the Investment ratio, $\mathcal{I}$

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APPENDIX

A.

A.1 Best Replies

We first show that a symmetric equilibrium is not feasible. Let $j, i \in \{\alpha, \beta\}$ and $B_i(y_j)$ be the set of $y_i^* \in [\epsilon, \overline{y}]$ such that, $y_i^* \in \arg\max_{y_i \in [\epsilon,\overline{y}]} \pi_j(y_i, y_j)$.

Lemma 1. *Let $y_j \in [\epsilon, \overline{y}]$ then $y_j \notin B_i(y_j)$.*

Proof. We show that given y_j platform i never chooses $y_i = y_j$ and therefore a symmetric equilibrium is not possible. A symmetric argument applies for the other platform. Assume $y_j \in B_i(y_j)$ so that $y_i = y_j$, then both platforms would make zero profits because of Bertrand competition on both sides of the market. We show that there exists profitable deviations for platform i. We consider two cases: *Case I* $\epsilon \le y_i = y_j < \overline{y}$. Let platform i increase its quality to $y_i + \delta < \overline{y}$, where $\delta > 0$; platform i becomes the high quality platform. Results from Theorem 1 imply that the resulting equilibrium price w_i for the high quality platform given the subgame $(\overline{\gamma}, y_i+\delta, y_j)$ is less than the utility earned by the highest quality content provider under all the market configurations, i.e., $w_i < \overline{\gamma}(q_j y_j + q_i y_i)$ for all $\overline{\gamma} > 1$, therefore $r_i > 0$. Moreover, from Theorem 1, we can show that $w_i > w_j \ge 0$. This implies that $\pi_i > 0$. Thus platform i would prefer to set quality $y_i = y_j + \delta$ instead of $y_i = y_j$.

Case II $\epsilon < y_i = y_j = \overline{y}$. Let platform i decrease its quality to $y_i - \delta < \overline{y}$; platform i becomes the low quality platform. Since the platforms are now differentiated, the price charged to consumers by platform i is $p_i > 0$ and $q_i = 1/3$. Therefore $p_i q_i + w_i r_i = \pi_i > 0$ since $w_i \ge 0$. We conclude that platform i would prefer to set quality $y_i = y_j - \epsilon$ instead of $y_i = y_j = \overline{y}$. □

Given quality choice y_α, platform β can choose a best reply that depends on whether it acts as a high quality or a low quality platform. In the former case it chooses a reply in the domain $(y_\alpha, \overline{y}]$ and in the latter case it chooses a reply in the domain $[\epsilon, y_\alpha)$. In order to avoid confusion when platform β is the high quality firm we will change notation as follows; we label the high(low) quality platform as $h(l)$ and the quality associated with it as $y_{h(l)}$.

A.1.1 Best reply in the domain $(y_l, \overline{y}]$

We will first analyze the best reply when platform β chooses a reply in the domain $(y_\alpha, \overline{y}]$. In this case platform β is the high quality platform and is labeled as y_h. We will show that the profit of the high quality firm is increasing in quality in every configuration. In Theorem 1 we have characterized the conditions for various market configurations to occur in the price subgame as a function of $\overline{\gamma}, y_l$ and y_h. These characterizations can be viewed as restrictions on y_h given $\overline{\gamma}$, y_l. When viewed as such the following hold,

1. Market is uncovered, with positive masses of consumers on both platforms, in the in the price subgame whenever,

$$y_h > -y_l \frac{9\overline{\gamma} - 5}{2(9\overline{\gamma} - 11)}. \tag{1}$$

2. Market is covered and a corner solution applies in the price subgame whenever,

$$y_h \in \left[-y_l \frac{21\overline{\gamma} - 17}{2(3\overline{\gamma} - 5)}, -y_l \frac{9\overline{\gamma} - 5}{2(9\overline{\gamma} - 11)}\right], \quad \text{if } 1 < \overline{\gamma} \le \tfrac{7}{6}, \tag{2}$$

$$y_h \in \left(-y_l \frac{3\overline{\gamma} - 1}{2(3\overline{\gamma} - 4)}, -y_l \frac{9\overline{\gamma} - 5}{2(9\overline{\gamma} - 11)}\right], \quad \text{if } \tfrac{7}{6} < \overline{\gamma} < \tfrac{22}{18}. \tag{3}$$

3. Market is covered and an interior solution applies in the price subgame whenever,

$$y_h \in \left(y_l, -y_l \frac{21\overline{\gamma} - 17}{2(3\overline{\gamma} - 5)}\right). \tag{4}$$

4. Market is preempted whenever,

$$y_h \in \left(y_l, -y_l \frac{3\overline{\gamma} - 1}{2(3\overline{\gamma} - 4)}\right], \text{ if } \tfrac{21}{18} \le \overline{\gamma} < \tfrac{24}{18}, \tag{5}$$

$$\overline{\gamma} \ge \frac{24}{18} \tag{6}$$

One can also deduce from the above that the uncovered configuration is possible only if $1 < \overline{\gamma} < \frac{22}{18}$; a covered market configuration with a corner solution is possible only if $1 < \overline{\gamma} < \frac{24}{18}$; a covered market configuration with an interior solution is possible only if $1 < \overline{\gamma} < \frac{7}{6}$; and preempted market configuration is possible only if $\overline{\gamma} \geq \frac{7}{6}$. The following lemma shows that the profit of a high quality platform is increasing in its quality in all configurations.

LEMMA 2. *Given y_l, $\overline{\gamma}$ and the domain $(y_l, \overline{y}]$, the profit function $\pi_h(y_l, y_h)$ is increasing in y_h for the market configurations CI, CII, and CIV.*

PROOF. We show that for each configuration the profit function $\pi_h(y_l, y_h)$ is increasing in y_h.

Uncovered Configuration CI: The profit function in this configuration is given by,

$$\pi_h^u = \frac{(y_h(24\overline{\gamma}+16) + y_l(12\overline{\gamma}-1))^2(y_h - y_l)}{54(y_l + 8y_h)^2}.$$

The derivative of the above function is positive for $\overline{\gamma} > 1$ and $y_h > y_l$, hence the profit function in this configuration is increasing in y_h.

Covered Configuration with corner solution CIII: The profit function in this configuration is given by,

$$\pi_h^{cc} = \frac{(y_h(16\overline{\gamma}+4) + y_l(3\overline{\gamma}-13))^2}{216(y_h - y_l)}.$$

The second derivative of the above function is given by,

$$\frac{\partial^2 \pi_h^{cc}}{\partial^2 y_h} = \frac{3y_l^2(1 - 2\overline{\gamma} + \overline{\gamma}^2)}{4(y_h - y_l)^3}.$$

The above derivative is positive for $\overline{\gamma} > 1$ and $y_h > y_l$, which implies that the function is convex under these restrictions. Moreover, the profit function π_h has a single root at $\tilde{y} = -\frac{y_l(3\overline{\gamma}-13)}{2(3\overline{\gamma}+2)}$. Since $\tilde{y} < \max\left\{-y_l\frac{3\overline{\gamma}-1}{2(3\overline{\gamma}-4)}, -y_l\frac{21\overline{\gamma}-17}{2(3\overline{\gamma}-5)}\right\}$ hence the profit function in this configuration is increasing in y_h.

Covered Configuration with interior solution CIII: The profit function in this configuration is given by,

$$\pi_h^{ci} = \frac{(6\overline{\gamma}+11)^2(y_h - y_l)}{486}.$$

The derivative of the above function is given by,

$$\frac{\partial \pi_h^{ci}}{\partial y_h} = \frac{(6\overline{\gamma}+11)^2}{486}.$$

and the derivative is positive. Therefore the profit function is increasing in y_h. $\square$

From Theorem 1 we can deduce that the preempted market configuration CIV is possible whenever $\overline{\gamma} \geq \frac{7}{6}$. Several price equilibria exist in this configuration depending on the tuple $(y_h, y_l, \overline{\gamma})$. Given y_l, and $\overline{\gamma}$ the correspondence $W : \mathbb{R}_+ \rightrightarrows \mathbb{R}_+$ gives the set of best response prices for each choice of $\tilde{y} \in (y_l, \overline{\gamma}]$. When $\frac{7}{6} \leq \overline{\gamma} < \frac{24}{18}$, we can partition the set in which y_h lies into two. These are,

$$(y_l, y_l(9\overline{\gamma}-8)] \text{ and } \left[y_l(9\overline{\gamma}-8), -y_l\frac{3\overline{\gamma}-1}{2(3\overline{\gamma}-4)}\right].$$

The set of prices in these two regions given the tuple y_l, y_h and $\overline{\gamma}$ are given below.

a. $\frac{7}{6} \leq \overline{\gamma} < \frac{24}{18}$ and $(y_l, y_l(9\overline{\gamma}-8)]$.
We deduce from Theorem 1[8] that given $\overline{\gamma}$ in the above range and y_h in the above partition many price equilibria exist. In particular, platform h will offer the price,

$$w_h = \frac{1}{3}(\overline{\gamma}-1)(y_l + 2y_h) - c,$$

where,

$$c \in [\frac{(9\overline{\gamma}-8)}{9}y_l - \frac{1}{9}y_h, \frac{(3\overline{\gamma}-1)}{9}y_l + \frac{(6\overline{\gamma}-8)}{9}y_h].$$

The choice of c depends on the price that platform l will pick. Specifically, the highest price charged by platform h for a particular y_h is by $\frac{1}{9}(y_h - y_l)(6\overline{\gamma}-5)$ and the lowest price is given by $\frac{1}{9}(5 - 3\overline{\gamma})(y_h - yl)$.

b. $\frac{7}{6} \leq \overline{\gamma} < \frac{24}{18}$ and $\left[y_l(9\overline{\gamma}-8), -y_l\frac{3\overline{\gamma}-1}{2(3\overline{\gamma}-4)}\right]$
In the second partition, we deduce from Theorem 1 that the highest price that platform h can charge is $w_h = \frac{1}{3}(\overline{\gamma}-1)(y_l+2y_h)$. The lowest price that platform h can charge at equilibrium for a particular y_h is given by $\frac{1}{9}(5 - 3\overline{\gamma})(y_h - yl)$.

When $\overline{\gamma} \geq \frac{24}{18}$ platform h will offer the price, $w_h = \frac{1}{3}(\overline{\gamma} - 1)(y_l + 2y_h) - c$, where,

$$c \in \left[\max\left\{\frac{(9\overline{\gamma}-8)}{9}y_l - \frac{1}{9}y_h, 0\right\}, \min\left\{\frac{(3\overline{\gamma}-1)}{9}y_l + \frac{(6\overline{\gamma}-8)}{9}y_h, (\overline{\gamma}-1)y_l\right\}\right],$$

where c depends on the price offered by platform l. Given $(y_l, \overline{\gamma})$ the correspondence $\Pi : \mathbb{R}_+ \rightrightarrows \mathbb{R}_+$ gives the set of profit values that platform h can attain for each choice $\tilde{y} \in (y_l, \overline{y}]$. We define the set of profit functions that have a maximum over the domain $(y_l, \overline{y}]$. Let

$$P(y_h) = \{\pi_h^p(y_h) | l(y_h) \leq \pi_h^p(y_h) \leq g(y_h)\}.$$

where $g(y_h) = \max\{\frac{1}{3}(\overline{\gamma}-1)(y_l + 2y_h), \frac{1}{9}(y_h - y_l)(6\overline{\gamma}-5)\}$ and $l(y_h) = \max\{\frac{1}{9}(5 - 3\overline{\gamma})(y_h - yl), \frac{2}{3}(\overline{\gamma}-1)(y_h - y_l)\}$. We will now show that if more than one market configuration is possible, for a given interval $(y_l, \overline{y}]$, given $\overline{\gamma}$, platform h will prefer to pick $y_h = \overline{y}$

LEMMA 3. *Given y_l, $\overline{\gamma}$ and an interval $(y_l, \overline{y}]$ assume that either (i) $\overline{\gamma} \leq \frac{7}{6}$, or (ii) $\overline{\gamma} > \frac{7}{6}$ and $\frac{\overline{y}}{y_l} \geq -\frac{3\overline{\gamma}-1}{2(3\overline{\gamma}-4)}$, then the best response, $B_h(y_l) = \overline{y}$.*

PROOF. It is sufficient to show that the profit function is increasing across the different market configurations under

[8]The full version of this Theorem gives the price characterizations and can be found in the LIDS report.

the above assumptions. We partition the domain in which $\overline{\gamma}$ is defined according to the types of market configurations that are possible for each $\overline{\gamma}$. We will then show that for each of these partitions the profit function is non decreasing in y_h.

Case I: $1 < \overline{\gamma} < \frac{21}{18}$; As previously stated, three market configurations are possible depending on the value of y_h and y_l; these are uncovered, CI, covered with a corner solution and covered with an interior solution, both of which are in $CIII$. Given a $\overline{\gamma}$ in the above range, the domain $(y_l, \infty]$ in which y_h lies can be partitioned into three sets; each of which corresponds to one of the three market configurations. These partitions are captured by the sets (1), (2) and (4). By lemma 2, we know that profits are increasing in y_h for each partition. We will first show that the value of the profit function in the partition defined in (1), is larger than any profit attained in the partition defined in (2) and then show that any profit attained in the partition defined in (4) is less than that attained in (2).

To show the first result we compare the infimum value of the profit function in the uncovered configuration to the highest profit attainable when platform h chooses y_h such that a covered market with a corner solution results (i.e, y_h is in the set specified by (2)). Let $y^{cc} = -y_l \frac{9\overline{\gamma}-5}{2(9\overline{\gamma}-11)}$, it follows that $\lim_{y_h \to y^{cc}} \pi_h^u(y_h, y_l) = \pi_h^{cc}(y^{cc}, y_l)$. By lemma 2, $\pi_h^u(y_h, y_l) > \pi_h^u(y^{cc}, y_l)$ whenever y_h lies in the set specified by the constraint (1). It also follows that $\pi_h^u(y_h, y_l) > \pi_h^{cc}(\tilde{y}, y_l)$ whenever $\tilde{y}$ is in the set specified by (2) since by lemma 2, $\pi_h^{cc}(y_{cc}, y_l) \geq \pi_h^{cc}(\tilde{y}, y_l)$.

To show the second result we compare the lowest value of the profit function in the covered configuration with a corner solution, to the supremum profit value attained when platform h chooses y_h such that a covered market with an interior solution results. The interval over which the covered configuration with an interior solution, $CIII$, is defined is open. Let $y^{ci} = -y_l \frac{21\overline{\gamma}-17}{2(3\overline{\gamma}-5)}$, since $\pi_h^{ci}(y, y_l)$ is right continuous, the supremum of $\pi_h^{ci}(y, y_l)$ over the range in which this configuration holds is given by $\pi_h^{ci}(y^{ci}, y_l)$. Moreover, it is the case that $\pi_h^{cc}(y^{ci}, y_l) = \pi_h^{ci}(y^{ci}, y_l)$. Therefore, it follows from lemma 2, that $\pi_h^{cc}(y_h, y_l) > \pi_h^{ci}(\tilde{y}, y_l)$ whenever y_h is in the set specified by (2) and $\tilde{y}$ is in the set specified by (4).

Case II: $\frac{7}{6} \leq \overline{\gamma} < \frac{22}{18}$; When $\frac{7}{6} \leq \overline{\gamma} < \frac{22}{18}$ three market configurations are possible depending on the value of y_h and y_l; these are uncovered, CI, covered with a corner solution, $CIII$, and a pre-empted market, CIV. Given a $\overline{\gamma}$ in the above range, the domain $(y_l, \infty]$ in which y_h lies can be partitioned into three sets each of which corresponds to one of the three market configurations. These partitions are captured by the sets (1), (3) and (5). We proceed in a similar manner as we did for case I. By lemma 2 we know that profits are increasing in y_h for each partition. We will first show that the value of the profit function in the partition defined by the inequality (1), is larger than any profit attained in the partition defined by (3) and then show that any profit attained in the partition defined by (5) is less than that attained in (3).

The first result is proved in the same way as it is done in case I. To show the second result we compare the infimum value of the profit function in the covered configuration with a corner solution to the profit value attained when platform h chooses y_h such that a pre-empted market results. Note the interval over which the covered configuration with a corner solution, $CIII$, is defined is open on its lower limit. Let $y^p = -y_l \frac{3\overline{\gamma}-1}{2(3\overline{\gamma}-4)}$, since $\pi_h^{cc}(y, y_l)$ is left continuous the infimum of $\pi_h^{cc}(y, y_l)$ over the range in which this configuration is defined is $\pi_h^{cc}(y^p, y_l)$. By plugging in $y_h = y^p$ into $\pi_h^{cc}(y_h, y_l)$ and $\pi_h^p(y_h, y_l)$ we determine that the $\lim_{y_h \to y^p} \pi_h^{cc}(y^p, y_l) = \pi_h^p(y^p, y_l)$. Note that for any $\pi_h^p(y_h) \in P(y_h)$ if $y_h \geq -y_l \frac{3\overline{\gamma}-1}{2(3\overline{\gamma}-4)}$ then $\pi_h^p(y_h)$ is single valued, see the price characterization in Theorem 1. Moreover, $\pi_h^p(y^p, y_l) \geq \tilde{\pi}_h^p(y_h, y_l)$ where $\tilde{\pi}_h^p(y_h, y_l) \in P(y_h)$ and $y_h \leq -y_l \frac{3\overline{\gamma}-1}{2(3\overline{\gamma}-4)}$. Therefore, it follows from lemma 2, that $\pi_h^{cc}(y_h, y_l) > \pi_h^p(\tilde{y}, y_l)$ when y_h is in the set specified by (3) and $\tilde{y}$ is in the set specified by (5).

Case III: $\frac{22}{18} \leq \overline{\gamma} < \frac{24}{18}$; When $\overline{\gamma}$ falls in the above range two market configurations are possible depending on the value of y_h and y_l; these are covered with a corner solution, $CIII$ and a pre-empted market, CIV. The domain $(y_l, \infty]$ in which y_h lies can be partitioned into two sets each of which corresponds to one of the two market configurations. These partitions are captured by sets (3) and (5). By lemma 2, we know that profits are increasing in y_h in the partition where a configuration $CIII$ is defined, i.e in the set (3). Showing that the value of the profit function in the partition defined by (3), is larger than any profit attained in the partition defined in (5) employs the same proof that is used to show the second result for case II above. Therefore, given that platform l picks $y_l < \overline{y}$ and platform h chooses to be a high quality platform, platform h chooses its best response to be $\overline{y}$. □

LEMMA 4. *Given y_l, $\overline{\gamma}$ and an interval $(y_l, \overline{y}]$. Assume that*

i. $\frac{7}{6} \leq \overline{\gamma} < \frac{4}{3}$ *then the best response,* $B_h(y_l) \in [f(\overline{y}, y_l), \overline{y}]$ *where*

$$f(\overline{y}, y_l) = \begin{cases} \frac{(9-3\overline{\gamma})\overline{y}+(6\overline{\gamma}-7)y_l}{3\overline{\gamma}+2} & \text{if } \overline{y} \in (y_l, y_l \frac{4\overline{\gamma}+3-9\overline{\gamma}^2}{\overline{\gamma}-3}], \\ \frac{(9-3\overline{\gamma})\overline{y}-(3\overline{\gamma}-1)y_l}{3\overline{\gamma}+1} & \text{if } \overline{y} \in [y_l \frac{4\overline{\gamma}+3-9\overline{\gamma}^2}{\overline{\gamma}-3}, -y_l \frac{3\overline{\gamma}-1}{2(3\overline{\gamma}-4)}]. \end{cases}$$

ii. $\overline{\gamma} \geq \frac{4}{3}$ *then the best response,* $B_h(y_l) \in [f(\overline{y}, y_l), \overline{y}]$ *where*

$$f(\overline{y}, y_l) = \begin{cases} \frac{(1+3\overline{\gamma})\overline{y}+y_l}{3\overline{\gamma}+2} & \text{if } \overline{y} \in (y_l, y_l \frac{-6\overline{\gamma}-17+27\overline{\gamma}^2}{3\overline{\gamma}+1}], \\ \frac{(1+3\overline{\gamma})\overline{y}-(\overline{\gamma}-1)9y_l}{3\overline{\gamma}+1} & \text{if } \overline{y} \in [y_l \frac{-6\overline{\gamma}-17+27\overline{\gamma}^2}{3\overline{\gamma}+1}, \infty). \end{cases}$$

PROOF. We give a general outline on how to prove each of the above cases. Given $\overline{y} \in [Cy_l, Ky_l]$, where C and K are the relevant constants given in the hypothesis. The least profit that platform h can make is given by $l(\overline{y})$ where $l(y_h) \in P(y_h)$. This follows from the fact that $l(y_h)$ is an increasing function of y_h. Let $\tilde{P}(y_h) = \{\pi_h^p(y_h) | \max_{y_h} \pi_h^p(y_h) \geq l(\overline{y})\}$. We find the minimum value of $\tilde{y}$ in the domain $(y_l, \overline{y}]$ such that $\max_{y_h} \pi_h^p(y_h) \geq l(\overline{y})$ where $\pi_h^p(y_h) \in \tilde{P}(y_h)$, i.e.

$$\tilde{y} = \min y_h$$
$$\text{s.t.} \quad \sum_{\pi_h^p(y_h) \in \tilde{P}(y_h)} 1_{\pi_h^p(y_h) \geq l(\overline{y})} > 0$$

Since the correspondence Π is convex valued (this follows from the convexity of W) we find that $\tilde{y} = y_h$ where $g(y_h) = l(\overline{y})$. Therefore the best response $B_h(y_l) = f(\overline{y}, y_l)$ lies in the set given by $[\tilde{y}, \overline{y}]$, where,$f(\overline{y}, y_l) = \arg\max \pi_h^p(y_h)$ and $\pi_h^p(y_h) \in \tilde{P}(y_h)$. □

A.1.2 Best reply in the domain $[\epsilon, y_h)$

We will follow a similar approach to that used in section A.1.1. Given y_h, we will compute firm l's best reply. We will show that the profit for the low quality firm is decreasing in y_l across all configurations which are possible whenever $1 < \overline{\gamma} \leq 7/6$ or $\overline{\gamma} > 4/3$. This will help us infer that the low quality platform chooses ϵ as its best response in those ranges. For the range, $21/18 \leq \overline{\gamma} < 24/18$, we show that the lower quality platform chooses the maximum between ϵ and a fraction of the quality chosen by the high quality platform.

Since the choice of y_l by the low quality firm determines the market configuration we define the critical limits for which the various configurations exist given y_h.

1. Market is uncovered, with positive masses of consumers on both platforms, in the in the price subgame whenever,

$$y_l < -y_h \frac{2(9\overline{\gamma} - 11)}{9\overline{\gamma} - 5}. \quad (7)$$

2. Market is covered and a corner solution applies in the price subgame whenever,

$$y_l \in \left[-y_h \frac{2(9\overline{\gamma} - 11)}{9\overline{\gamma} - 5}, -y_h \frac{2(3\overline{\gamma} - 5)}{21\overline{\gamma} - 17}, \right], \quad (8)$$
$$\text{if } 1 < \overline{\gamma} \leq \tfrac{7}{6},$$
$$y_l \in \left[-y_h \frac{2(9\overline{\gamma} - 11)}{9\overline{\gamma} - 5}, -y_h \frac{2(3\overline{\gamma} - 4)}{3\overline{\gamma} - 1}\right), \quad (9)$$
$$\text{if } \tfrac{7}{6} < \overline{\gamma} \leq \tfrac{22}{18}.$$

3. Market is covered and an interior solution applies in the price subgame whenever,

$$y_l \in \left(-y_h \frac{2(3\overline{\gamma} - 5)}{21\overline{\gamma} - 17}, y_h\right). \quad (10)$$

4. Market is preempted whenever,

$$y_l \in \left[-y_h \frac{2(3\overline{\gamma} - 4)}{3\overline{\gamma} - 1}, y_h\right), \text{ if } \tfrac{21}{18} \leq \overline{\gamma} < \tfrac{24}{18}, \quad (11)$$
$$\overline{\gamma} \geq \frac{24}{18} \quad (12)$$

LEMMA 5. *Given y_h, $\overline{\gamma}$ and $y_l \in [\epsilon, y_h]$, the profit function $\pi_l(y_l, y_h)$ is decreasing in y_l for all market configurations for which it is defined.*

PROOF. We show that for each configuration the profit function $\pi_l(y_l, y_h)$ is decreasing in y_l.

Uncovered Configuration CII: The denote the profit function in this configuration by π_l^u, One can show that if the quality parameter is in the range $1 < \overline{\gamma} < \frac{22}{18}$ $\frac{\partial \pi_l^u}{\partial y_l} < 0$. Hence the profit function in this configuration is decreasing in y_l.

Covered Configuration with corner solution CIII: The profit function in this configuration is denoted by π_l^{cc}, One can show that the above derivative is negative when y_l lies in the set specified in (8) is satisfied and $\overline{\gamma} > 1$. Hence the profit function in this configuration is decreasing in y_l.

Covered Configuration with interior solution CIII: The profit function in this configuration is given by, $\pi_l^{ci} = \frac{(103+36\overline{\gamma}-84\overline{\gamma})}{486(y_h-y_l)}$. The derivative of the above function is given by, $\frac{\partial \pi_l^{ci}}{\partial y_l} = -\frac{103}{486} - \frac{2}{27}\overline{\gamma}^2 + \frac{14}{81}\overline{\gamma}$. The above derivative is negative therefore the profit function is decreasing in y_l when y_l lies in the set specified in (10).

Pre-empted Configuration CIV: The profit function in this configuration is given by, $\pi_l^p = \frac{2}{9}y_h - \frac{2}{9}y_l$. The derivative of the above function is given by, $\frac{\partial \pi_l^p}{\partial y_l} = \frac{-2}{9}$. The above derivative is negative therefore the profit function is decreasing in y_l when this configuration is defined. □

We will now show that given y_h and $\overline{\gamma}$, and the strategy space $E_l = [\epsilon, y_h)$ platform l will prefer to pick ϵ whenever $1 < \overline{\gamma} < \frac{7}{6}$.

LEMMA 6. *Given y_h, $1 < \overline{\gamma} \leq \frac{7}{6}$, and a strategy space E_l then $B_l(y_h) = \epsilon$.*

PROOF. As previously stated in section A.1.1 three market configurations are possible when $1 < \overline{\gamma} < \frac{21}{18}$; these are uncovered ($CI$), covered with a corner solution and covered with an interior solution (both of which are in configuration $CIII$). Given a $\overline{\gamma}$ in the above range, the domain $[\epsilon, y_h)$ in which y_l lies can be partitioned into three sets, each of which corresponds to one of the three market configurations. These partitions are captured in (7), (8) and (10). By lemma 5, we know that profits are decreasing in y_l for each partition. We will first show that the value of the profit function in the partition defined in (7), is larger than any profit attained in the partition defined in (8) whenever both partitions are defined given ϵ and y_h. Similarly, we show that any profit attained when y_l lies in the partition defined by the constraint in (10) is not greater than that attained when y_l lies in the partition specified by (8).

To show the first result we compare the infimum value of the profit function in the uncovered configuration to the highest possible profit attained when platform l chooses y_l such that a covered market with a corner solution results (i.e, y_l is in the set specified in (8)). Let $y_l^{cc} = -y_h \frac{2(9\overline{\gamma}-11)}{9\overline{\gamma}-5}$, it follows that $\lim_{y_l \to y_l^{cc}} \pi_l^u(y_l, y_h) = \pi_l^{cc}(y_l^{cc}, y_h)$ (Since π_l^u is right continuous, the limit exists). Since $\pi_l^u(y_l, y_h) > \pi_l^u(y_l^{cc}, y_h)$ when y_l satisfies the inequality in (7), it also follows from lemma 5 that $\pi_l^u(y_l, y_h) > \pi_l^{cc}(\tilde{y}, y_h)$ when $\tilde{y}$ lies in the set specified in (8).

To show the second result, we compare the lowest value of the profit function in the covered configuration with a corner solution to the supremum profit value attained when platform l chooses y_l such that a covered market with an interior solution results. The interval over which the covered configuration with an interior solution, $CIII$, is defined is open. Let $y_l^{ci} = -y_l \frac{2(3\overline{\gamma}-5)}{21\overline{\gamma}-17}$, we define the supremum of $\pi_l^{ci}(y_l, y_h)$ over the range in which this configuration is defined as $\pi_l^{ci}(y_l^{ci}, y_h)$. We note that y_l^{ci} is the infimum of the interval over which this configuration is defined, therefore $\lim_{y_l \to y_l^{ci}} \pi_l^{ci}(y_l, y_h) = \pi_l^{ci}(y_l^{ci}, y_h)$ since $\pi_l^{ci}(y_h, y_l)$ is left continuous. By plugging in $y_l = y_l^{ci}$ into $\pi_l^{cc}(y_l, y_h)$ we note that $\pi_l^{cc}(y_l^{ci}, y_h) = \pi_l^{ci}(y_l^{ci}, y_h)$. Therefore, it follows from lemma 5, that $\pi_l^{cc}(y_h, y_l) > \pi_l^{ci}(\tilde{y}, y_h)$ when y_l satisfies the constraint in (8) and $\tilde{y}$ satisfies equation (10). Therefore, given that platform h picks y_h, platform l chooses to be a low quality platform and picks ϵ as its response. □

LEMMA 7. *Given y_h, $\frac{7}{6} < \overline{\gamma} < \frac{22}{18}$ and the strategy space E_l then $B_l(y_h)$* $= \max\{\epsilon, -y_h \frac{2(3\overline{\gamma}-4)}{3\overline{\gamma}-1}\}$.

PROOF. As shown in section A.1.1 when $\frac{7}{6} \leq \overline{\gamma} < \frac{22}{18}$ three market configurations are possible depending on the value of y_h and y_l; these are uncovered, CI, covered with a corner solution, $CIII$, and a pre-empted market, CIV. Given a $\overline{\gamma}$ in the above range, the domain $[\epsilon, y_h)$ in which y_l lies can be partitioned into three sets each of which corresponds to one of the three market configurations. These partitions are captured in (7), (9) and (11). We proceed in a similar manner as we did for the previous Lemma. By lemma 5 we know that profits are decreasing in y_l for each partition. We will first show that the value of the profit function in the partition defined by (7), is larger than any profit attained in the partition defined in (9). We then show that any profit attained in the partitions defined in (9) or (7) is less than that attained by the maximum profit in the partition defined in (11).

The first result is proved in the same way as it is done in Lemma 6. To show the second result we compare the infimum value of the profit function in the covered configuration with a corner solution to the profit value attained when platform l chooses a y_l such that a pre-empted market results. Note the interval over which the covered configuration with a corner solution, $CIII$, is defined is open on its upper limit. Let $y_l^p = -y_h \frac{2(3\overline{\gamma}-4)}{3\overline{\gamma}-1}$, we define the infimum of $\pi_l^{cc}(y, y_l)$ over the range in which this configuration is defined as $\pi_l^{cc}(y_l^p, y_h)$. Since $\pi_l^{cc}(y, y_h)$ is right continuous $\lim_{y_l \to y_l^p} \pi_l^{cc}(y_l, y_h) = \pi_l^{cc}(y_l^p, y_h)$. Note y_l^p is the supremum of the range. By plugging in $y_l = y_l^p$ into the profit functions under a covered market (with a corner solution) and a pre-empted market, we find that $\pi_l^{cc}(y_l^p, y_h) < \pi_l^p(y_l^p, y_h)$. This implies that the profit function is discontinuous across these two market configurations at this point. We now show that $\lim_{y_l \to 0} \pi_l^u(y_h, y_l) < \pi_l^p(y_l^p, y_h)$. We note that $\pi_l^u(y_h, \epsilon)$ is a continuous function in ϵ and the limit as $\epsilon \to 0$ exists. We define $\lim_{\epsilon \to 0} \pi_l^u(y_h, \epsilon) = \pi_l^u(0, y_h)$. It follows that $\pi_l^u(0, y_h) - \pi_l^p(y_l^p, y_h)$ is given by,

$$\frac{y_h(9\overline{\gamma}^2 - 105\overline{\gamma} + 106)}{54(3\overline{\gamma} - 1)},$$

which is negative when $\frac{7}{6} < \overline{\gamma} < \frac{22}{18}$. Therefore it follows from Lemma 5, that $\pi_l^{cc}(y_h, y_l) < \pi_l^p(y_l^p, y_h)$ when y_l is the partition specified in (9). Moreover, $\pi_l^p(y_l^p, y_h) > \pi_l^u(y_h, \tilde{y})$ for any $\tilde{y}$ that falls in the partition specified in (7). Therefore, the best response given y_h is the $\max\{\epsilon, -y_h \frac{2(3\overline{\gamma}-4)}{3\overline{\gamma}-1}\}$. □

LEMMA 8. *Given y_h, $\frac{22}{18} \leq \overline{\gamma} < \frac{24}{18}$ and the strategy space E_l, then $B_l(y_h)$* $= \max\left\{\epsilon, -y_h \frac{2(3\overline{\gamma}-4)}{3\overline{\gamma}-1}\right\}$.

PROOF. When $\overline{\gamma}$ falls in the above range two market configurations are possible depending on the value of y_h and y_l; these are a covered market with a corner solution, $CIII$ and a pre-empted market, CIV. Given a $\overline{\gamma}$ in the above range, the domain $[\epsilon, y_h)$ in which y_l lies can be partitioned into two sets each of which corresponds to one of the two market configurations. These partitions are captured in (9) and (11). We proceed in a similar manner as we did for the previous Lemma 7. We will show that any profit attained in the partition defined in Eq. (9) is less than that attained by the maximum profit in the partition defined in Eq. (11). This proof is analogous to the proof for the second result in lemma 7. Therefore the same results apply, in particular, when $\epsilon < -y_h \frac{2(3\overline{\gamma}-4)}{3\overline{\gamma}-1}$ platform l picks $y_l = -y_h \frac{2(3\overline{\gamma}-4)}{3\overline{\gamma}-1}$ and when $\epsilon \geq -y_h \frac{2(3\overline{\gamma}-4)}{3\overline{\gamma}-1}$ platform l picks $y_l = \epsilon$. □

For $\overline{\gamma} > \frac{24}{18}$ only the pre-empted market configuration exists and by Lemma 5 the profit is decreasing in y_l. Therefore given that platform h picks $y_h > 0$, platform l best response in the domain $[\epsilon, y_h)$ is ϵ.

A similar analysis to that carried out in the previous two subsections also applies when determining the set in which platform $\alpha' s$ best replies lie given platform $\beta' s$ choice.

A.2 Subgame Perfect Equilibrium

We show for each of these regions the sets in which the best reply responses lie and where they intersect thus determining the subgame perfect equilibria.

1. $1 < \overline{\gamma} < \frac{7}{6}$. Platform's β best reply given y_α is defined as,

$$B_\beta(y_\alpha) = \begin{cases} \epsilon, & \text{if } \epsilon < y_\alpha < \overline{y} \text{ and } \pi_\beta^l(\epsilon, y_\alpha) \geq \pi_\beta^h(\overline{y}, y_\alpha), \\ \overline{y}, & \text{if } \epsilon < y_\alpha < \overline{y} \text{ and } \pi_\beta^l(\epsilon, y_\alpha) \leq \pi_\beta^h(\overline{y}, y_\alpha), \\ \epsilon, & \text{if } y_\alpha = \overline{y}, \\ \overline{y}, & \text{if } y_\alpha = \epsilon. \end{cases}$$

   Platform's $\alpha' s$ best reply given y_β is similarly defined. Note that π_β^h and π_β^l refer to the relevant profit functions when platform β acts as a high and low quality platform respectively.

   It follows that any subgame perfect equilibrium when $1 < \overline{\gamma} < \frac{7}{6}$ entails one firm choosing $\overline{y}$ and the other choosing ϵ. The resulting market configuration depends on the investment ratio $\mathcal{I} = \frac{\overline{y}}{\epsilon}$ and the value of $\overline{\gamma}$. Figure 2, shows the sets that contain the best responses and the points where they intersect.

2. $\frac{7}{6} \leq \overline{\gamma} < \frac{24}{18}$ and $\frac{\overline{y}}{\epsilon} \geq -\frac{(3\overline{\gamma}-1)}{2(3\overline{\gamma}-4)}$. Platform's β best reply

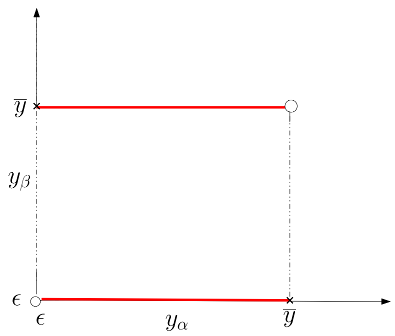

Figure 2: The red line (dotted line) is the set in which best responses for platform $\beta(\alpha)$ lie. These sets intersect only at $(\epsilon, \overline{y})$ and $(\overline{y}, \epsilon)$.

given y_α is defined as,

$$B_\beta(y_\alpha) = \begin{cases} \epsilon, & \text{if } \epsilon \le y_\alpha < -\epsilon\frac{3\overline{\gamma}-1}{2(3\overline{\gamma}-4)} \\ & \text{and } \pi^l_\beta(\epsilon, y_\alpha) \ge \pi^h_\beta(\overline{y}, y_\alpha), \\ y_\alpha\frac{2(3\overline{\gamma}-4)}{3\overline{\gamma}-1}, & \text{if } -\epsilon\frac{3\overline{\gamma}-1}{2(3\overline{\gamma}-4)} \le y_\alpha \le \overline{y} \\ & \text{and } \pi^l_\beta(\epsilon, y_\alpha) \ge \pi^h_\beta(\overline{y}, y_\alpha), \\ \overline{y}, & \text{if } \epsilon \le y_\alpha < \overline{y}\frac{2(3\overline{\gamma}-4)}{3\overline{\gamma}-1} \\ & \text{and } \pi^l_\beta(\epsilon, y_\alpha) \le \pi^h_\beta(\overline{y}, y_\alpha), \\ f(\overline{y}, y_\alpha), & \text{if } \overline{y}\frac{2(3\overline{\gamma}-4)}{3\overline{\gamma}-1} \le y_\alpha < \overline{y} \\ & \text{and } \pi^l_\beta(\epsilon, y_\alpha) \le \pi^h_\beta(\overline{y}, y_\alpha), \end{cases}$$

Where $f(\overline{y}, y_\alpha)$ is as defined in Lemma 4.

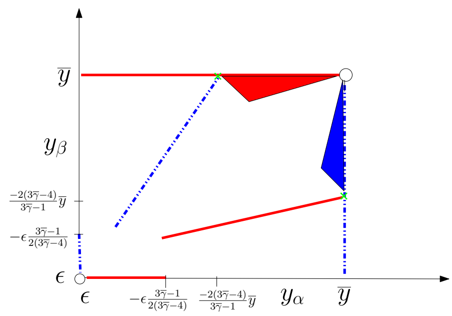

Figure 3: The red (blue) points show the sets in which best responses for platform $\beta(\alpha)$ lie. These sets intersect only at $(-\overline{y}\frac{2(3\overline{\gamma}-4)}{3\overline{\gamma}-1}, \overline{y})$ and $(\overline{y}, -\overline{y}\frac{2(3\overline{\gamma}-4)}{3\overline{\gamma}-1})$.

Figure 3 represents the sets where the best responses lie in the case $\frac{\overline{y}}{\epsilon} = \mathcal{I} \ge -\frac{3\overline{\gamma}-1}{2(3\overline{\gamma}-4)}$. These sets intersect only at the points $(-\overline{y}\frac{2(3\overline{\gamma}-4)}{3\overline{\gamma}-1}, \overline{y})$ and $(\overline{y}, -\overline{y}\frac{2(3\overline{\gamma}-4)}{3\overline{\gamma}-1})$ as shown in the diagram. These points form a SPE; this follows from Lemma 7 and 3 together with the fact that $\frac{\overline{y}}{\epsilon} \ge -\frac{(3\overline{\gamma}-1)}{2(3\overline{\gamma}-4)}$. The high quality platform invests in the highest possible quality whilst the low quality platform is a fraction of the investment by the high quality platform. Therefore, the investment quality pair at the SPE is given by $\{\overline{y}, -\overline{y}\frac{2(3\overline{\gamma}-4)}{3\overline{\gamma}-1}\}$.

3. $\frac{7}{6} \le \overline{\gamma} < \frac{24}{18}$ and $\frac{\overline{y}}{\epsilon} \le -\frac{(3\overline{\gamma}-1)}{2(3\overline{\gamma}-4)}$. Platform's β best reply given y_α is defined as,

$$B_\beta(y_\alpha) = \begin{cases} \epsilon, & \text{if } \epsilon \le y_\alpha < \overline{y} \text{ and } \pi^l_\beta(\epsilon, y_\alpha) \ge \pi^h_\beta(\overline{y}, y_\alpha), \\ \tilde{y}, & \text{if } \epsilon \le y_\alpha < \overline{y} \text{ and } \pi^l_\beta(\epsilon, y_\alpha) \le \pi^h_\beta(\overline{y}, y_\alpha), \end{cases}$$

Where $\tilde{y} \in [f(\overline{y}, y_\alpha), \overline{y}]$ and $f(\overline{y}, y_\alpha)$ is as defined in Lemma 4.

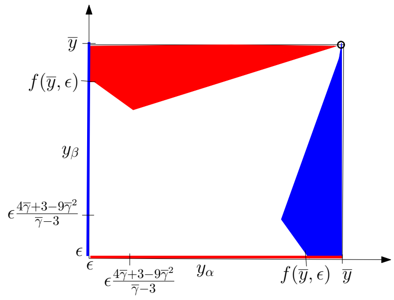

Figure 4: The red (blue) points show the sets in which best responses for platform $\beta(\alpha)$ lie. These sets intersect only at $(\tilde{y}, \epsilon)$ and $(\epsilon, \tilde{y})$ where $\tilde{y} \in [f(\overline{y}, \epsilon), \overline{y}]$.

Figure 4 represents the sets where the best responses lie in the case $\frac{\overline{y}}{\epsilon} = \mathcal{I} < -\frac{3\overline{\gamma}-1}{2(3\overline{\gamma}-4)}$. These sets intersect only at $(\tilde{y}, \epsilon)$ and $(\epsilon, \tilde{y})$ where $\tilde{y} \in [f(\overline{y}, \epsilon), \overline{y}]$ as shown in the diagram. These points form a SPE; this follow from Lemma 7 and 4 together with the fact that $\frac{\overline{y}}{\epsilon} < -\frac{(3\overline{\gamma}-1)}{2(3\overline{\gamma}-4)}$. The high quality platform invests in a $\tilde{y} \in [f(\overline{y}, \epsilon), \overline{y}]$ whilst the low quality platform chooses the lowest quality available.

4. $\overline{\gamma} \ge \frac{4}{3}$. Platform's β best reply given y_α is defined as,

$$B_\beta(y_\alpha) = \begin{cases} \epsilon, & \text{if } \epsilon \le y_\alpha < \overline{y} \text{ and } \pi^l_\beta(\epsilon, y_\alpha) \ge \pi^h_\beta(\overline{y}, y_\alpha), \\ \tilde{y}, & \text{if } \epsilon \le y_\alpha < \overline{y} \text{ and } \pi^l_\beta(\epsilon, y_\alpha) \le \pi^h_\beta(\overline{y}, y_\alpha), \end{cases}$$

Where $\tilde{y} \in [f(\overline{y}, y_\alpha), \overline{y}]$ and $f(\overline{y}, y_\alpha)$ is as defined in Lemma 4.

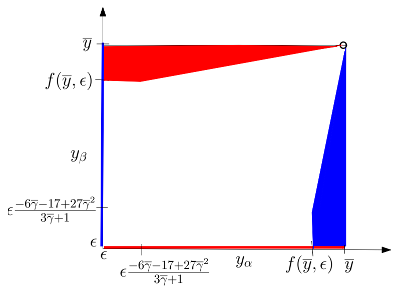

Figure 5: The red (blue) points show the sets in which best responses for platform $\beta(\alpha)$ lie. These sets intersect only at $(\tilde{y}, \epsilon)$ and $(\epsilon, \tilde{y})$ where $\tilde{y} \in [f(\overline{y}, \epsilon), \overline{y}]$.

Figure 5 represents the sets where the best responses lie. These sets intersect only at $(\tilde{y}, \epsilon)$ and $(\epsilon, \tilde{y})$ where $\tilde{y} \in [f(\overline{y}, \epsilon), \overline{y}]$ as shown in the diagram. These points form a SPE; this follow from Lemma 7 and 4. The high quality platform invests in a $\tilde{y} \in [f(\overline{y}, \epsilon), \overline{y}]$ whilst the low quality platform chooses the lowest quality available. Platform's $\alpha's$ best reply given y_β is similarly defined. It follows that any subgame perfect equilibrium when $\overline{\gamma} \geq \frac{24}{18}$ entails one firm choosing $\tilde{y}$ and the other choosing ϵ.