Optimization and Stochastic Control of MANETs

Asu Ozdaglar

Electrical Engineering and Computer Science
Massachusetts Institute of Technology

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Cross-Layer Mechanisms for Wireless Networks

- Scheduling, Routing, Network Coding:
  - Queue-length based backpressure policies

- Rate Control:
  - Utility maximization framework for representing different QoS requirements
  - Convex optimization duality and subgradient methods used to develop distributed algorithms for rate allocation.

- Cross layer mechanisms have been designed by combining the two frameworks through the association:
  - Differential queue length on each link provides information about the Lagrange Multiplier (or the shadow cost) of that link

Challenges and Ongoing Work

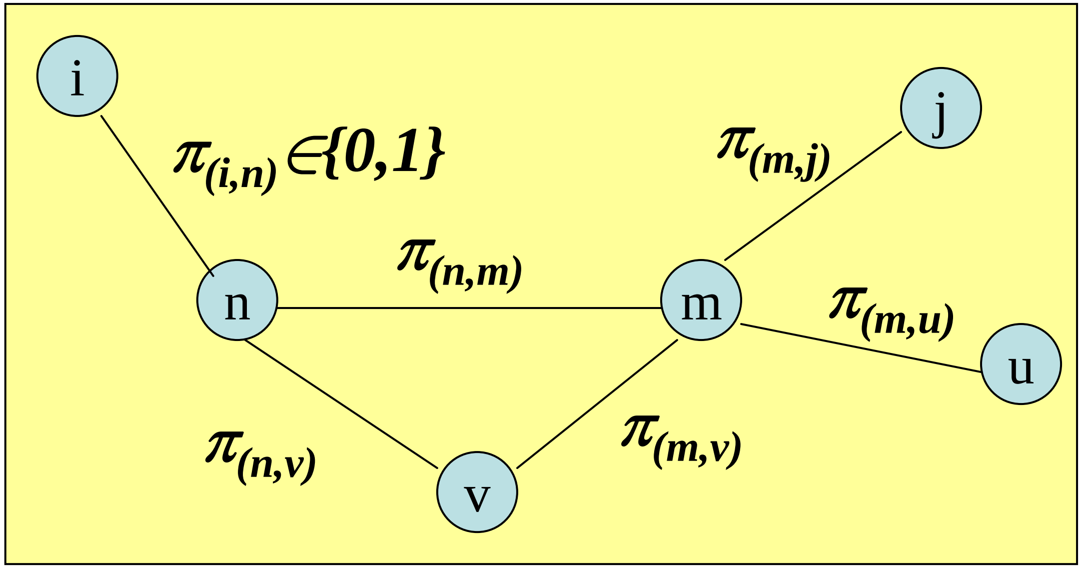
- Backpressure Policies for Wireless Networks:
  - Traditionally have relied on a centralized scheduler
  - Delay performance not well-understood.
- Subgradient Methods:
  - Convergence can be established using different stepsize rules.
  - Very few applicable results on the rate of convergence in the dual and primal space
  - In practice, we care about constructing near-optimal (near feasible) primal solutions
    - Not studied systematically for convex programs.
  - Nonconvex utility maximization (representing inelastic preferences)

This Talk

- Polynomial Complexity Distributed Algorithms for Coding-Scheduling - Rate Control Decisions in Wireless Networks [Eryilmaz, Ozdaglar, Modiano 07]

Wireless Network Model

- The network is represented by a graph: $\mathcal{G} = (\mathcal{N}, \mathcal{L})$.

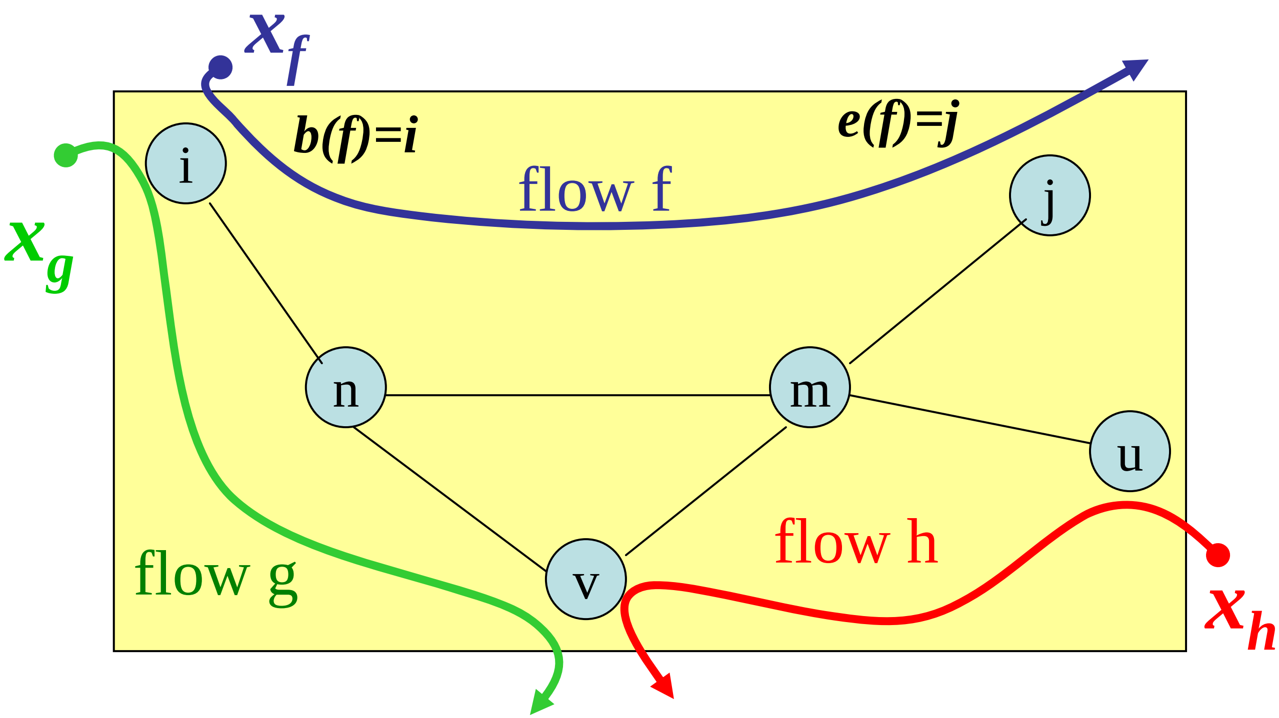


- Π = Convex hull of the set of link rates that are allowable in a time slot, i.e., we have:

$$\pi[t] \in \Pi, \quad \forall t.$$

Traffic Model

- $\mathcal{F}$: The set of flows that share the network.
- Each Flow-*f* is defined by a pair of nodes: (*b(f)*,*e(f)*).



- Let x_f denote the mean rate of flow $f \in \mathcal{F}$.
- Link level interference constraints Π impose constraints on the maximum flow rate region Λ.

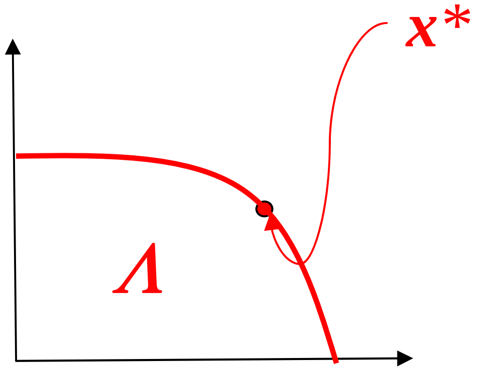
- Then, $U_f(x_f)$ is a strictly concave function that measures the utility of Flow-*f* as a function of x_f.

Problem Statement

• We aim to design a ***distributed*** *scheduling-routing-flow control* mechanism that

- guarantees stability of the queues,
- and allocates the mean flow rates, $\{ x_f \}$, so that they attain the unique optimum of the following problem:

$$\max_{\mathbf{x}} \quad \sum_{f \in \mathcal{F}} U_f(x_f)$$
$$s.t. \quad \mathbf{x} \in \mathbf{\Lambda}$$



• We use x^* to denote the optimizer of the above problem, and also call it the *fair allocation*.

Background and Related Work [partial]

- *Tassiulas and Ephremides ('92,'93)*
  - Introduced the queue-length based scheduling-routing mechanism
  - Central Controller necessary for Wireless Networks
  - Only *throughput-optimality*, no rate control
- *Tassiulas ('98)*
  - Introduced a Randomized Algorithm for switches
  - No rate control & assumed central information
- *Eryilmaz, Srikant, Lin et al., Neely et al., Stolyar ('05)*
  - Solved the utility maximization problem with rate control
  - Required centralized controller
- *Lin et al., Wu et al., Chaporkar et al. ('05)*
  - Considered distributed algorithms that <u>sacrifice a portion</u> of the capacity
- *Modiano, Shah, Zussman ('06)*
  - Proposed Gossip algorithms for distributed implementation
  - Focused on the switch scenario & no rate control

Joint Scheduling-Routing-Flow Control Policy

- $q_{n,d}[t]$ = queue at node ***n*** with packets destined to node ***d*** in slot ***t.***

1. Generic Randomized Scheduling-Routing Policy:

- Define link weights for each link *(n,m)*: $w_{(n,m)}[t]$

Examples to link weights

- Unicast sessions without coding :
  - $q_{n,d}[t]$ = queue at node ***n*** with packets destined to node ***d*** in slot ***t.***
  - $$w_{(n,m)}[t] = \max_{d} \left| q_{n,d}[t] - q_{m,d}[t] \right|$$
- Multicast sessions with intra-session coding [*Ho et al. (`05)]*
  - $q_{n,d}^{D}[t]$ = queue at node ***n*** with packets destined to $d \in D$ in slot ***t.***
  - $$w_{(n,m)}[t] = \max_{D} \sum_{d \in D} \left(q_{n,d}^{D}[t] - q_{m,d}^{D}[t] \right)^{+}$$
- Unicast sessions with inter-session coding [*Eryilmaz and Lun (`06)]*
  - $$w_{(n,m)}[t] = \max \left\{ \max_{D \in \{D_1, D_2\}} \sum_{d \in D} \left(q_{n,d}^{D}[t] - q_{m,d}^{D}[t] \right)^{+}, \sigma_{(n,m)}^{D_1, D_2} \right\}$$

Joint Scheduling-Routing-Flow Control Policy

1. Generic Randomized Scheduling-Routing Policy:

- Define link weights for each link *(n,m)*: $w_{(n,m)}[t]$
- Let $\pi^*[t] \in \arg\max_{\pi \in \Pi} \sum_{(n,m) \in \mathcal{L}} \pi_{(n,m)} w_{(n,m)}[t]$ $\quad \pi^*[t] \in \arg\max_{\pi \in \Pi} (\pi \cdot w[t])$
- Assuming that there exists a randomized policy ***(R)*** satisfying

$$P\left(\pi^{(R)}[t] = \pi^*[t] \mid \mathbf{q}[\mathbf{t}]\right) \geq \delta > 0 \qquad \forall \mathbf{q}[\mathbf{t}]. \quad (1)$$

- Repeat:

$$\pi^{(R)}[t] \leftarrow \text{Pick a random allocation satisfying (1);}$$

PICK

$$\pi[t+1] = \begin{cases} \pi[t] & \text{if } (\mathbf{w}[t] \cdot \pi[t]) \geq (\mathbf{w}[t] \cdot \pi^{(R)}[t]) \\ \pi^{(R)}[t] & \text{otherwise} \end{cases};$$

COMPARE

$$t \leftarrow t+1;$$

Lagrangian Duality for Joint Scheduling-Routing-Flow Control

2. Dual Flow-Control Policy:

- At each slot t, Flow-f updates its arrival rate as

$$x_f[t] = U_f'^{-1}\left(\frac{q_{b(f),e(f)}[t]}{K}\right),$$

where K is a positive design parameter.

- The scheduling-routing policy is randomized but not yet distributed.

- The flow-control policy is decentralized and uses only local queue-length information.

Optimality of the Joint Algorithm

- **Theorem 1:** Given any randomized algorithm that satisfies **(1)**, there exists positive constants, C_1 and C_2, for which

$$\sum_{n\in\mathcal{N}}\sum_{d\in\mathcal{N}}\overline{q_{n,d}} \leq C_1 K,$$

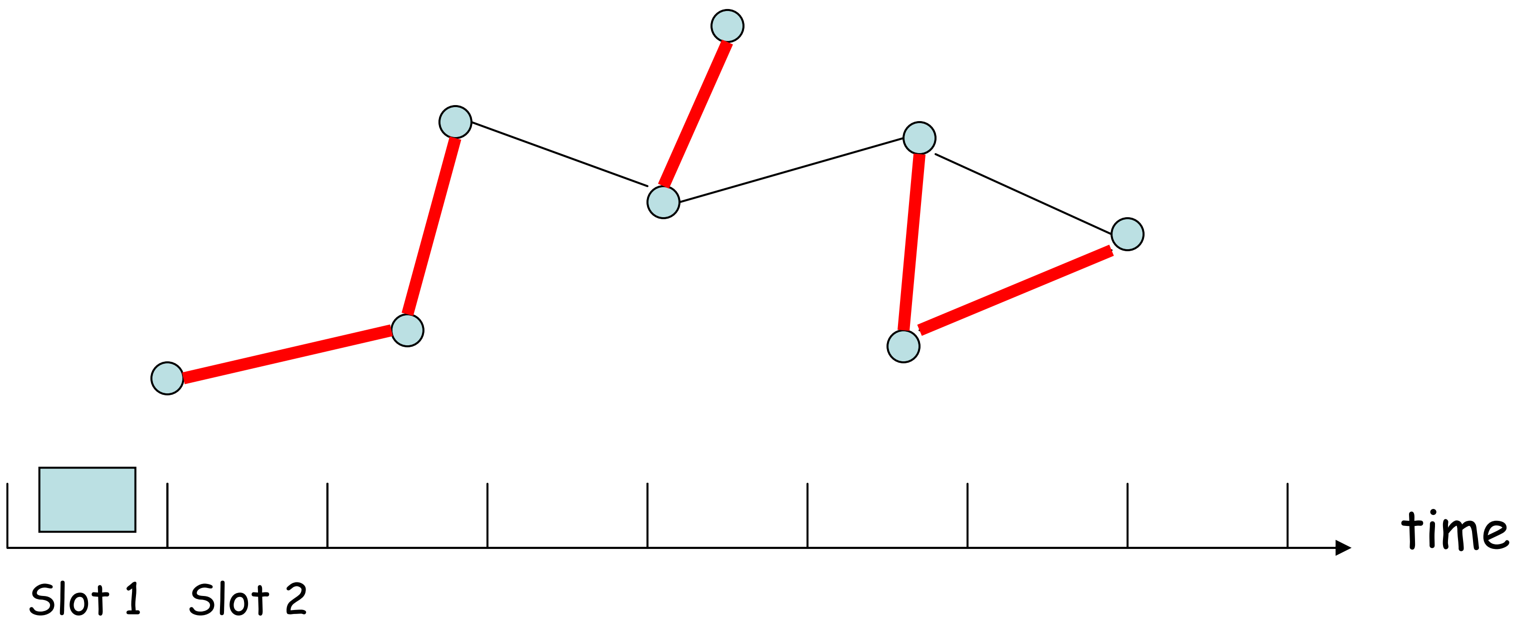
$$\sum_{f\in\mathcal{F}} U_f(\bar{x}_f) \geq \sum_{f\in\mathcal{F}} U_f(x_f^*) - \frac{C_2}{K},$$

where $\bar{x}_f := \lim_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1} E[x_f[t]],$ and similarly for $\overline{q_{n,d}}$.

- Trade-off between average delay and fairness
- The result applies to a large class of interference models

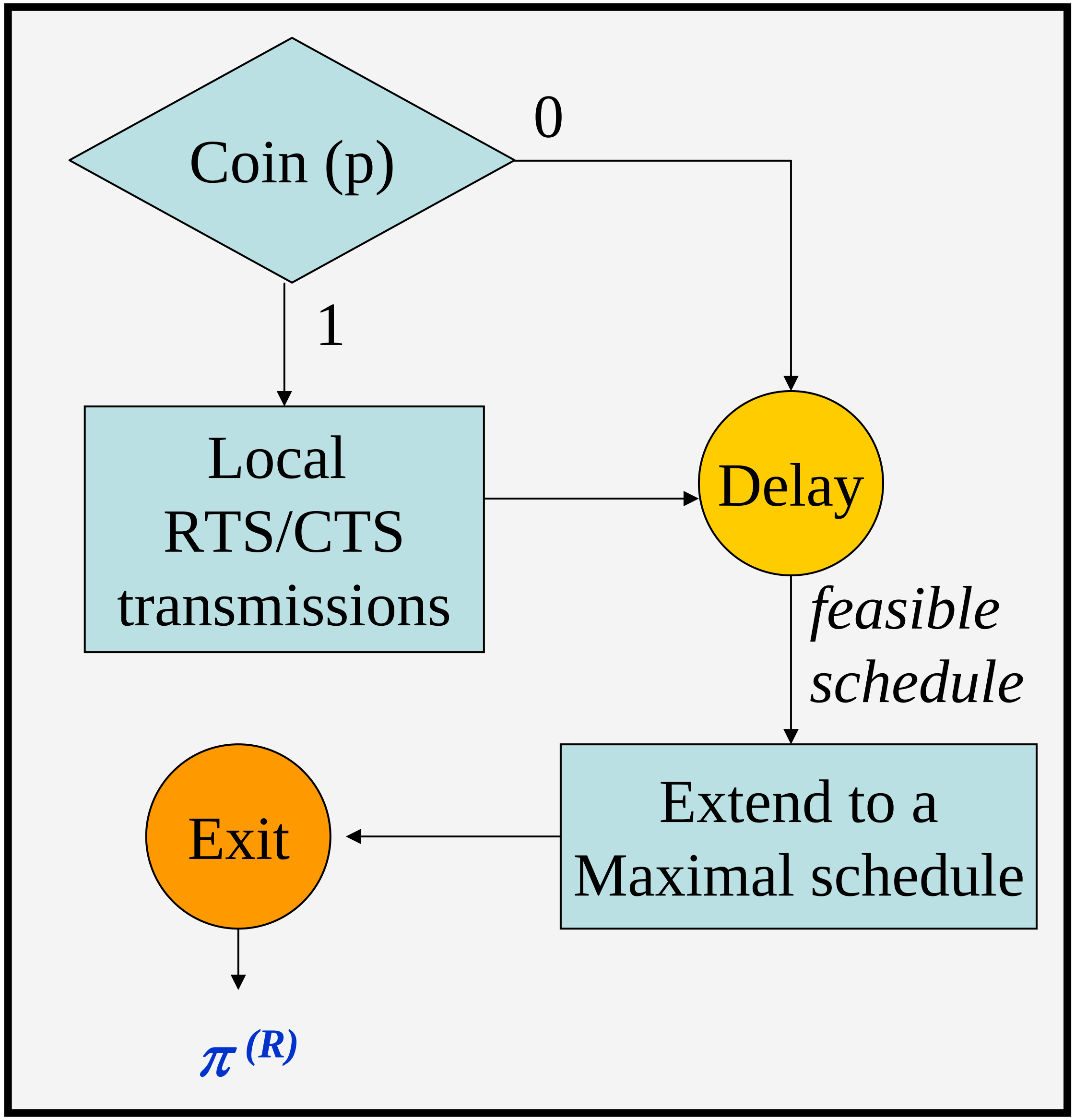
Distributed Algorithm for 2nd Order Interference

- We say a link is *active* if there is a transmission over it.
- A transmission over link ***(n,m)*** is successful iff no neighbors of n and m have an active link incident to them.



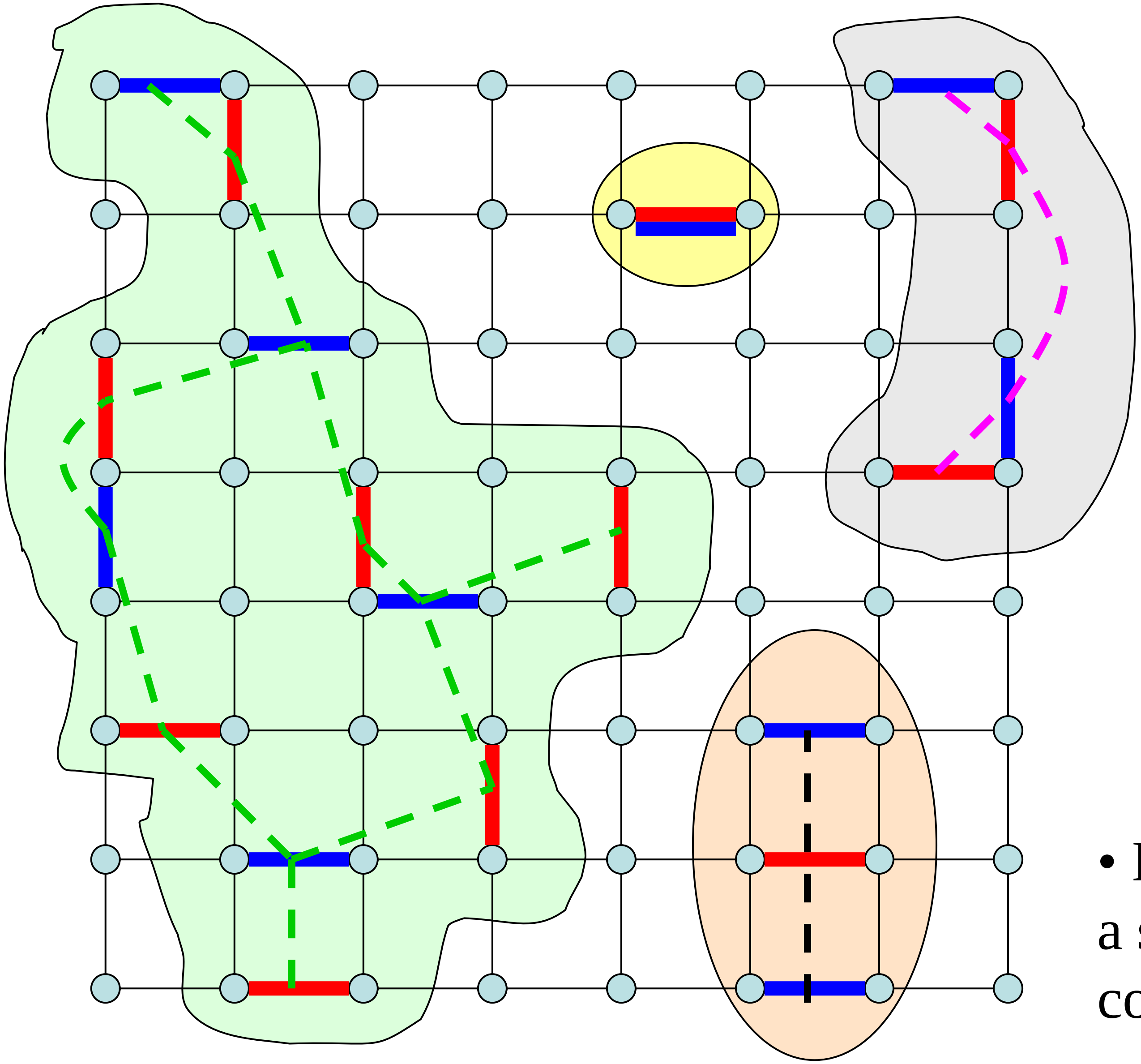
- Remark: For this model, finding the maximum weight schedule is an **NP-hard** problem.

PICK Algorithm



- Every link has a unique ID number.
- The algorithm finds a (maximal) feasible schedule for which (1) is satisfied.
- At the end of the algorithm, each link knows the ID number of its neighboring nodes that are in $\pi^{(R)}$ in the conflict graph.

COMPARE Algorithm

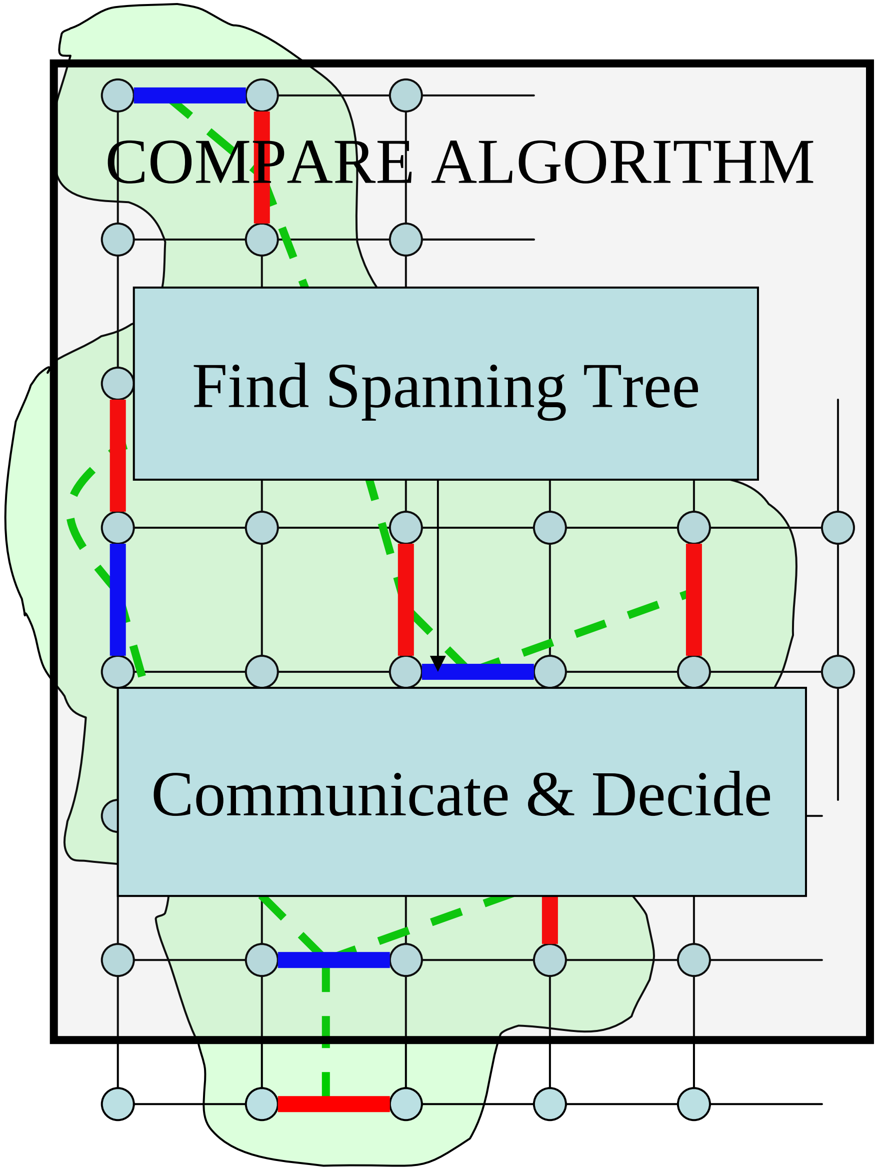


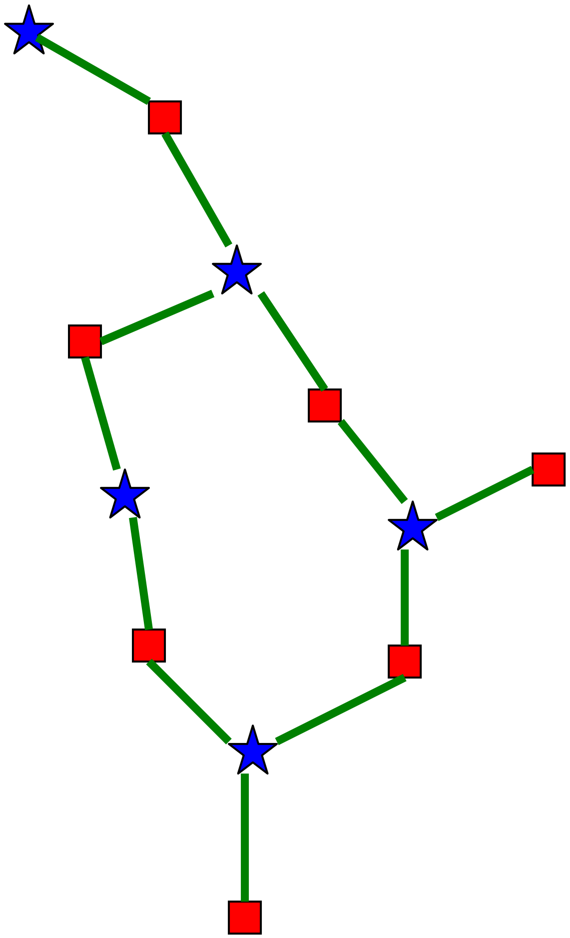
- Grid Topology
- Two schedules:

  π^{old} , $\pi^{(R)}$

- Connecting interfering links
- Several disjoint connected components
- It suffices to consider a single disconnected component.

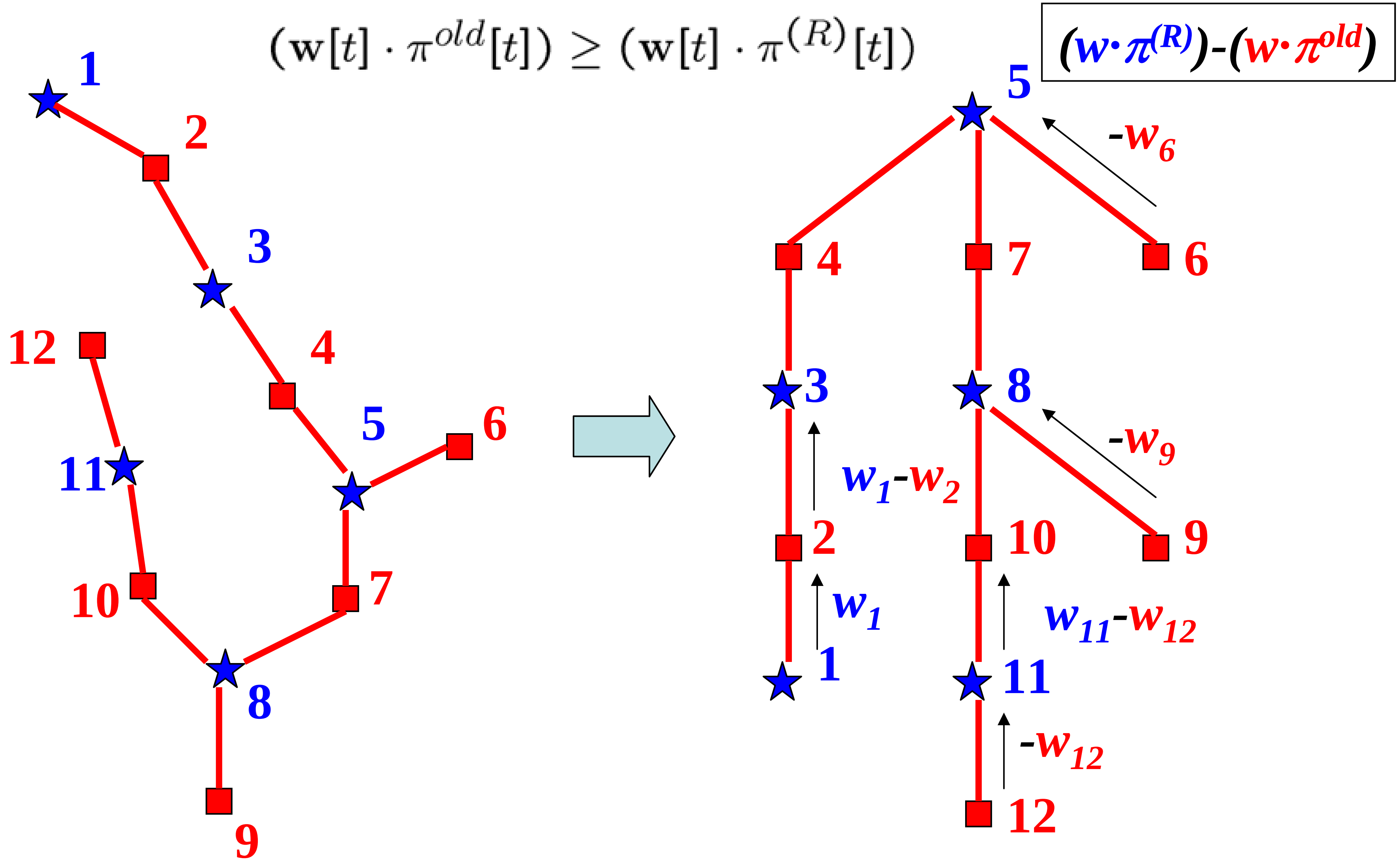
Conflict graph





COMMUNICATE & DECIDE Procedure

$$(\mathbf{w}[t] \cdot \pi^{old}[t]) \geq (\mathbf{w}[t] \cdot \pi^{(R)}[t])$$

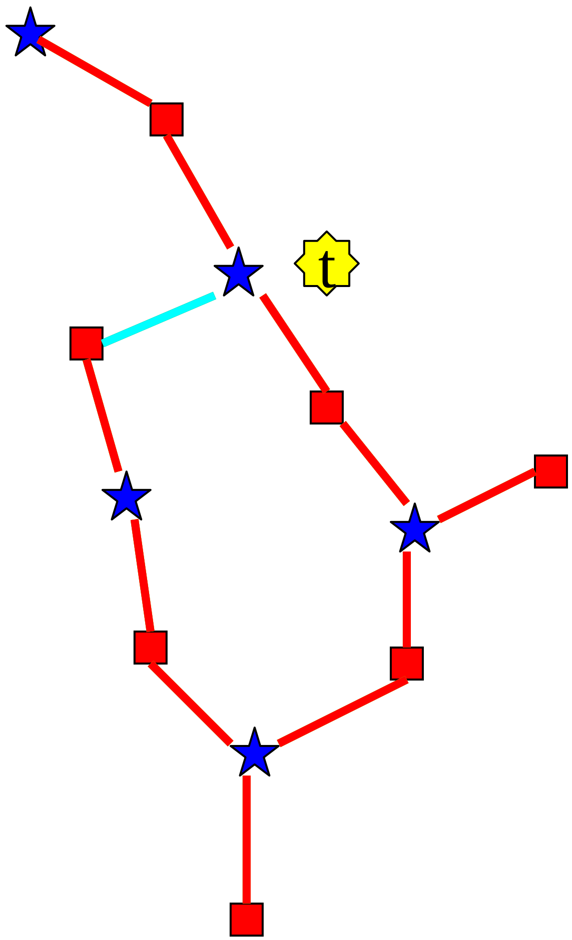


Result

Theorem 2: The distributed implementations of PICK and COMPARE Algorithms designed for the second order interference model asymptotically (in ***K***) achieve ***full utilization of network resources*** with $\boldsymbol{O(N^3)}$ time and $\boldsymbol{O(N^2)}$ message exchanges per node, per stage.

- Thus, the randomized algorithm is guaranteed to achieve full utilization of the resources with polynomial complexity.
- This result is particularly interesting when we note that the maximum weight problem is an NP-hard problem for the second order interference model.

FIND SPANNING TREE Procedure



- Token based procedure
- Assume there is a single token generated in the graph.
  - Every node accepts the token if it has not traversed it so far.
  - If the node has already forwarded the token, it accepts it only if it is returning from the neighbor to which the token was forwarded.
  - A token is returned to the parent of a node only if all the other neighbors are tried.

Proof idea [based on Tassiulas (`98)]

- Show the negative mean drift of the Lyapunov function for

***y* = (*q* , $\pi^{(R)}$):**

$$V(\mathbf{y}) = \sum_{n\in\mathcal{N}} \sum_{d\in\mathcal{N}} q_{n,d}^2 + \left(\sum_{l\in\mathcal{L}} w_l((\pi^*_{\mathbf{w}})_l - \pi_l^{(R)}) \right)^2$$

Measures the stability level of the queues.

Measures the accuracy of the random schedule to the optimal one.