The Marginal User Principle for Resource Allocation in Wireless Networks

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Resource and Traffic Management in Communication Networks

- Traditional Network Optimization: Focus on a central objective, devise synchronous/asynchronous, centralized/distributed algorithms.
  - Assumes all users are homogeneous with no self interest
  - Relies on communication between central controller and agents (generally slow with high informational requirements)
- New Paradigm: Analysis of resource allocation among heterogeneous self-interested agents with decentralized information.
  - Suggests using game theory and economic market mechanisms.
  - Utility-based framework of economics used to represent user preferences.

Related Work

- Existing literature focuses on:
  - resource allocation among competing heterogeneous users
  - social welfare (aggregate utility) maximization [Kelly]
- Pricing used as a means of regulating selfish user behavior and achieving social optimum in a distributed manner.
- Our work takes a different viewpoint:
  - Networks operated by for-profit service providers.
  - Pricing used to make profits or for service differentiation.
  - Combined study of pricing and resource allocation essential.
- With a few exceptions [Walrand, Basar, Mitra], this game theoretic interaction neglected.
- This talk presents a new approach to resource allocation under flat fee pricing.

Resource Allocation in Wireless Networks

- Motivating Model: Downlink power control and pricing in a cellular wireless system
  - Model and results more generally applicable for resource allocation with interference/congestion effects.
- Existing research focus: Power (resource) allocation schemes that maximize aggregate utility, or satisfy various fairness objectives [Shroff, Mazumdar, Saraydar, Mandayam, Goodman]
  - At each time period, base station measures the channel gains and allocates the resources (ex: in a proportionally fair manner).
- Problem: Unmotivated from the point of view of SP or equilibrium.
  - Interested in considering the effects of SP incentives in resource allocation.

Towards a New Approach

- SP sets the entry price and chooses a rule for power (transmission rate) allocation as a function of users' channel conditions.
- SP's goal: Design prices and power allocation policy to maximize profits, recognizing the effects of his decision on the choice of users to participate and pay.
- Formally, analyze a two stage game and consider the subgame perfect equilibrium
- Difference from existing models:
  - Use of fixed access prices
  - SP also chooses allocation policies
- Compare with currently used ad hoc mechanisms and potential social optimum.

Model

- Focus on a single base station with potential users, $i \in \mathcal{N} = \{1, \ldots, N\}$, with utility function $u_i(x)$.
- $u_i(x)$ measures both willingness to pay and also potentially the demand for immediacy, related to concavity.
- Total power constraint on the base station

$$\sum_{i=1}^{N} p_i \leq P_T,$$

  where p_i is the transmission power allocated by the base station to user i.

- Reliable transmission rate to user i is given by

$$x_i = \frac{1}{2} \log \left\{ 1 + \frac{h_i p_i}{\sigma^2} \right\},$$

  where h_i is the channel gain of user i, and σ^2 is the background noise.

Allocation Rules

- The channel gain h_i is a random variable that depends on the location of the user in the cell and shadowing.
- We assume that the channel gains of potential users is characterized by a permutation invariant cumulative dist.
  - Implies anonymity, where the SP cannot discriminate among users, except on the basis of their channel gains.
- With M part. users, let H_M be a largest cardinality set in $\Re^M$ st if $\mathbf{h},\ \tilde{\mathbf{h}} \in H_M$, $\mathbf{h}$ and $\tilde{\mathbf{h}}$ are not permutations of each other.
- Let $F(\mathbf{h}_M, M)$ be the distribution function over $\mathbf{h}_M \in H_M$.
- Allocation rule with M users:

$$x_M : \Re \times H_{M-1} \mapsto \Re$$

  - Identity of the user and ordering of channel gains of other users irrelevant.

User Equilibrium

- Given M participating users and an allocation rule $x_m(\cdot)$, user preferences are represented by the expected utility function
$$U_i(x_m(\cdot), M) = E_{\mathbf{h}_M}\left[u_i(x_M(\mathbf{h}_M))\right].$$
- For a given price q, the net utility of user i is
$$e_i(U_i(x_m(\cdot), M) - q),$$
where $e_i \in \{0, 1\}$ is a participation decision variable.
- Given a price q and a class of allocation rules $\{x_M(\cdot)\}_{M \in \mathcal{N}}$, a vector $[\{e_i\}_{i \in \mathcal{N}}, M]$ is a user equilibrium if

$$M = \max_{m \in \mathcal{N}} \left\{ \sum_{i \in \mathcal{N}} e_i \;\middle|\; e_i = 1 \text{ only if } U_i(x_m(\cdot), m) \geq q \right\}.$$

Service Provider Problem

- The service provider sets the prices and the allocation rules to maximize his profits

$$\text{maximize}_{q,\{x_M(\cdot)\}} \quad q\sum_{i=1}^{N} e_i$$

$$\text{subject to} \quad g_M(k) \leq P_T, \qquad \forall\ M,\ \forall\ k \in H_M,$$

where $g_M(k) = \sum_i \frac{\sigma^2}{k_i}\left(e^{x_M(h=k_i,\hat{\mathbf{h}}=\mathbf{k_{-i}})} - 1\right)$.

- The model outlined corresponds to a dynamic game with the following timing of events:
  - The SP announces an admission price q and a class of allocation rules $\{x_M(\cdot)\}_{M\in\mathcal{N}}$.
  - Users simult. decide whether or not to enter the network.
  - The channel gains of all participating users, $\mathbf{h}_M$ is realized and power allocated according to $x_M(\mathbf{h}_M)$.

SP Equilibrium

- Characterizing the optimal prices and the allocation rule corresponds to finding the subgame perfect equilibrium(SPE) of the game [every $(q, \{x_M(\cdot)\})$ defines a different subgame].

- For our purposes, we represent the SPE as a tuple $(q^*, x^*_{M^*}(\cdot), \{e_i\}_{i\in\mathcal{N}}, M^*)$ that maximizes

$$\begin{aligned}
\text{maximize}_{q, x_M(\cdot, \{e_i\}, M)} \quad & q \sum_{i=1}^{N} e_i \\
\text{subject to} \quad & g_M(\mathbf{k}) \leq P_T, \qquad \forall\, \mathbf{k} \in H_M, \\
& e_i = 1 \text{ only if } U_i(x_M(\cdot), M) \geq q \\
& \sum_{i=1}^{N} e_i = M.
\end{aligned}$$

- We refer to $(q^*, x^*_{M^*}(\cdot), M^*)$ as an SP equilibrium.

Analysis

- We consider the special case where the utility functions of the users satisfy

$$u_1(x) \geq \ldots \geq u_N(x), \qquad \forall\, x \in [0, \infty).$$

- In view of the permutation invariant assumption on the distribution function, the expected utility function for user i given M participating users can be expressed as

$$U_i(x_M(\cdot), M) = \int_{H_M} \left[\frac{1}{M} \sum_{i=1}^{M} u_i(x(h = k_i, \hat{\mathbf{h}} = \mathbf{k_{-i}})) \right] dF(\mathbf{k}, M),$$

where $\mathbf{k} = (k_i, \mathbf{k_{-i}}) \in H_M$.

Analysis

- Proposition: Let $(q^*, x^*_{M^*}(\cdot), M^*)$ be an SPE. Then $x^*_{M^*}$ can be obtained pointwise, i.e., for each $\mathbf{k} \in H_{M^*}$, the M^* values, $x^*_{M^*}(h = k_i, \hat{\mathbf{h}} = \mathbf{k_{-i}})$, $i = 1, \ldots, M^*$, are found by solving the M^*-dimensional problem

$$\begin{aligned} \text{maximize} \quad & \frac{1}{M^*} \sum_{i=1}^{M*} u_{M^*}\Big(x_{M^*}(h = k_i, \hat{\mathbf{h}} = \mathbf{k_{-i}})\Big) \\ \text{subject to} \quad & g_{M^*}(\mathbf{k}) \leq P_T, \end{aligned}$$

and $q^* = U_{M^*}(x^*_{M^*}(\cdot), M^*)$.

- Intuition: In view of the ordered structure of the utility functions, it can be seen that at the SPE:
  - The set of participating users will be $\{1, \ldots, M^*\}$.
- We refer to M^* as the equilibrium marginal user (M^* is indifferent between joining the network or not.)

Optimal Power Allocation Policy

- Marginal User Principle: The SP allocates the power levels such that the utility of the marginal user is maximized, where a marginal user refers to the user that is indifferent between joining the network or not.
- Implication 1: If marginal user has log utility, profit maximizing policy is proportional fairness.
- Implication 2: Equilibrium allocation differs from maximizing sum of the utilities. Two sources of distortion relative to social optimum:
  - Admission control
  - SP maximizes utility of marginal user, not all users
- While motivation drawn from power allocation, the marginal user principle generalizes to other resource allocation problems.

Conclusions and Extensions

- Extend flat pricing model
  - Nonlinear pricing schemes
  - Different entry fees for different levels of service
- Consider competition between multiple providers
- Resource allocation for multi-hop wireline networks