Competition and Efficiency in Congested Networks

Asu Ozdaglar

Joint work with Daron Acemoglu, Dept. of Economics, MIT

Radcliffe Exploratory Seminar on Dynamic Networks

Electrical Engineering and Computer Science

Massachusetts Institute of Technology

Introduction

- Price competition with congestion-sensitive service provision.
- Motivated by the structure of (congested) communication networks or transport problems where we have ownership of resources.
  - More data or traffic on a particular route exerts a negative externality on existing data or traffic (e.g. by increasing delay or probability of packet loss).
- New Feature: A higher price results in traffic moving to an alternative route, but also increases congestion there, making it less attractive.
  - New source of markup in oligopolistic competition.

Motivation: Comm/Transport Networks

- Analysis of resource allocation in the presence of decentralized information, selfish users/administrative domains.
- Instead of a central control objective, model as a multi-agent decision problem: Game theory and economic market mechanisms.
- Existing literature focus:
  - Optimization framework: Prices as control parameters to achieve network objectives in a distributed way *[Kelly], [Low], [Srikant]*
  - Competitive equilibrium framework: User/traffic equilibrium *[Beckmann, Mcguire, Winsten], [Dafermos, Nagurney, Sparrow]*
  - Game theory framework: Strategic interactions among competing heterogeneous users
- Recent interest: Quantification of efficiency loss, "Price of Anarchy, Stability", *[Koutsoupias and Papadimitriou]* in "user games".
- Question: Effects of prices/tolls on performance when they are set (partly) for profit maximization

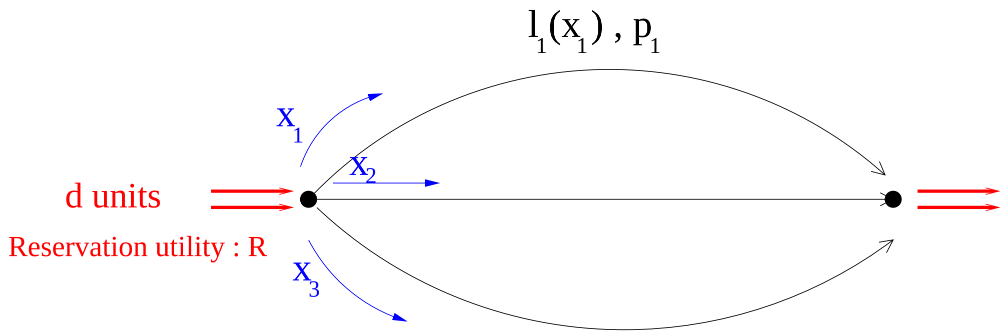
Motivation: Economics

- Interest in implications of competition.
- Typical result: greater competition improves efficiency.
  - With large number of oligopolists, equilibria close to Walrasian equilibrium *[Hart]*, *[Novshek]*, *[Roberts and Postlewaite]*.
- Little analysis of competition in the presence of negative externalities.
- New result: Greater competition may reduce inefficiency.
  - Intuition: The derivative of the latency of competing routes adds to equilibrium markups.
  - Source of differential monopoly power, distorting the pattern of traffic.
- Despite these distortions, it is possible to characterize the extent of inefficiency.

Our Work

- A model of price competition in the presence of congestion externalities.
- Main Results: Tight bounds on the performance losses relative to optimal routing irrespective of the number of routes and service providers and market structure.
  - Price of Anarchy for price competition and selfish routing

Model for Decentralized System - Parallel Links



- I parallel links.
- Interested in routing d units of traffic ("inelastic traffic")
  - Nonatomic users: Aggregate flow of many "small" users.
- Users have a reservation utility R and decide not to send their flow if the effective cost exceeds the reservation utility.
- Multiple service provider owns the links: charges a price p_i per unit bandwidth on link i.

Assumptions

- Each link $i \in \mathcal{I}$ has a latency function $l_i(x_i)$ that represents the delay as a function of the flow x_i on link i.
  - Assume $l_i(x_i)$: convex, continuously differentiable, nondecreasing.
- Wardrop's principle: Flows are routed along paths with minimum "effective cost".
  - Used extensively for equilibrium studies in transportation and communication networks.

Wardrop Equilibrium

- Definition: Given $p \geq 0$, x^* is a Wardrop Equilibrium (WE) iff

$$\begin{aligned} l_i(x_i^*) + p_i &= \min_j \{l_j(x_j^*) + p_j\}, \qquad \forall\, i \text{ with } x_i^* > 0, \\ l_i(x_i^*) + p_i &\leq R, \qquad \forall\, i \text{ with } x_i^* > 0, \end{aligned}$$

and $\sum_{i \in \mathcal{I}} x_i^* \leq d$, with $\sum_{i \in \mathcal{I}} x_i^* = d$ if $\min_j \{l_j(x_j) + p_j\} < R$.

We denote the set of WE at a given p by $W(p)$.

– For any $p \geq 0$, the set $W(p)$ is nonempty.

– If the l_i are strictly increasing, $W(p)$ is a singleton and a continuous function of p.

Social Problem and Optimum

- Definition: A flow vector x^S is a *social optimum* if it is an optimal solution of the *social problem*

$$\text{maximize}_{\substack{x\geq 0 \\ \sum_{i\in\mathcal{I}} x_i \leq d}} \sum_{i\in\mathcal{I}} (R - l_i(x_i))x_i,$$

- $x^S \in \mathbb{R}_+^I$ is a social optimum iff

$$\begin{aligned} l_i(x_i^S) + x_i^S l_i'(x_i^S) &= \min_{j\in\mathcal{I}}\{l_j(x_j^S) + x_j^S l_j'(x_j^S)\}, \qquad \forall\ i \text{ with } x_i^S > 0, \\ l_i(x_i^S) + x_i^S l_i'(x_i^S) &\leq R, \qquad \forall\ i \text{ with } x_i^S > 0, \end{aligned}$$

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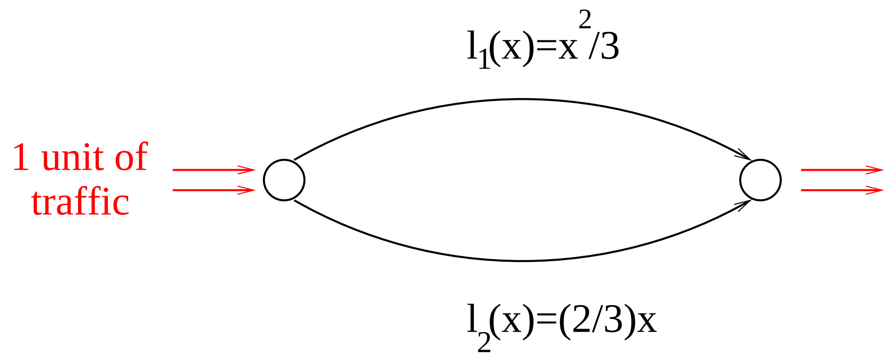
- $(l_i)'(x_i^S)x_i^S$: Marginal congestion cost, Pigovian tax.

Oligopoly Equilibrium

- Assume that each of the links is owned by a different service provider.
- Given the prices of other providers $p_{-i} = [p^j]_{j \neq i}$, SP i sets p_i to maximize his profit
$$\Pi_i(p_i, p_{-i}, x) = p_i x_i,$$
where $x \in W(p_i, p_{-i})$.
- We refer to the game among SPs as the price competition game.
- Definition: A vector $(p^{OE}, x^{OE}) \geq 0$ is a (pure strategy) Oligopoly Equilibrium (OE) if $x^{OE} \in W(p_i^{OE}, p_{-i}^{OE})$ and for all $i \in \mathcal{I}$,
$$\Pi_i(p_i^{OE}, p_{-i}^{OE}, x^{OE}) \geq \Pi_i(p_i, p_{-i}^{OE}, x), \qquad \forall\, p_i \geq 0,\ \forall\, x \in W(p_i, p_{-i}^{OE}). \tag{1}$$
We refer to p^{OE} as the *OE price*.
- Equivalent to the subgame perfect equilibrium notion.

Example



- Social Optimum: $x_1^S = 2/3, \quad x_2^S = 1/3$
- WE: $x_1^{WE} = 0.73 > x_1^S, \quad x_2^{WE} = 0.27$
- Single Provider: $x_1^{ME} = 2/3, \quad x_2^{ME} = 1/3$
- Multiple Providers: $x_1^{OE} = 0.58, \quad x_2^{OE} = 0.42$
  - The monopolist internalizes the congestion externalities.
  - Increasing competition decreases efficiency!
  - There is an additional source of "differential power" in the oligopoly case that distorts the flow pattern.

Existence and Price Characterization

- Proposition: Assume that the latency functions are linear. Then the price competition game has a (pure strategy) OE.

- Existence of a mixed strategy equilibrium can be established for arbitrary convex latency functions.
- Oligopoly Prices: Let (p^{OE}, x^{OE}) be an OE. Then,

$$p_i^{OE} = (l_i)'(x_i^{OE})x_i^{OE} + \frac{\sum_{j \in \mathcal{I}_s} x_j^{OE}}{\sum_{j \notin \mathcal{I}_s} \frac{1}{l_j'(x_j^{OE})}}$$

- In particular, for two links, the OE prices are given by

$$p_i^{OE} = x_i^{OE}(l_1'(x_1^{OE}) + l_2'(x_2^{OE})).$$

  - Increase in price over the marginal congestion cost.

Efficiency Bound for Parallel Links

- Recall our efficiency metric: Given a set of latency functions $\{l_i\}$ and an equilibrium flow x^{OE}, we define the efficiency metric as

$$r(\{l_i\}, x^{OE}) = \frac{R\sum_{i=1}^{I} x_i^{OE} - \sum_{i=1}^{I} l_i(x_i^{OE})x_i^{OE}}{R\sum_{i=1}^{I} x_i^{S} - \sum_{i=1}^{I} l_i(x_i^{S})x_i^{S}}.$$

- Thm: Consider a parallel link network with inelastic traffic. Then

$$r(\{l_i\}, x^{OE}) \geq \frac{5}{6}, \qquad \forall \ \{l_i\}_{i\in\mathcal{I}}, \ x^{OE},$$

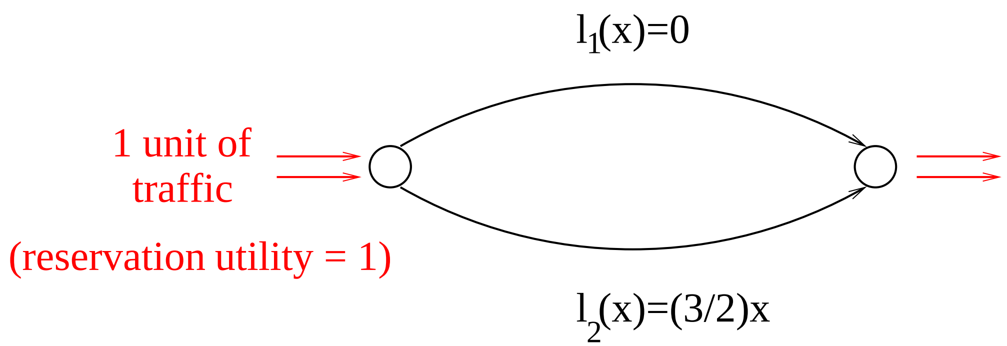
  and the bound is tight.

- *Proof Idea:*
  - Lower bound the infinite dimensional optimization problem by a finite dimensional problem.
  - Use the special structure of parallel links to analytically solve the optimization problem.

- Contrasts (superficially) with the intuition that with large number of oligopolists equilibrium close to competitive.

Bound is tight

- Consider a two link network,



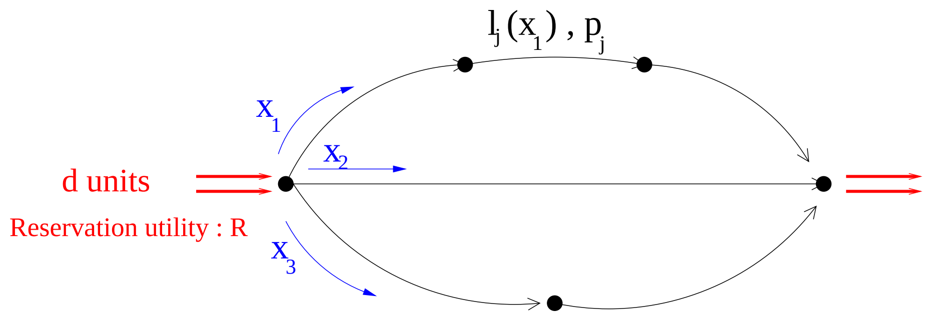
- Social Optimum: Send one unit along upper link.
- Unique OE: Send 2/3 units along upper link, 1/3 units along lower link.

$$r_2(\{l_i\}, x^{OE}) = \frac{1 - \frac{1}{3} \cdot \frac{1}{2}}{1} = \frac{5}{6}.$$

- Hence, we have

$$\min_{\{l_i\}} \min_{x^{OE}} r_2(\{l_i\}, x^{OE}) = \frac{5}{6}.$$

Parallel-Serial Link Topology



- Greater inefficiency due to double marginalization.
- OE arbitrarily ineff due to "coordination failure" of serial providers.
- Example: Consider a two path network:
  - 3 links on path 1 with $l_1^i = 0$ for $i = 1, 2, 3$,
  - 1 link on path 2 with $l_2(x_2) = kx_2$, $k \geq 0$.
- The unique social optimum is $x^S = (1, 0)$.
- $p_1^i = 1$ and $p_2 = \frac{1}{2}$ are OE prices, with flows $x^{OE} = (0, \frac{1}{2k})$.
- The efficiency metric $r_2(\{l_j\}, x^{OE}) = \frac{1}{4k} \to 0$ as $k \to \infty$.

Strict OE and Price Characterization

- Can be avoided by considering "coalition-proof" subgame perfect Nash equilibria, where serial providers form coalitions.
  - Instead we consider a stronger equilibrium: strict OE
- Definition: A vector $(p^{OE}, x^{OE}) \geq 0$ is a *strict OE* if $x^{OE} \in W \ \ p_j^{OE}, p_{-j}^{OE}$ and for all $i \in \mathcal{I}$, $j \in \mathcal{N}_i$,

$$\Pi_j(p_j^{OE}, p_{-j}^{OE}, x^{OE}) > \Pi_j(p_j, p_{-j}^{OE}, x), \qquad \forall\, p_j \neq p_j^{OE},\ x \in W(p_j, p_{-j}^{OE}).$$

- Strict OE: Unique best response, all path flows positive.
- Proposition: Assume that the latency functions are linear and strictly increasing. Then the price competition game has a strict OE.
- Theorem: Consider a parallel-serial link network with $l_i(0) = 0$. Then, the efficiency metric for strict OE satisfies

$$r(\{l_j\}, x^{OE}) \geq \frac{1}{2}, \qquad \forall\ \{l_j\},\ x^{OE}.$$

Fixed Cost - Positive Latency at 0 Congestion

- Relax the assumption $l_i(0) = 0$.
- Parallel topology: Tight bound of $2\sqrt{2} - 2 \approx 5/6$.
- Parallel-serial topology: no bound.
- Example: Consider a two path network:
  - n links on path 1 with identically 0 latency functions
  - 1 link on path 2 with $l(x_2) = \epsilon x_2 + b$ for some $\epsilon > 0$ and $b > 0$
- The flows at the unique OE are given by

$$\bar{x}^{OE} = \Big[\frac{2\epsilon + b}{\epsilon(n+2)}, \frac{\epsilon n - b}{\epsilon(n+2)}\Big].$$

Let $\epsilon = b/\sqrt{n}$. Then, as $b \to 1$ and $n \to \infty$, $\bar{x}^{OE} \to (0, 1)$, and $r_2(\{l_j\}, x^{OE}) \to 0$.

Extensions I: Elastic Traffic

- Elastic traffic adds the standard monopoly distortions.
- No bounds in general for elastic traffic.
- Non-tight bounds for concave marginal utility in *[Hayrapetyan, Tardos, Wexler 05]*.
- Tight bound of 2/3 for concave marginal utility in *[Ozdaglar 06]*.

Extensions II: Capacity Investments

- How far are investments in network capacities and infrastructure from optimum?
- Study price and capacity competition.
- *[Weintraub, Johari, Van Roy 06]* efficiency in symmetric and simultaneous move price and capacity game.
- *[Acemoglu, Bimpikis, Ozdaglar 06]* nonsymmetric capacitated networks:
  - Unbounded price of anarchy, bound of $2\sqrt{2}-2 \approx 5/6$ for price of stability.
- Price of stability can be implemented via Stackleberg game.

Extensions III: Alternative Routing Paradigms

- Partially optimal routing: *[Acemoglu, Johari, Ozdaglar 06]*.
  - End-to-end route selection selfish
    - Transmission follows minimum latency route for each source.
  - Network providers route traffic within their own network to achieve minimum intradomain latency.
- Performance related to presence of Braess' paradox.
- New efficiency bounds on partially optimal routing.