The Marginal User Principle for Resource Allocation in Wireless Networks

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Resource and Traffic Management in Communication Networks

- **Traditional Network Optimization:** Focus on a central objective, devise synchronous/asynchronous, centralized/distributed algorithms.
  - Assumes all users are homogeneous with no self interest
  - Relies on communication between central controller and agents (generally slow with high informational requirements)

- **New Paradigm:** Analysis of resource allocation among heterogeneous self-interested agents with decentralized information.
  - Suggests using game theory and economic market mechanisms.
  - Utility-based framework of economics used to represent user preferences.
Related Work

- Existing literature focuses on:
  - resource allocation among competing heterogeneous users
  - social welfare (aggregate utility) maximization [Kelly]

- Pricing used as a means of regulating selfish user behavior and achieving social optimum in a distributed manner.

- Our work takes a different viewpoint:
  - Networks operated by for-profit service providers.
  - Pricing used to make profits or for service differentiation.
  - Combined study of pricing and resource allocation essential.

- With a few exceptions [Walrand, Basar, Mitra], this game theoretic interaction neglected.

- This talk presents a new approach to resource allocation under flat fee pricing.
Resource Allocation in Wireless Networks

- **Motivating Model:** Downlink power control and pricing in a cellular wireless system
  - Model and results more generally applicable for resource allocation with interference/congestion effects.

- **Existing research focus:** Power (resource) allocation schemes that maximize aggregate utility, or satisfy various fairness objectives [Shroff, Mazumdar, Saraydar, Mandayam, Goodman]
  - At each time period, base station measures the channel gains and allocates the resources (ex: in a proportionally fair manner).

- **Problem:** Unmotivated from the point of view of SP or equilibrium.
  - Interested in considering the effects of SP incentives in resource allocation.
Towards a New Approach

• SP sets the entry price and chooses a rule for power (transmission rate) allocation as a function of users’ channel conditions.

• SP’s goal: Design prices and power allocation policy to maximize profits, recognizing the effects of his decision on the choice of users to participate and pay.

• Formally, analyze a two stage game and consider the subgame perfect equilibrium

• Difference from existing models:
  – Use of fixed access prices
  – SP also chooses allocation policies

• Compare with currently used ad hoc mechanisms and potential social optimum.
Model

- Focus on a single base station with potential users, \( i \in \mathcal{N} = \{1, \ldots, N\} \), with utility function \( u_i(x) \).
- \( u_i(x) \) measures both willingness to pay and also potentially the demand for immediacy, related to concavity.
- Total power constraint on the base station

\[
\sum_{i=1}^{N} p_i \leq P_T,
\]

where \( p_i \) is the transmission power allocated by the base station to user \( i \).
- Reliable transmission rate to user \( i \) is given by

\[
x_i = \frac{1}{2} \log \left\{ 1 + \frac{h_i p_i}{\sigma^2} \right\},
\]

where \( h_i \) is the channel gain of user \( i \), and \( \sigma^2 \) is the background noise.
Allocation Rules

- The channel gain $h_i$ is a random variable that depends on the location of the user in the cell and shadowing.
- We assume that the channel gains of potential users is characterized by a permutation invariant cumulative dist.
  - Implies anonymity, where the SP cannot discriminate among users, except on the basis of their channel gains.
- With $M$ part. users, let $H_M$ be a largest cardinality set in $\mathbb{R}^M$ st if $h, \tilde{h} \in H_M$, $h$ and $\tilde{h}$ are not permutations of each other.
- Let $F(h_M, M)$ be the distribution function over $h_M \in H_M$.
- Allocation rule with $M$ users:
  $$x_M : \mathbb{R} \times H_{M-1} \mapsto \mathbb{R}$$
  - Identity of the user and ordering of channel gains of other users irrelevant.
User Equilibrium

- Given $M$ participating users and an allocation rule $x_m(\cdot)$, user preferences are represented by the expected utility function

$$U_i(x_m(\cdot), M) = E_{h_M}[u_i(x_M(h_M))].$$

- For a given price $q$, the net utility of user $i$ is

$$e_i(U_i(x_m(\cdot), M) - q),$$

where $e_i \in \{0, 1\}$ is a participation decision variable.

- Given a price $q$ and a class of allocation rules $\{x_M(\cdot)\}_{M \in \mathcal{N}}$, a vector $[\{e_i\}_{i \in \mathcal{N}}, M]$ is a user equilibrium if

$$M = \max_{m \in \mathcal{N}} \left\{ \sum_{i \in \mathcal{N}} e_i \mid e_i = 1 \text{ only if } U_i(x_m(\cdot), m) \geq q \right\}.$$
Service Provider Problem

• The service provider sets the prices and the allocation rules to maximize his profits

\[
\begin{align*}
\text{maximize}_{q,\{x_M(\cdot)\}} & \quad q \sum_{i=1}^{N} e_i \\
\text{subject to} & \quad g_M(k) \leq P_T, \quad \forall M, \forall k \in H_M,
\end{align*}
\]

where \(g_M(k) = \sum_i \frac{\sigma_i^2}{k_i} \left( e^{x_M(h=k_i,\hat{h}=k_i-k_i)} - 1 \right)\).

• The model outlined corresponds to a dynamic game with the following timing of events:
  
  – The SP announces an admission price \(q\) and a class of allocation rules \(\{x_M(\cdot)\}_{M \in \mathcal{N}}\).
  
  – Users simult. decide whether or not to enter the network.
  
  – The channel gains of all participating users, \(h_M\) is realized and power allocated according to \(x_M(h_M)\).
SP Equilibrium

- Characterizing the optimal prices and the allocation rule corresponds to finding the subgame perfect equilibrium (SPE) of the game [every \((q, \{x_M(\cdot)\})\) defines a different subgame].

- For our purposes, we represent the SPE as a tuple \((q^*, x^*_M(\cdot), \{e_i\}_{i \in \mathcal{N}}, M^*)\) that maximizes

\[
\begin{align*}
\text{maximize}_{q, x_M(\cdot), \{e_i\}, M} & \quad q \sum_{i=1}^{N} e_i \\
\text{subject to} & \quad g_M(k) \leq P_T, \quad \forall k \in H_M, \\
& \quad e_i = 1 \text{ only if } U_i(x_M(\cdot), M) \geq q \\
& \quad \sum_{i=1}^{N} e_i = M.
\end{align*}
\]

- We refer to \((q^*, x^*_M(\cdot), M^*)\) as an SP equilibrium.
Analysis

- We consider the special case where the utility functions of the users satisfy
  \[ u_1(x) \geq \ldots \geq u_N(x), \quad \forall \ x \in [0, \infty). \]

- In view of the permutation invariant assumption on the distribution function, the expected utility function for user \( i \) given \( M \) participating users can be expressed as

\[
U_i(x_M(\cdot), M) = \int_{H_M} \left[ \frac{1}{M} \sum_{i=1}^{M} u_i(x(h = k_i, \hat{h} = k_{-i})) \right] dF(k, M),
\]

where \( k = (k_i, k_\cdot) \in H_M \).
Analysis

• Proposition: Let \((q^*, x^*_M(\cdot), M^*)\) be an SPE. Then \(x^*_M\) can be obtained pointwise, i.e., for each \(k \in H_{M^*}\), the \(M^*\) values, \(x^*_M(h = k_i, \hat{h} = k_{-i})\), \(i = 1, \ldots, M^*\), are found by solving the \(M^*\)-dimensional problem

\[
\begin{align*}
\text{maximize} & \quad \frac{1}{M^*} \sum_{i=1}^{M^*} u_{M^*} \left( x_{M^*}(h = k_i, \hat{h} = k_{-i}) \right) \\
\text{subject to} & \quad g_{M^*}(k) \leq P_T, \\
\text{and} & \quad q^* = U_{M^*}(x^*_M(\cdot), M^*). 
\end{align*}
\]

• Intuition: In view of the ordered structure of the utility functions, it can be seen that at the SPE:

– The set of participating users will be \(\{1, \ldots, M^*\}\).

• We refer to \(M^*\) as the equilibrium marginal user (\(M^*\) is indifferent between joining the network or not.)
Optimal Power Allocation Policy

- **Marginal User Principle:** The SP allocates the power levels such that the utility of the marginal user is maximized, where a marginal user refers to the user that is indifferent between joining the network or not.

- **Implication 1:** If marginal user has log utility, profit maximizing policy is proportional fairness.

- **Implication 2:** Equilibrium allocation differs from maximizing sum of the utilities. Two sources of distortion relative to social optimum:
  - Admission control
  - SP maximizes utility of marginal user, not all users

- While motivation drawn from power allocation, the marginal user principle generalizes to other resource allocation problems.
Conclusions and Extensions

• Extend flat pricing model
  – Nonlinear pricing schemes
  – Different entry fees for different levels of service

• Consider competition between multiple providers

• Resource allocation for multi-hop wireline networks